1. The number of binary bit strings[1] of length n (≥ 6) that contain exactly three occurrences of '01' is $^{n+1}C_7$ [2].

Partitions into 8 ~~places~~ ~~cast~~ the $(n-6)$ boxes. In the 1^{st} one fill 1's, 2^{nd} part fill 0's, then write a (01) & next 1's, 0's, 01 & 1's & 0's, then a 01, then 1's, & 0's. No. of such ways = No. of req. bit strings

(not in a partition, but b/w them)

$\therefore$ No. of such ways $= {}^{n-6+7}C_{n-6} = {}^{n+1}C_7$

3

[3 Marks]

2. In any sequence of n natural numbers, there exist(s) atleast 1 [3] sub-sequence(s) of consecutive terms with a sum divisible by n.

Consider, the sub-sequences starting from 1^{st} element. Total subsequences possible $= n$, Denote their sums by S_i (For length i). If one of them is divisible by 'n', we already got one sub-sequence. Or else, the remainders possible are 1 to $(n-1)$. But there are 'n' 'S_i's. So, two have same remainder. So the sub-seq b/w S_j & S_i is the required one. So, there is atleast '1'.

2

[3 Marks]

3. The statement: "from any ten distinct two-digit numbers, one can always choose two disjoint non-empty subsets, so that their elements have the same sum" is True [4].

[1] Binary bit strings are strings made of zeros and ones.

[2] Fill in this blank with an appropriately simplified expression involving n.

[3] Fill in this blank with the largest integer constant that renders the statement true for any $n \in \mathbb{N}$. Needless to say, if you fill the blank with the number zero, then it means that such a set may not exist for some case.

[4] Fill this blank with "true" or "false".

As it is a subset of 10 distinct 2-digit no's, the least possible value for an element is '10'. For non-empty subset, the sum let it, S_i $(1 \le i \le 2^{10}-2)$, the min. value is 10 (for the set {10}.) and the max. value is $\underbrace{(99+98 \cdots 91)}_{9 \text{ elements}}$ (we don't 10 because, the other disjoint part shouldn't be empty i.e., 765. So $i \in [1, 1022]$. So,

$10 \le S_i \le 765$

from pigeon-hole principle, $\exists i,j$ $(i \neq j)$ such that $S_i = S_j$. If the subsets have any common elements, we can remove them, because the sums remain equal. So, we get 2 disjoint sets

3

[3 Marks]

4. The statement: "there exists some number consisting of all sevens (*i.e.*, 77 . . . 7) that is a multiple of 2017" is True[5].

Consider set of 2018 elements: 7, 77, 777 . . . $\underset{(2018 \text{ times})}{(7 \ldots 7)}$. As there are only 2017 remainders possible with 2017. Two of them have same remainder. By subtracting them we get $(77..7)\underset{(n)}{(0..0)}$. divisible by 2017 But 10^n doesn't have any common factor with 2017. So $(77..7)$ is divisible by 2017 for some no of 7's

2

[2 Marks]

5. The no. of ways in which n students can be assigned to k identical computers such that each student is assigned exactly one computer[6] is given by $\frac{K^n}{K!}$ [7].

Let the 'k' computers be distinct. Then no. of ways of assigning ~~them to each of n st~~, each student to 1 computer is k^n. But as they are indistinguishable, ~~k! of the above assignments~~ each actual assignment is repeated $(K!)$ times. So, no. of required ways of assignment $= \frac{k^n}{K!}$

0

[5] Fill this blank with "true" or "false".

[6] assume, $n \geq k$

[7] Fill in this blank with an appropriately simplified expression involving n, k

[2 Marks]

6. Let n (> 1), k $(< n)$ be two natural numbers. The number of $k-$tuples $(A_1, A_2 \ldots, A_{k-1}, A_k)$ such that $A_1 \subseteq A_2 \subseteq \ldots \subseteq A_k \subseteq \{1, 2, \ldots, n-1, n\}$ is $\underline{(K+1)^n}$ [8].

An element can belong to either A_1 or $A_1^c \cap A_2$ or $A_2^c \cap A_3 \ldots A_{k-1}^c \cap A_k$ or A_k^c. So, for an element we have $(K+1)$ possibilities. For n elements $(K+1)^n$. (all distinct)

2

[2 Marks]