Neutrino Mixing by Modifying the Yukawa couplings of Constrained Sequential Dominance

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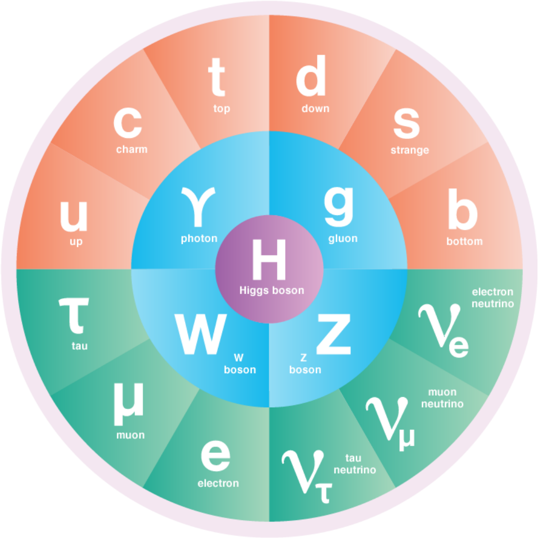
Standard Model of Particle Physics

It is based on $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge groups.



Neutrino Mass in Standard Model

- All the quarks and charged leptons get their masses through the Yukawa couplings with the Higgs field $H = (H^+, H^0)^T$

$$-\mathcal{L}_Y = h^u_{ij}\overline{Q_{Li}}u_{Rj}\tilde{H} + h^d_{ij}\overline{Q_{Li}}d_{Rj}H + f^e_{ij}\overline{L_{Li}}e_{Rj}H + h.c. \tag{1}$$

- Mass matrices for quarks and charged leptons are

$$(m_u)_{ij} = h^u_{ij}v, \quad (m_d)_{ij} = h^d_{ij}v, \quad (m_e)_{ij} = f^e_{ij}v. \tag{2}$$

- There are no right handed neutrinos $\implies$ Neutrinos are mass less.

Evidence for Neutrino Mass and Mixing

Neutrino Oscillation $\Longrightarrow$ 1) Neutrinos have small but non-zero mass.
2) Neutrino flavor eigenstates are different from mass eighenstates that is lepton mixing.

So,

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U^*_{\alpha i}|\nu_i\rangle, \tag{3}$$

$\alpha = e, \mu, \tau$ are flavour index and $i = 1, 2, 3$ are mass indices. U is defined as the mixing matrix.

A PDG parametrisation of U is

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\beta_1} & 0 \\ 0 & 0 & e^{i\beta_2} \end{pmatrix}. \tag{4}$$

Experimental Data

Parameter	Best-fit	3σ
$\Delta m^2_{21}[10^{-5}eV^2]$	7.37	6.93 - 7.96
$\Delta m^2_{31(23)}[10^{-3}eV^2]$	2.56(2.54)	2.54 - 2.69 (2.42 - 2.66)
$\sin^2\theta_{12}$	0.297	0.250 - 0.354
$\sin^2\theta_{23}, \Delta m^2_{31(32)} > 0$	0.425	0.381 - 0.615
$\sin^2\theta_{23}, \Delta m^2_{32(31)} < 0$	0.589	0.384 - 0.636
$\sin^2\theta_{13}, \Delta m^2_{31(32)} > 0$	0.0215	0.0190 - 0.0240
$\sin^2\theta_{13}, \Delta m^2_{32(31)} < 0$	0.0216	0.0190 - 0.0242
$\frac{\delta}{\pi}$	1.38(1.31)	2σ :(1.0 - 1.9) (0.92 - 1.88)

Different PMNS Matrix

The most promising mixing matrix upto 2012 is

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{5}$$

It predicts $s_{12} = \frac{1}{\sqrt{3}}$, $s_{23} = \frac{1}{\sqrt{2}}$ and $s_{13} = 0$.

- Experimentally $s_{13} \neq 0 \implies U_{TBM}$ is ruled out.

