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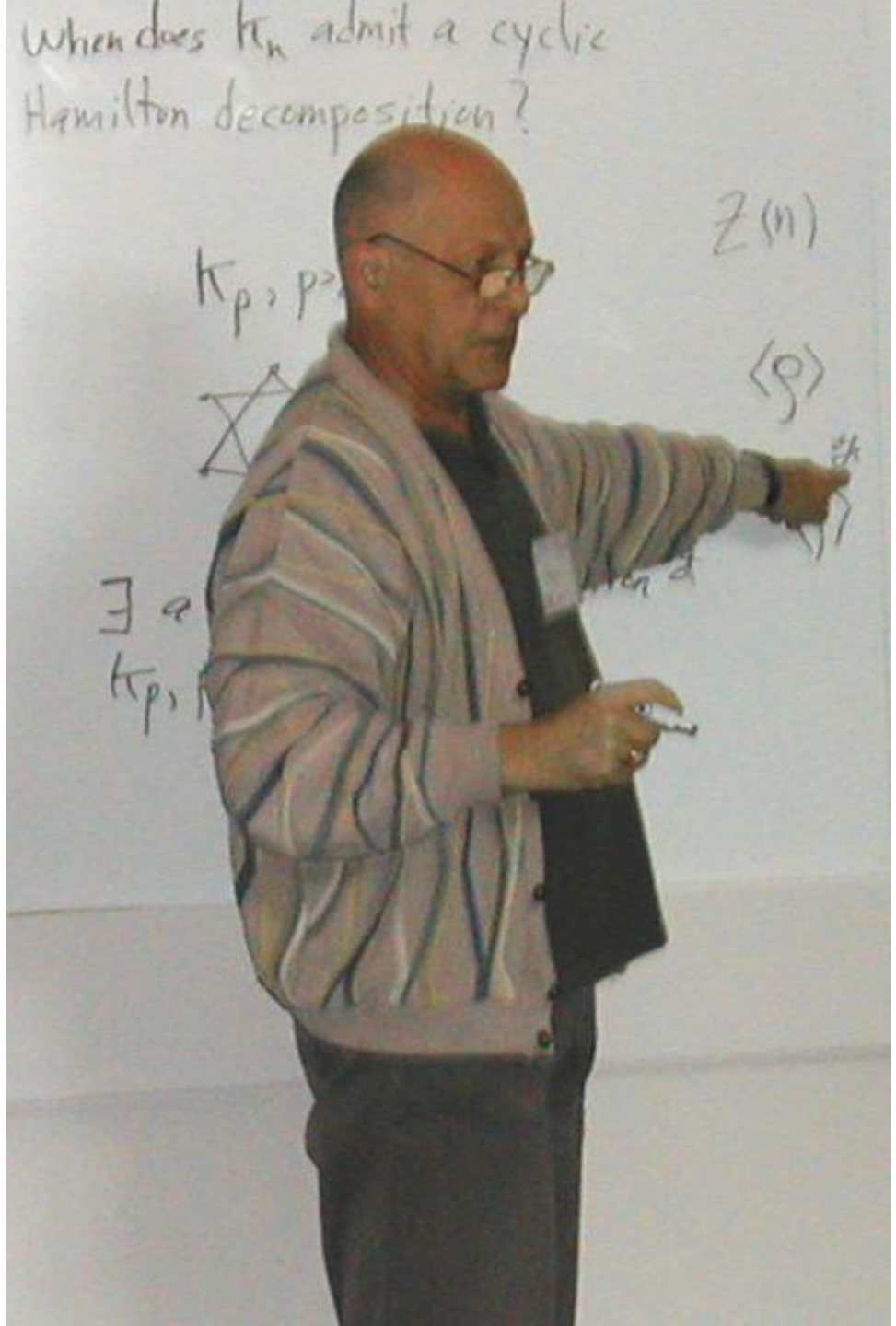
When does K_n admit a cyclic
Hamilton decomposition?
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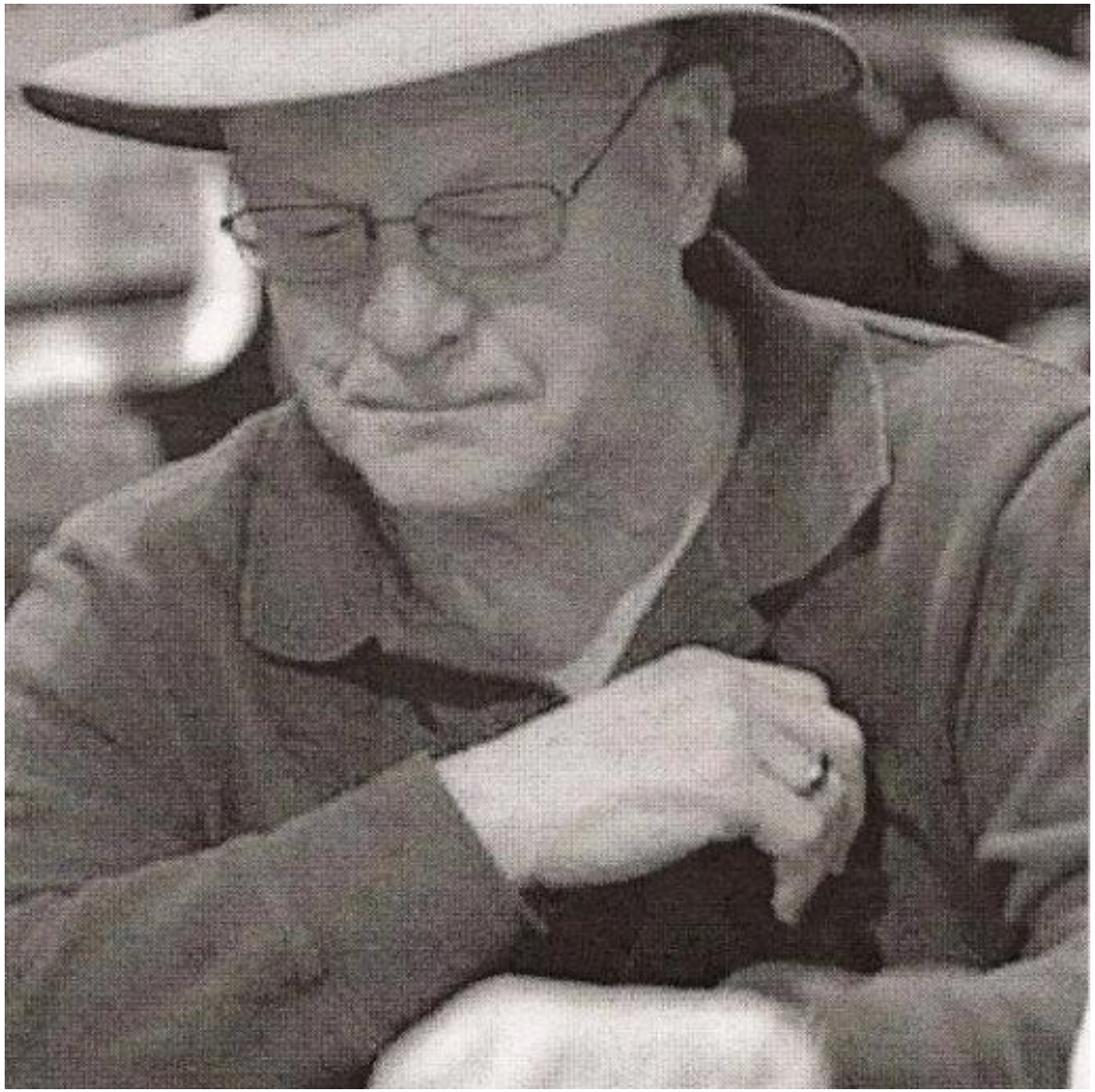

