# Diversity Gain Using a Repeater in a Wireless Personal Area Network

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***Abstract*— In this paper we show that a one-hop repeater in an ultra-wideband (UWB) communication system for a wireless personal area network (WPAN) can be used to achieve diversity gain in a Rayleigh fading channel. We use maximal ratio combining (MRC) in the transmitter-repeater link and space-time coding (STC) in the repeater-receiver link, where the repeater has two antennas for transmission as well as reception. The repeater transmits at a higher rate than the transmitter so that the symbols are properly synchronized. We then derive the closed-form expression for the probability of bit error and show that there exists a trade-off between the bit error rate (BER) performance and the transmit powers of the transmitter and the repeater. Based on our BER curves, we then show that the performance of the proposed system is much better than a conventional repeater as well as the traditional communication system.**

*Keywords* – **Ultra-wideband, Repeater, Space-Time Coding, Maximal Ratio Combining.**

## I. INTRODUCTION

Home networking is an emerging application area in which all devices at home (TVs, PCs, monitors, security cameras, telephones, etc.) will be connected without wires. With the FCC regulations [1] in force, the unlicensed $3.1 - 10.6$ GHz band – called Ultra-Wideband (UWB) – is now being targeted by the IEEE P802.15 [2], [3], [4], [5], [6] working group for wireless personal area networks (WPAN) for home applications. (UWB is not considered a technology anymore but an unlicensed spectral band. The FCC defines UWB as any signal that occupies more than 500 MHz in the 3.1 – 10.6 GHz with certain power restrictions. Impulse radio is not the only technology satisfying UWB requirements. Recently, several multiband techniques have been proposed for UWB [7].) UWB radios should be capable of low power high speed data transmissions over short ranges ($\approx$ 10m). Hence UWB has generated a lot of interest among researchers.

Indoor multipath channel models have been studied in [8], [9], [10]. Due to the transmitted data being spread over a broad frequency band that is much greater than the coherence bandwidth of the multipath channel, the receiver performance is robust even in the presence of multipath interference [11]. Rake receivers for UWB systems have been proposed in [12], [13]. Other advantages include a simple design which translates into a low implementation cost. Though time-domain UWB systems have been widely discussed in literature, frequency-domain designs have become more popular in recent years.

A high transmitter power can lead to interference with existing narrow-band communication systems. To overcome this problem, a one-hop repeater with pulse position modulation (PPM) was proposed in [14]. However, the focus here is more on relaying strategy rather than system performance and related power consumption.

It has been shown that MIMO techniques provide several advantages like improved bit-rate, reduced transmission power, etc [15]. For MIMO techniques to be effective, the spacing of antennas should be of the order of $\lambda/2$ where $\lambda$ is the carrier wavelength [16]. This requirement increases the size of the devices which, in turn, would increase the implementation costs. Thus, in a WPAN, it would be impractical to use diversity techniques at the transmitting and receiving devices.

In this paper, we propose a repeater architecture which provides the diversity gains without increasing the form factors of end devices. The repeater contains multiple antennas which can be used both for transmission and reception. Through our analysis, we show that a repeater can not only be used for reducing interference with narrow-band devices, but also for providing diversity gain, if appropriate transmit and receive diversity techniques are used. However, this is achieved with some computational complexity at the repeater and receiver.

The paper is organized as follows. The system model is presented in Section II. In Section III we discuss the BER performance. Our Results are summarized in Section IV. Conclusions are presented in Section V.

## II. SYSTEM MODEL

A short range repeater system is shown in Fig. 1. The repeater has two antennas which can be used for transmission as well as reception. Both the transmitter and repeater use antipodal signaling. The transmitter transmits symbols with amplitude $A_1$ . The two antennas at the repeater are used for reception and MRC [17] is used to decode the transmitted symbol. The decoded symbol at the repeater is again transmitted using the Alamouti scheme with amplitude $A_2$.

