

INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

ASSIGNMENT-I

MA 4090 : Combinatorics

Max Marks: 50

1. For each of the following five permutations σ_i on $[9]$ that are written in their one line notations, find the corresponding permutations $\bar{\sigma}_i$ in the gaurd representations. [5]
- (a) $\sigma_1 = 372418965$
 - (b) $\sigma_2 = 419635827$
 - (c) $\sigma_3 = 182497635$
 - (d) $\sigma_4 = 827594613$
 - (e) $\sigma_5 = 725498163$

2. Let $n = 9$ and π^* denote the image of permutation π (in its one line notation) written in the leader representation. Write π^* where π equals [5]
- (a) 423968157
 - (b) 941683257
 - (c) 827651394
 - (d) 613782594
 - (e) 432167985

In each case, write down π^* in one line form and find the number of descents of π^* .

3. Let n be a fixed positive integer ≥ 2 and let $1 \leq k \leq n$ be a fixed integer. Let $m = n - k$.
- (a) Show that the sum $\sum a_1 a_2 \dots a_m$ where each summand is a product of m distinct integers in the set $[n - 1]$ and the sum is taken over all the m -subsets of $[n - 1]$ and the sum is taken over all the m - subsets of $[n - 1]$ is equal to $c(n, k)$ [3]
 - (b) Show that the sum in (a) actually counts the number of pairs (S, f) where S is a subset of $[n - 1]$ with $|S| = n - k$ and $f: S \rightarrow [n - 1]$ is a function such that $f(i) \leq i$ for all i . [2]
4. What is the number of permutations on $[n]$ that contains 1 in a k -cycle? Show the somewhat surprising result that this does not depend on k (but depends only on n). You may use the gaurd representation or prove the result by direct counting. How many permutations contain any j in a k -cycle? What is the number of permutations that contain 1 and 2 in the same cycle? [5]
5. Observe that the only permutation on $[n]$ that has no descents is the identity permutation. Use this to show that the number of permutations with exactly one descent= $A(n, 2) = 2^n - (n + 1)$. [5]
6. The function f is defined from the set of natural numbers to the set of real numbers by the following equation: [5]

$$\sum_{d|n} f(d) = \log n$$

Find f .

7. Three workmen a, b, c have to be assigned a job each out of the available jobs 1,2,3,4. Further, 'a' is not qualified for jobs 2 and 3, 'b' is not qualified for the jobs 2 and 4 and 'c' is not qualified for the job 1. Using Rook polynomials, find the number of ways in which each of them is assigned a distict job for which he is qualified. [5]

8. Prove that

$$\frac{\phi(n)}{n} = \sum_{d|n} \frac{\mu(d)}{d}$$

[5]

9. (a) In how many ways can the integers $1, 2, \dots, 9$ be permuted so that no odd integer will be in its original position?

[3]

(b) How many positive integers less than 720 are coprime to 720?

[2]

10. Suppose n people leave their hats as well as bags outside a hall and while stepping out of the hall, each person picks up one hat and one bag. What is the number of ways in which every person neither gets back his own hat nor his own bag?

[5]