Seesaw with Three Right Handed neutrinos

Assuming the charged lepton mass matrix to be diagonal, we add three right handed neutrinos ν_R^{sol}, ν_R^{atm}, ν_R^{dec} to SM, Yukawa Lagragian for Neutrino mass is

$$\begin{aligned}\mathcal{L}^{Yuk} = \; & (\tfrac{H_u}{v_u})(d\overline{L}_e + e\overline{L}_\mu + f\overline{L}_\tau)\nu_R^{atm} + (\tfrac{H_u}{v_u})(a\overline{L}_e + b\overline{L}_\mu + c\overline{L}_\tau)\nu_R^{sol} \\ & +(\tfrac{H_u}{v_u})(a'\overline{L}_e + b'\overline{L}_\mu + c'\overline{L}_\tau)\nu_R^{dec} + H.c.\end{aligned} \tag{6}$$

Majorana Lagrangian is given by

$$\mathcal{L}_\nu^{\mathcal{M}} = M_{sol}\overline{\nu_R^{sol}}(\nu_R^{sol})^c + M_{atm}\overline{\nu_R^{atm}}(\nu_R^{atm})^c + M_{dec}\overline{\nu_R^{dec}}(\nu_R^{dec})^c. \tag{7}$$

So,

$$M_R = \begin{pmatrix} M_{atm} & 0 & 0 \\ 0 & M_{sol} & 0 \\ 0 & 0 & M_{dec} \end{pmatrix}, \quad m_D = \begin{pmatrix} d & a & a' \\ e & b & b' \\ f & c & c' \end{pmatrix}, \tag{8}$$

$$m^\nu = M_D M_R^{-1} M_D^T. \tag{9}$$

Sequential Dominance

$$M_{\rm atm} \ll M_{\rm sol} \ll M_{\rm dec}, \quad \frac{(e,f)^2}{M_{\rm atm}} \gg \frac{(a,b,c)^2}{M_{\rm sol}} \gg \frac{(a',b',c')^2}{M_{\rm dec}}. \tag{10}$$

- Third column of m_D and m_R can be decoupled.
- Three right handed neutrino model $\implies$ Two right handed neutrino model.

Constrained Sequential Dominance (CSD)

$$d = 0, \quad e = f, \quad , a = b = -c. \tag{11}$$

After performing above mentioned decoupling and above condition, Dirac and Majorana mass matrices take the form

$$m_D = \begin{pmatrix} 0 & a \\ e & a \\ e & -a \end{pmatrix}, \quad M_R = \begin{pmatrix} M_{atm} & 0 \\ 0 & M_{sol} \end{pmatrix}. \tag{12}$$

Putting this m_D and M_R in m_ν of Eq.(8), we find

$$U_{TBM}^T m_\nu U_{TBM} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3a^2}{M_{sol}} & 0 \\ 0 & 0 & \frac{2e^2}{M_{atm}} \end{pmatrix}. \tag{13}$$

Our model and deviation from CSD

We take

$$m_D^{'} = m_D + \Delta m_D, \quad m_D = \begin{pmatrix} 0 & a \\ e & a \\ e & -a \end{pmatrix}, \quad \Delta m_D = \begin{pmatrix} e\epsilon_1 & a\epsilon_4 \\ e\epsilon_2 & a\epsilon_5 \\ e\epsilon_3 & a\epsilon_6 \end{pmatrix}, \qquad (14)$$

This $m_D^{'}$ in general should give deviation from TBM pattern.
Hence the Seesaw formula for active neutrinos will be

$$m_\nu^s = m_D^{'} M_R^{-1} (m_D^{'})^T. \qquad (15)$$

This m_ν^s should be diagonalised by U_{PMNS}.

$$m_\nu^d = U_{PMNS}^T m_\nu^s U_{PMNS} = diag(m_1, m_2, m_3). \qquad (16)$$

Our Model and deviation from CSD

- In order to simplify our calculations, we parametrise s_{12} and s_{23} as

$$s_{12} = \frac{1}{\sqrt{3}}(1 + r), \quad s_{23} = \frac{1}{\sqrt{2}}(1 + s). \tag{17}$$

- r, s and s_{13} will be become non-zero in our model if we allow small non-zero ϵ_i.
- The matrices m_ν^s and U_{PMNS} depend on r, s, s_{13} and ϵ_i which are small and of the same order. So, m_ν^s and U_{PMNS} can be expanded in terms of these small parameters.
- After doing that we can see that m_ν^d need not be in diagonal form. We put off-diagonal elements to zero give ϵ_i in terms of r, s, s_{13}. Diagonal elements give masses of neutrinos in terms of model parameters.