We assume that the channel experiences quasi-static Rayleigh fading over a duration of $2T$, where $T$ is the symbol duration. The channel gains from the transmitter to the repeater are $h_{11}$ and $h_{12}$, while those from the repeater to the receiver are $h_{21}$ and $h_{22}$, as shown in Fig. 1. Also, $h_{ij}$, $i = 1, 2$, $j = 1, 2$ are complex circular Gaussian random variables with zero mean

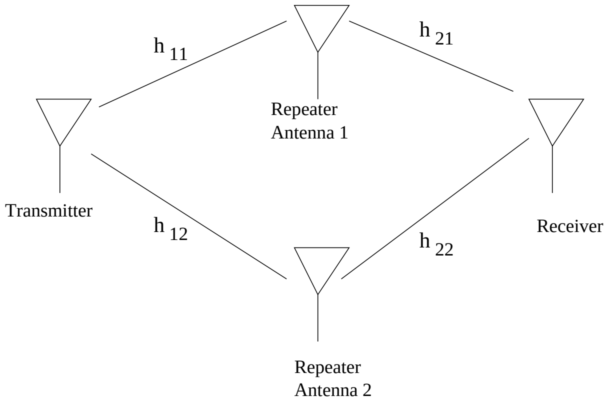

Fig. 1. System model, showing the transmitter, repeater (with two antennas) and receiver with the corresponding channel gains

and variance $E[|h_{ij}|^2] = \alpha^2$. Further, the transmitted signal is corrupted by AWGN with zero mean and one sided power spectral density $N_o$. The channel estimates at the repeater and the receiver are assumed to be perfectly.

### A. The Transmitter-Repeater Link

If $s$ be the transmitted symbol, the received symbol at the first antenna, $r_{11}$ and the symbol at the second antenna $r_{12}$ of the repeater are

$$\begin{aligned} r_{11} &= A_1 h_{11} s + n_{11} \\ r_{12} &= A_1 h_{12} s + n_{12} \end{aligned} \tag{1}$$

respectively.

We assume that the channel gains for the transmitter-repeater link are perfectly known at the repeater. We then use MRC, wich is a very simple receive diversity combining technique. The advantage of using MRC is that the phase shift in the channel is effectively compensated, which results in an optimum decision. The decision variable $U$ at the repeater is given by

$$U = h_{11}^* r_{11} + h_{12}^* r_{12} \tag{2}$$

The decoded symbol $\hat{s}$ is then obtained as [18]

$$\hat{s} = min_s d(U, s) \tag{3}$$

where,

$$d^2(\mathbf{x}, \mathbf{y}) = (\mathbf{x} - \mathbf{y})(\mathbf{x}^* - \mathbf{y}^*) \tag{4}$$

### B. The Repeater-Receiver Link

The symbol $\hat{s}$ is then transmitted by the repeater using the Alamouti scheme [18], which, for our system, is

$$\begin{pmatrix} \hat{s} & \hat{s} \\ -\widehat{s^*} & \widehat{s^*} \end{pmatrix} \tag{5}$$

In (5), the transmission time slots are given by the row index of the matrix and the transmit antenna index is given by the column index The symbol received at time $t$ is

$$r_{21} = A_2 h_{21} \hat{s} + A_2 h_{22} \hat{s} + n_{21}. \tag{6}$$

The received symbol at time $t + T$ is

$$r_{22} = -A_2 h_{21} \hat{s}^* + A_2 h_{22} \hat{s}^* + n_{22} \tag{7}$$

Combining the received symbols according to the Alamouti scheme gives the decision metrics

$$\begin{aligned} V &= h_{21}^* r_{21} + h_{22} r_{22}^* \\ W &= h_{22}^* r_{21} - h_{21} r_{22}^* \end{aligned} \tag{8}$$

We find the symbol that minimizes $d(V, s)$ to get an estimate of the transmitted symbol $\tilde{s}$ in the first time slot. We repeat the above for the metric $d(W, s)$ to estimate the symbol $\tilde{s}$ transmitted in the second time slot. It is worth noting that in case of an error, the symbols decoded in consecutive time slots at the receiver may be different, though the same symbol was transmitted on both time slots through both antennas at the repeater.

