INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

Problem Sheet 2

Date : 19.02.17   MA 1140 : Linear Algebra

1. Does the vector b lie in the column space of A, where

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } b = (3,4,5)^T?$$

2. Find the column space, row space, and the null space of the following matrix.

$$\begin{bmatrix} 2 & 4 & 8 & 16 \\ 3 & 9 & 27 & 81 \\ 4 & 8 & 16 & 64 \end{bmatrix}.$$

3. Consider two $n \times n$ matrices A and B such that $AB = 0$. Show that

   - The column space of B is contained in the null space of A.
   - The row space of A is contained in the left null space of B.

4. Let $v_1 = (2,0,-1), v_2 = (3,1,0), v_3 = (1,-1,c)$ for some $c \in \mathbb{R}$. Under what conditions on c, the set $\{v_1, v_2, v_3\}$ forms a basis of $\mathbb{R}^3$?

5. Find the rank of the following matrix :

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 3 & -1 & 1 \\ 5 & -4 & 2 & 0 \end{bmatrix}.$$

6. Determine the basis and dimension of the following :

   - The space of cubic polynomials $p(x) \in \mathbb{R}[X]$ such that $p(3) = 0$.
   - The space of cubic polynomials $p(x) \in \mathbb{R}[X]$ such that $p(3) = p(5) = 0$.

7. Determine the rank and nullity of matrix A, where

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ 3 & -7 & -5 & 5 & 1 \\ 1 & -5 & -3 & 2 & 0 \\ 7 & 5 & -1 & 9 & 5 \end{bmatrix}$$

   Verify rank - nullity theorem for the matrix A.

8. Determine the rank, nullity for A^T, where A is given in above problem. Verify the rank - nullity theorem for the matrix A^T. What is the relation between the rank of A and A^T. What about the nullity?