SYNOPSIS
OF THE PH.D. THESIS OF

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“A FRAMEWORK FOR LOSSLESS RULE REDUCTION IN FUZZY RULE BASED SYSTEMS”

Introduction:

The field of Fuzzy Control Systems is one of the most active and fruitful areas of research in which fuzzy sets theory is applied. Fuzzy set theory was proposed by Lotfi.A.Zadeh to formalise qualitative concepts that have no precise boundaries. For example, there are no meaningful landmark values representing the boundaries between low and normal, or normal and high. Rather, such linguistic terms are formalized by referring to fuzzy sets of numbers.

A fuzzy controller consists of a collection of control laws whose inputs and outputs are both fuzzy values. All controller rules are fired in parallel and the recommended actions are combined according to fuzzy value combination rules, weighted by the degree of satisfaction of the antecedent. Some process of defuzzification is required to convert the resulting fuzzy set description of an action into a specific value for a control variable.

The ability to control a system in uncertainty or unknown environments is one of the characteristics of any intelligent control system. Fuzzy inferencing procedures are becoming, therefore, increasingly crucial to the process of managing uncertainty. Fuzzy set theory provides a systematic framework for dealing with different types of uncertainty within a single conceptual framework.

What makes a Fuzzy Controller Different?

There are three main aspects of a Fuzzy Controller that cannot be found in their non-fuzzy counterparts designed via conventional linear / non-linear control methods:

1. The use of if-then rules
2. The universal approximation property
3. The ability to cope with set-wise inputs
The first aspect concerns the representation of the experts’ knowledge about the behaviour of the plant under control and heuristics-based manual control algorithms. The manual control algorithm, expressing available direct control or a supervisory control strategy is formulated in terms of fuzzy (if-then) rules. In the same way, the plant’s behaviour also can be expressed by a set of rules. Then the major problem is to identify these rules and their parameters so that the operator’s control or supervisory actions and the plant’s response are sufficiently well described and thus, certain control objectives and/or performance criteria can be achieved.

The second aspect of Universal Approximation means that a fuzzy system with certain types of membership functions, inference and defuzzification can approximate any real continuous function on a compact set to arbitrary accuracy.

The third aspect, coping with set-wise inputs, is relevant for control tasks where the controller inputs take fuzzy values instead of crisp ones. In contrast to classical controllers, fuzzy controllers can also deal with fuzzy values and even the mixture of crisp and fuzzy values become possible, by considering ‘crisp’ values as a special type of fuzzification, viz., singleton fuzzification.

The Issue under consideration:

An increase in the number of input variables and/or the number of membership functions in the input domains quickly leads to a Combinatorial Explosion in the number of rules. This uncurbed increase in the number of rules affects the efficiency both in time and space. Thus techniques for reduction of these rules command utmost and immediate attention.

How is our approach different:

Most of the techniques that have been proposed towards reducing the number of rules are adhoc in nature and they lack the analytical background that would allow a thorough analysis. Also very little work has been done on rule reduction techniques that preserve the inference, i.e., the outputs of the original and the reduced rule bases are identical. In this work, we propose a novel rule reduction technique for Fuzzy Systems by combining the antecedents of the rules with same consequents. This rule reduction is lossless with respect to inference.

To this end, we propose a general framework for Fuzzy Systems and show how the well-established Mamdani and Takagi–Sugeno (TSK) models of fuzzy
systems fit into this framework. We then propose some conditions on this general framework that facilitate lossless rule reduction. We then explore these conditions in the setting of Fuzzy Logic with the different Fuzzy Logic operations. We find that R- and S-implications play a very critical role. We have also solved one of the open problems posed by Michal Baczynski. We apply the results generated to different models of Fuzzy Systems both in Fuzzy Control and in Approximate Reasoning.

Outline of the Thesis:

This thesis is divided into the following 3 parts:

Part I: Introduction

In Chapter 1, we discuss the Structure and Inference and various stages and types of Fuzzy Rule Based Systems.

In Chapter 2, we introduce one of the most important issues that plagues any Fuzzy Rule Based System – Combinatorial Explosion of Rules. We also give a comprehensive survey of the Rule Reduction techniques proposed so far in the literature and their effectiveness and efficiency. The different rule reduction techniques are broadly classified into those that reduce rules while building a fuzzy system and those that act on a given rule base, and also those that are Lossless and Lossy with respect to inference.

Part II: Exploration

In Chapter 3, we propose a General Framework for Fuzzy Rule Based Systems. Every stage in the inference of a Fuzzy System is mapped to a corresponding function over appropriated domains. We propose conditions on these operations to enable Lossless Rule Reduction.

In Chapter 4, we explore the above conditions that enable Lossless Rule Reduction for different Fuzzy Logic operators. This leads to the study of distributivity of Fuzzy Implication Operators over T- and S-norms. We also study the distributivity of the different Matching Functions that have been proposed in the literature over T- and S-norms. We have also solved in this chapter one of the open problems posed by Michal Baczynski.
Part III : Application

In this section, we show how the results generated in Part II are applicable to many of the established and widely employed Fuzzy Systems – both in Fuzzy Control and Approximate Reasoning Systems – towards lossless rule reduction.

In Chapter 5, we show how the distributivity of R- and S-implications over t- and s-norms, as explored in Chapter 4, help us in rule reduction in the case of Mamdani Fuzzy Systems. We know that Mamdani Fuzzy Systems with ML-Implications or any t-norm is a Universal Approximator. But are Mamdani Fuzzy Systems with genuine fuzzy Implications Universal Approximators? We have answered this in the affirmative in this chapter.

The two most important ways of inferencing in Approximate Reasoning are Zadeh’s Compositional Rule of Inference (CRI) and Compatibility Modification Inference (CMI). In Chapter 6, we discuss lossless rule reduction procedures for these Approximate Reasoning Systems.

In Chapter 7, we show a few relatively simple ways of rule reduction in Takagi-Sugeno-Kang and Wang-Mendel Fuzzy Systems. When the given fuzzy system is consistent we show how the antecedents can be combined so as to reduce the number of rules and still retain its earlier accuracy.

By way of Conclusion, Chapter 8 summarises the different aspects of the work. We also discuss some of the possible future extensions to this work and the work already done along these lines but not included in this thesis.

Appendices:

There are two Appendices to this thesis. Appendix A gives an example of a Fuzzy Control System. It explains the working of a very simple Fuzzy Logic Controller for an Air-Conditioner. Appendix B gives a practical application, that of a NASA database, where rule reduction has been applied and found to be extremely beneficial.