Graph homomorphisms: structure and symmetry

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Abstract

This paper is the first part of an introduction to the subject of graph homomorphism in the mixed form of a course and a survey. We give the basic definitions, examples and uses of graph homomorphisms and mention some results that consider the structure and some parameters of the graphs involved. We discuss vertex-transitive graphs and Cayley graphs and their rather fundamental role in some aspects of graph homomorphisms. Graph colourings are then explored as homomorphisms, followed by a discussion of various graph products.

1 Introduction

Homomorphisms provide a way of simplifying the structure of objects one wishes to study while preserving much of it that is of significance. Most mathematicians remember the isomorphism theorems we learn in a first course on group theory, and certainly anyone involved in some way with mathematics or computer science knows about integers modulo some n. It is not surprising that homomorphisms also appeared in graph theory, and that they have proven useful in many areas.

We do not claim to provide the definitive survey but only an introductory course. Of necessity, we had to omit most proofs and, sadly, even many results and some aspects of the subject matter. But we do try to provide an extensive bibliography and the interested reader will be able to find the missing pieces with little trouble. For the rest, we will all have to wait for the book that Hell and Nešetřil are reportedly writing. The course/survey is divided into two parts. The present paper is concerned with structure and symmetry, the sequel [56] deals with computational aspects of graph homomorphisms and surveys the computer science roots of the subject.

Graph homomorphisms in the current sense were first studied by Sabidussi in the late fifties and early sixties, with results published in the paper on *Graph derivatives* [109] and

G. Hahn and G. Sabidussi (eds.), Graph Symmetry, 107–166.