INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

Problem Sheet 1

Date : XX.02.18 MA 1140 : Linear Algebra

1. For what value of q, the following system is singular.

$$\begin{aligned} x + 4y - 2z &= 1 \\ x + 7y - 6z &= 6 \\ 3y - qz &= 1. \end{aligned}$$

2. Find a,b and c, using Gauss elimination method.

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

3. Write LU- decomposition form of the following matrix:

$$\begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix}.$$

4. Find x_1, x_2 and x_3 using LU-decomposition

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix}$$

5. Find A^{-1} and use it solve $Ax = b$, where

$$A = \begin{bmatrix} 2 & -2 & 2 \\ 2 & 2 & 2 \\ 2 & 4 & 8 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}.$$

6. Consider a set V such that $V = \{(x, y) : x, y \in \mathbb{R}\}$. Let us define the operation of addition : $(a, b) \oplus (x, y) = (a + x + 1, b + y + 1)$ and operation of multiplication : $\alpha \odot (x, y) = (\alpha\, x, \alpha\, y)$, where $\alpha, a, b \in \mathbb{R}$ and $(a, b) \in V$.

   Is this set V a vector space? If not, which properties it should satisfy to be a vector space?

7. Determine the inverse of the matrix :

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & -1 & -2 \\ 1 & -1 & -1 & 2 \\ 1 & -2 & 1 & -1 \end{bmatrix}.$$

8. Determine whether the vector v_1 lies in the subspace generated by the vectors v_2, v_3, v_4 :

   - $v_1 = (1, 4, -3); v_2 = (1, 0, 1); v_3 = (1, 1, 0); v_4 = (3, 1, 2)$.
   - $v_1 = (1, 1, 2); v_2 = (0, 1, 0); v_3 = (3, 5, 6); v_4 = (1, 2, 1)$.

9. Consider the set of vectors $\{(3, -2, 4, 5), (0, 0, 2, 8), (0, 1, 2, 5), (0, 2, -3, -4)\}$. Is this set linearly dependent?

10. Determine whether the following set of functions is linearly independent or not :

$$f(x) = x^2 + 3x - 2$$
$$g(x) = x + 7$$
$$h(x) = 2x^2 - 5x + 1$$