Our model and Deviation from CSD

In limit where ϵ_i, r, s, s_{13} tend to zero, we get the leading order expressions

$$m_1 = 0, \quad m_2 = \frac{3a^2}{M_{sol}}, \quad m_3 = \frac{2e^2}{M_{atm}}. \tag{18}$$

- m_1 is zero at leading order. m_1 will also zero at subleading orders. This is a consequence of the fact that two right handed neutrino model is proposed.
- So, we can have only normal mass hierarchy. Then m_2 and m_3 can be fitted into $\sqrt{\Delta m^2_{sol}}$ and $\sqrt{\Delta m^2_{atm}}$.
- We can fit

$$\frac{3a^2}{M_{sol}} \sim \sqrt{\Delta m^2_{sol}}, \quad \frac{2e^2}{M_{atm}} \sim \sqrt{\Delta m^2_{atm}}. \tag{19}$$

- It is noticed $\sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}} \sim s_{13}$.

Our Model and Deviation from CSD

We rexpress Eq.(15)

$$\frac{1}{\sqrt{\Delta m^2_{atm}}} m^d_\nu \equiv \frac{1}{\sqrt{\Delta m^2_{atm}}} (U^T_{PMNS} m^s_\nu U_{PMNS}) \tag{20}$$
$$= diag(\frac{m_1}{\sqrt{\Delta m^2_{atm}}}, \frac{m_2}{\sqrt{\Delta m^2_{atm}}}, \frac{m_3}{\sqrt{\Delta m^2_{atm}}}).$$

So, $\frac{1}{\sqrt{\Delta m^2_{atm}}} m^d_\nu$ can be expanded in power series of ϵ_i, r, s, s_{13} and $\sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}}$.

- Upto first order in ϵ_i, m_ν^s can be expanded as

$$m_\nu^s = m_{\nu(0)}^s + m_{\nu(1)}^s, \quad (21)$$

$$m_{\nu(0)}^s = m_D M_R^{-1} m_D^T, \quad m_{\nu(1)}^s = m_D M_R^{-1}(\Delta m_D) + \Delta m_D M_R^{-1} m_D^T. \quad (22)$$

- Similarly upto first order in r, s, s_{13}, U_{PMNS} can be expanded as

$$U_{PMNS} = U_{TBM} + \Delta U, \quad (23)$$

$$\Delta U = \begin{pmatrix} -\frac{r}{\sqrt{6}} & \frac{r}{\sqrt{3}} & e^{-i\delta_{CP}} s_{13} \\ \frac{-r+s}{\sqrt{6}} - \frac{e^{i\delta_{CP}} s_{13}}{\sqrt{3}} & -\frac{r+2s+\sqrt{2}e^{i\delta_{CP}} s_{13}}{2\sqrt{3}} & \frac{s}{\sqrt{2}} \\ \frac{r+s}{\sqrt{6}} - \frac{e^{i\delta_{CP}} s_{13}}{\sqrt{3}} & \frac{r-2s-\sqrt{2}e^{i\delta_{CP}} s_{13}}{2\sqrt{3}} & -\frac{s}{\sqrt{2}}. \end{pmatrix}. \quad (24)$$

Our Model and Deviation from CSD

- Diagonal elements give three neutrino masses

$$m_1 = 0, \quad m_2 = \frac{3a^2}{M_{sol}}, \quad m_3 = \frac{2e^2}{M_{atm}} + \frac{2e^2(\epsilon_2 + \epsilon_3)}{M_{atm}}. \tag{25}$$

- By equating off diagonal elements to zero gives

$$\epsilon_1 = \sqrt{2}e^{i\delta_{CP}} s_{13}, \quad , \epsilon_2 - \epsilon_3 = 2s. \tag{26}$$

12 element is automatically zero up to first order. Above two results are from 13 and 23 elements.