## III. BER PERFORMANCE ANALYSIS

From the previous section, we know that the transmitted symbol is $s$, the symbol decoded and re-transmitted at the repeater is $\hat{s}$ and the symbol decoded at the receiver is $\tilde{s}$. We define

$$\begin{aligned} P(\hat{s} = 1 | s = 0) &= P_1, \\ P(\tilde{s} = 1 | \hat{s} = 0) &= P_2. \end{aligned} \tag{9}$$

From (9),

$$\begin{aligned} P(\hat{s} = 0 | s = 0) &= 1 - P_1, \\ P(\tilde{s} = 0 | \hat{s} = 0) &= 1 - P_2. \end{aligned} \tag{10}$$

The conditional probability of error

$$\begin{aligned} P(\tilde{s} = 1 | s = 0) &= P(\tilde{s} = 1, \hat{s} = 0 | s = 0) \\ &\quad + P(\tilde{s} = 1, \hat{s} = 1 | s = 0) \end{aligned} \tag{11}$$

From elementary probability theory, we have

$$\begin{aligned} P(\tilde{s} = 1 | s = 0) &= P(\tilde{s} = 1 | \hat{s} = 0, s = 0) P(\hat{s} = 0 | s = 0) \\ &\quad + P(\tilde{s} = 1 | \hat{s} = 1, s = 0) P(\hat{s} = 1 | s = 0). \end{aligned} \tag{12}$$

The received symbol $\tilde{s}$ is independent of the transmitted symbol $s$. Hence,

$$\begin{aligned} P(\tilde{s} = 1 | \hat{s} = 0, s = 0) &= P(\tilde{s} = 1 | s = 0) \\ P(\tilde{s} = 1 | \hat{s} = 1, s = 0) &= P(\tilde{s} = 1 | \hat{s} = 1) \end{aligned} \tag{13}$$

and

$$\begin{aligned} P(\tilde{s} = 1 | s = 0) &= P(\tilde{s} = 1 | \hat{s} = 0) P(\hat{s} = 0 | s = 0) \\ &\quad + P(\tilde{s} = 1 | \hat{s} = 1) P(\hat{s} = 1 | s = 0). \end{aligned} \tag{14}$$

The Alamouti scheme and MRC give the same BER performance [18]. Using this knowledge, we consider our system to be a two-stage MRC. Hence the probability of error for the transmitter-repeater link (one transmit antenna two receive antennas, with MRC) is the same as that for the repeater-receiver link. Thus, from [17],

$$P_1 = \frac{1}{4}(1 - \mu_1)^2 (2 + \mu_1), \tag{15}$$

$$P_2 = \frac{1}{4}(1 - \mu_2)^2 (2 + \mu_2) \tag{16}$$

where

$$\mu_1 = \frac{A_1\alpha}{\sqrt{N_0 + A_1^2\alpha^2}}, \mu_2 = \frac{A_2\alpha}{\sqrt{N_0 + A_2^2\alpha^2}}$$

and $A_1$, $A_2$, $\alpha$ and $N_0$ are as defined in Section II.

Assuming that the transmitted bits are equiprobable, from (9), (10) and (14), the closed form expression for the average probability of bit error for our system is obtained as

$$\begin{aligned} P_e &= P_1(1 - P_2) + P_2(1 - P_1) \qquad (17) \\ &= P_1 + P_2 - 2P_1P_2, \end{aligned}$$

where $P_1$ and $P_2$ are given by (15) and (16) respectively.

## IV. RESULTS AND DISCUSSION

The BER performance of the communication system proposed in this paper is shown in Fig. 2 for different instances of the transmit powers of the transmitter and repeater, i.e., for $\frac{A_1}{A_2} = 1$, $\frac{A_1}{A_2} > 1$ and $\frac{A_1}{A_2} < 1$. We use (17) to evaluate the probability of error and then verify these results by simulation. We find that the BER performance is best when the transmit power is more than the repeater transmission power. The BER performance degrades when the repeater transmits at a lesser power than the transmitter.