Our Model and Deviation from CSD

Above results shows follwing

- $\sin\theta_{13}$ will be non-zero if$\epsilon_1 \neq 0$.
- $\sin\theta_{23}$ will deviate from its TBM value if either ϵ_2 or ϵ_3 is non zero.
- However deviation of $\sin\theta_{12}$, which is quantified in terms of r, is undetermined at this order. As a result, ϵ_4, ϵ_5 and ϵ_6 are undetermined at this level.
- These parameters can be determined in the second order correction to the diagonalisation of our seesaw formula.

Second Order Correction

We expand $\frac{1}{\sqrt{\Delta m^2_{atm}}} m^d_\nu$ in Eq.(19) up to second order in ϵ_i, r, s, s_{13} and $\sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}}$. Expanding U_{PMNS} and m^s_ν, up to second order in ϵ_i, r, s and s_{13}

$$m^s_\nu = m^s_{\nu(0)} + m^s_{\nu(1)} + m^s_{\nu(2)}, \quad m^s_{\nu(2)} = \Delta m_D M_R^{-1} (\Delta m_D)^T, \tag{27}$$

$$U_{PMNS} = U_{TBM} + \Delta U + \Delta^2 U, \tag{28}$$

$$\Delta^2 U = \begin{pmatrix} -\frac{2s_{13}^2+r^2}{2\sqrt{6}} & -\frac{s_{13}^2}{2\sqrt{3}} & 0 \\ \frac{2s_{13}e^{i\delta_{\rm CP}}(r-2s)+\sqrt{2}(2rs+s^2)}{4\sqrt{3}} & \frac{-r^2+2rs-2s^2-2\sqrt{2}s_{13}e^{i\delta_{\rm CP}}(r+s)}{4\sqrt{3}} & -\frac{s_{13}^2+s^2}{2\sqrt{2}} \\ \frac{s_{13}e^{i\delta_{\rm CP}}(r+2s)+\sqrt{2}rs}{2\sqrt{3}} & \frac{r^2-2\sqrt{2}s_{13}e^{i\delta_{\rm CP}}(r-s)+2rs}{4\sqrt{3}} & -\frac{s_{13}^2+s^2}{2\sqrt{2}} \end{pmatrix}.$$

$\frac{1}{\sqrt{\Delta m_{atm}^2}} m_\nu^s$ can be computed by putting above m_ν^s and U_{PMNS} into Eq. (19) up to second order in ϵ_i, r, s, s_{13} and $\sqrt{\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}}$.

- Equating the diagonal elements on both sides, we find

$$m_1 = 0, \quad m_2 = \frac{3a^2}{M_{sol}} + \frac{3a^2}{M_{sol}}(\epsilon_4 + \epsilon_5 - \epsilon_6), \tag{29}$$

$$m_3 = \frac{2e^2}{M_{atm}} + \frac{2e^2}{M_{atm}}(\epsilon_3 + s) + \frac{2e^2}{M_{atm}}(s_{13}^2 + \epsilon_3^2 + 2\epsilon_3 s + 3s^2). \tag{30}$$

Second Order Correction

After demanding the off-diagonal elements $\frac{1}{\sqrt{\Delta m^2_{atm}}} m^s_\nu$ should be zero, we get the following three relations

$$2\epsilon_4 - \epsilon_5 + \epsilon_6 = 3r, \quad (31)$$

$$4\epsilon_3 s + 5s^2 - 4\sqrt{2} s_{13} e^{i\delta_{\rm CP}} (\epsilon_3 + s) = 0, \quad (32)$$

$$3e^{-i\delta_{\rm CP}} e^{i\phi} \sqrt{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} [2s_{13} + \sqrt{2} e^{i\delta_{\rm CP}} (\epsilon_5 + \epsilon_6 + 2s)] = 2s_{13} e^{i\delta_{\rm CP}} (\epsilon_3 + s). \quad (33)$$

- Deviation of $\sin\theta_{12}$ in terms of r is also found in terms of ϵ_4,ϵ_5 and ϵ_6.
- Solving above three equations, two of the ϵ_4, ϵ_5 can be determined. One is free parameter.

A Model for the Dirac Mass matrix

A flavour model of $SO(3) \times SO(3)' \times Z_3$. Charge assignments to the fiels which are relevant for neutrino sector

	ϕ_{atm}	ϕ_{sol}	ϕ'_{atm}	ϕ'_{sol}	χ^{atm}	χ^{sol}	ν_R^{atm}	ν_R^{sol}	L	H
$SO(3)$	3	3	1	1	1	1	1	1	3	1
$SO(3)'$	1	1	3	3	1	1	1	1	3	1
Z_3	ω	ω^2	ω	ω^2	ω^2	ω	ω^2	ω	1	1

The invariant Lagrangian in the neutrino sector is

$$\mathcal{L} = \frac{\phi_{atm}}{M_P}\bar{L}\nu_R^{atm}H + \frac{\phi_{sol}}{M_P}\bar{L}\nu_R^{sol}H + \frac{\phi'_{atm}}{M_P}\bar{L}\nu_R^{atm}H + \frac{\phi'_{sol}}{M_P}\bar{L}\nu_R^{sol}H + \frac{\chi^{atm}}{2}\overline{(\nu_R^{atm})^c}\nu_R^{atm} + \frac{\chi^{sol}}{2}\overline{(\nu_R^{sol})^c}\nu_R^{sol} + h.c. \quad (34)$$

Here, M_P is the Planck scale, which is the cut-off scale of the model. We have taken M_P as the cut-off scale but grand unified scale can also be taken as the cut-off of the model.

A Model for Dirac Mass Matrix

we can see that neutrinos acquire Dirac mass terms, once the following scalar fields acquire vevs: $\phi_{atm}, \phi_{sol}, \phi'_{atm}, \phi'_{sol}$.

- The vevs of ϕ_{atm}, ϕ_{sol} spontaneously break the flavour symmetry $SO(3)$,ϕ_{atm}, ϕ_{sol} have the following pattern

$$\frac{\langle\phi_{atm}\rangle}{M_P} = y_a \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \frac{\langle\phi_{sol}\rangle}{M_P} = y_s \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \tag{35}$$

  Here, y_a, y_s are dimensionless quantities.

- $SO(3)'$ is spontaneously broken by $\langle\phi'_{atm}\rangle, \langle\phi'_{sol}\rangle$. They may take the following form

$$\frac{\langle\phi'_{atm}\rangle}{M_P} = y'_a \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix}, \quad \frac{\langle\phi'_{sol}\rangle}{M_P} = y'_s \begin{pmatrix} \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{pmatrix}. \tag{36}$$

  Here, y'_a, y'_s are dimensionless quantities.

A Model for Dirac Mass Matrix

- We can get our desired Dirac Mass matrix provided $y_a = y'_a$ and $y_s = y'_s$.
- Let us assume the symmetries $SO(3)$ and $SO(3)'$ are broken at scales Λ and Λ' respectively,

$$\langle \phi_{atm} \rangle, \langle \phi_{sol} \rangle \sim \Lambda, \quad \langle \phi'_{atm} \rangle, \langle \phi'_{sol} \rangle \sim \Lambda' \tag{37}$$

- We propose $\Lambda' \sim 0.1 \times \Lambda$, so that $\epsilon_i \sim 0.1$

Conclusion

- We have attempted to explain the neutrino mixing in order to be consistent with the current neutrino oscillation data.
- Earlier to explain the TBM pattern CSD model has been proposed. Here we have considered a phenomenological model, where we have modified the neutrino Yukawa couplings of CSD model by introducing small ϵ_i parameters which are complex.
- Real and imaginary parts of the ϵ_i are assumed to be less than or the order of $\sin\theta_{13} \sim \sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}}$.
- Thereafter we follwed an approximate process of diagonalising the seesaw mass matrix of our model and compute expression, up to second order level, to neutrino masses and mixing angles in terms of small ϵ_i.
- Useing these expressions we have demonstrated that neutrino mixing angles can deviate from TBM values by appropriately choosing ϵ_i.
- Finally we have constructed a model in order to justify the neutrino Yukawa coupling structure of our model.

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Thank You