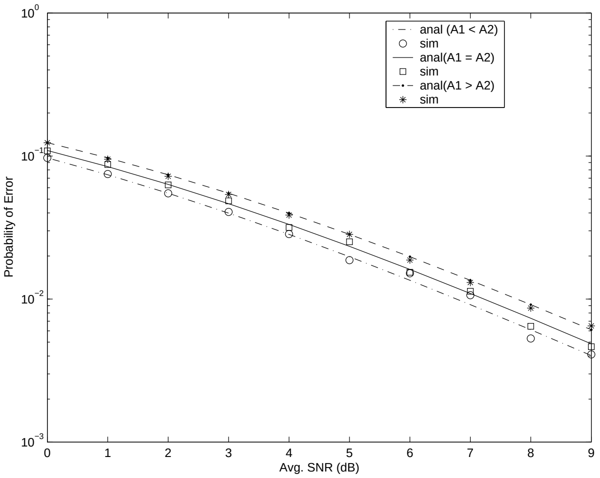

Fig. 2. BER curves for a one-hop repeater system with diversity. Analysis and simulation results for various transmission (at the transmitter) and re-transmission (at the repeater) powers.

In Fig. 3, we show the BER performance with respect to the signal to noise ratio (SNR) for

1) a communication system that employs a repeater with multiple antennas and diversity combining
2) a conventional repeater
3) a traditional communication system without any repeater or space-time diversity.

We find that the performance of a communication system employing a conventional repeater is worse than one without it, provided the repeater transmission power is less than that of the transmitter. This is because of the effect of fading both on the transmitter-repeater link and the repeater-receiver link. However, the communication system proposed in this paper is shown to perform much better than a traditional communication system without a repeater. In fact, even if the transmit power of the repeater is reduced by 3 dB ($A_1 > A_2$) relative to the transmitter, for higher SNRs it still performs better than the traditional communication system in a Rayleigh fading channel.

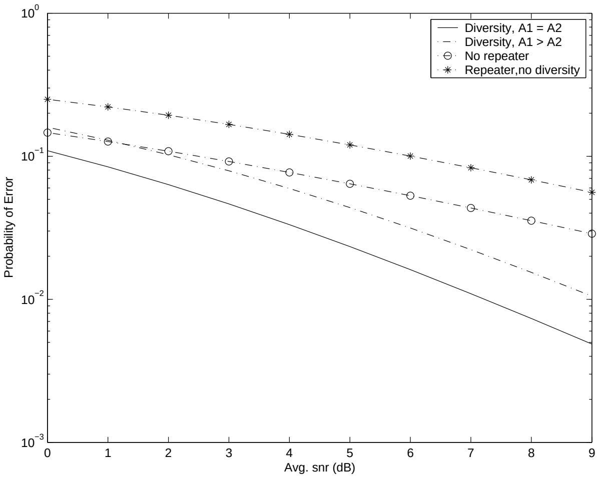

Fig. 3. Comparison of the BER performance of various short-range communication systems. A traditional single transmit/receive antenna based communication system, a system employing a repeater equipped with only one antenna and a multi- antenna repeater based communication system exploiting the benefits of transmit/receive diversity.

## V. CONCLUSIONS

In this paper, we have considered a system which uses a one-hop repeater with receive and transmit diversity to boost the overall performance. We derive a closed form expression for the BER when receive diversity with MRC and transmit diversity with STC are employed at the repeater. We find that even if the repeater re-transmission power is significantly reduced, the performance is still better than the traditional communication system for higher values of the SNR.

Though we have considered only two antennas at the repeater for reception and re-transmission to demonstrate the advantage of diversity combining in reducing transmit power and improving quality of transmission, more number of antennas can be used for a better performance. Also, the architecture proposed in this paper can be extended to include multipath delay diversity combining techniques like the Rake demodulator (on the transmitter-repeater link).

In our analysis, we have assumed that the channel experiences Rayleigh fading. But many indoor multipath channels experience Nakagami as well as log-normal fading. We feel that the proposed repeater architecture should help reduce transmit power without loss in performance even in those environments. This is an issue that needs serious investigation.

In a deployment scenario, directional antennas can be used toward the receiver to minimize interference.

The repeater architecture can be interpreted in a wider context. There is an increase in the number of functionalities available on (and expected from) wireless devices. The functionalities include music playback, video capture, GPS, etc. The additional functionalities are likely to increase the size of the wireless device. Hence grid-based architectures have been proposed for wireless peripherals [19], [20]. The wireless grids consist of peripherals - cameras, displays, keyboards, etc. The handheld devices uses the devices depending on need and availability. UWB is the most likely protocol for wireless grids. This paper shows that repeater can be used as an additional resource on the grid.

We are currently exploring the effect of modulation, synchronization and interference issues for the proposed scheme.

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