DYNAMIC FRICTION MODEL

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INTRODUCTION

Vehicle motion is primarily determined by the friction forces transferred to the road by the tires. Therefore, a proper tire friction model is one of the key elements of a complete vehicle model intended for the use in different vehicle dynamics simulation studies for handling applications.

Static friction models are appropriate when we have steady-state conditions for the linear and angular velocities. Dynamic friction models attempt to capture the transient behavior of the tire-road contact forces under time-varying velocity conditions.
DAHL MODEL

Dahl’s model starts with the stress-strain curve in classical solid mechanics. When subject to stress, the friction force increases gradually until rupture occurs.

SIMULATION MODEL:

\[
\frac{dF}{dx} = \sigma_0 \left(1 - \frac{F}{F_c} \text{sgn}(v_r)\right)^\beta
\]

Where \(x\) = relative displacement, \(\sigma_0\) = stiffness coefficient, \(F_c\) = maximum friction force (Coulomb force) and \(\beta\) is a parameter that determines the shape of the stress-strain curve.

\(v_r = \frac{dx}{dt}\) is the relative velocity. \((v_r = r\omega - v)\)
For time-domain model,

\[
\frac{dF}{dt} = \frac{dF}{dx} \frac{dx}{dt} = \frac{dF}{dx} v_r = \sigma_0 \left( 1 - \frac{F}{F_c} \text{sgn}(v_r) \right)^\beta v_r
\]

The most commonly used value of \( \beta \) is 1. Therefore,

\[
\frac{dF}{dt} = \sigma_0 \left( v_r - \frac{F |v_r|}{F_c} \right)
\]
MODEL DESCRIPTION:

\[ \frac{dz}{dt} = v_r - \frac{\sigma_0 z |v_r|}{F_c} \]

\[ F = \sigma_0 z \]

Where \( z \) is the relative displacement of the bristles.
NORMALIZED TOTAL FRICTION FORCE VS TIME

Dahl Model

Panel

Coulomb force (F_c) 1
Stiffness coefficient (aΩ) 10

Friction force for \( v = 0.002 \)

Time
Friction force

0 0.2 0.4 0.6 0.8 1
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
SOME COMMON TERMS

The longitudinal force $F$ is usually described as a static function of the longitudinal slip rate $s$ which is defined as,

$$s = \begin{cases} \frac{v - r\omega}{r\omega - v}, & \text{for } r\omega < v \neq 0 \text{ (driving)} \\ \frac{v}{v - r\omega}, & \text{for } r\omega \leq v \neq 0 \text{ (braking)} \end{cases}$$

Where $v =$ vehicle speed, $\omega =$ wheel angular velocity, $r =$ effective tire radius.
It is assumed that the tire/road contact is realized through a lot of tiny, massless and elastic elements called bristles. The contact patch has a rectangular form with the length $L$. The uniform normal pressure distribution is assumed in the paper. The relative speed between the bristle base point attached to the belt, and the tip which adheres to the ground is (considering traction) 

$$v_r = r\omega - v$$

The bristles deform producing the tire longitudinal force $F$. 
A lumped friction model assumes a point tire-road friction contact. As a result, the mathematical model describing such a model is an ordinary differential equations that can be easily solved by time integration.

Distributed friction models, on the other hand, assume the existence of a contact patch between the tire and the ground with an associated normal pressure distribution. This formulation results in a partial differential equation, that needs to be solved both in time and space.
ONE-WHEEL SYSTEM WITH LUMPED FRICTION (LEFT), AND DISTRIBUTED FRICTION (RIGHT)
When the thickness of the film is large enough to completely separate the bodies in contact, the friction coefficient may increase with velocity as hydrodynamic effects become significant. This is called the Stribeck effect.
The LuGre model is an extension of the Dahl model that includes the Stribeck effect.

SIMULATION OF MODEL:

\[
\frac{dz}{dt} = v_r - \frac{\sigma_0 |v_r|}{g(v_r)} z
\]

\[
F = \left( \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v_r \right) F_n
\]

\[
g(v_r) = \theta \left[ \mu_c + (\mu_s - \mu_c) \times \exp \left( -\left( \frac{v_r}{v_s} \right)^\alpha \right) \right]
\]
Where $\sigma_0 =$ rubber longitudinal lumped stiffness

$\sigma_1 =$ rubber longitudinal lumped damping (bristle damping coefficient),

$\sigma_2 =$ viscous relative damping (friction viscous coefficient),

$F_n =$ normal force.

$\mu_c =$ normalized Coulomb friction

$\mu_s =$ normalized Static friction

$\nu_s =$ Stribeck relative velocity

$\alpha$ is a constant. Here we use $\alpha = \frac{1}{2}$.

$g(\nu_r)$ is the Stribeck-type tire/road sliding friction function.
The Lumped LuGre Model

Friction force at steady state condition

Friction force for v = 0.002

Panel
- Stiffness (a0) = 10000
- Damping coefficient (c1) = 336.22
- Viscous friction coefficient (c2) = 0.4
- Coulomb friction level (Fc) = 1
- Stiction force (Fs) = 1.5
- Stribeck velocity (Vs) = 0.001

Push Button
DISTRIBUTED LUGRE MODEL

In this model, we extend the point friction model to a distributed friction model along the patch by letting \( z(\zeta, t) \) denote the friction state of the bristle element at a certain time \( t \).

At every time instant \( z(\zeta, t) \) provides the deflection distribution along the contact patch.
SIMULATION MODEL:

The equation for this model can be written as (using first two equations of Lumped LuGre model)

\[
\frac{d z(\zeta, t)}{dt} = v_r - \frac{\sigma_0 |v_r|}{g(v_r)} z
\]

\[
F(t) = \int_0^L dF(\zeta, t)
\]

Where \( g(v_r) \) is same as defined in lumped model.
\[ dF(\zeta, t) = \left( \sigma_0 z(\zeta, t) + \sigma_1 \frac{\partial z(\zeta, t)}{\partial t} + \sigma_2 v_r \right) dF_n(\zeta, t) \]

Where \( dF(\zeta, t) \) is the differential friction force developed in the element \( d\zeta \) and \( dF_n(\zeta, t) \) is the differential normal force applied in the element \( d\zeta \) at time \( t \).

This model assumes that the contact velocity of each differential state element is equal to \( v_r \).
RELATION WITH MAGIC FORMULA:

Steady state expressions depend on \( v \) and \( w \). Therefore two cases are formed accordingly.

**Driving case:**

In this case \( v < r \omega \), therefore the force at steady state is given by

\[
F_d(s) = sgn(v_r)F_n g(s) \left[ 1 + \frac{g(s) \left( e^{-\sigma_0 L|s|/g(s)} - 1 \right)}{\sigma_0 L|s|} \right] + F_n \sigma_2 r \omega s
\]

\[
g(s) = \mu_c + (\mu_s - \mu_c) \cdot \exp \left( -\left| r \frac{\omega s}{v_s} \right|^{\alpha} \right)
\]
**Braking case:**

In this case $v > r\omega$, therefore the force at steady state is given by

$$F_b(s) = sgn(v_s)F_ng(s) \left[ 1 + \frac{g(s)|1 + s|}{\sigma_0 L |s|} \left( e^{-\sigma_0 L |s|/g(s)|1 + s|} - 1 \right) \right] + F_n \sigma_2 vs$$

$$g(s) = \mu_c + (\mu_s - \mu_c) \cdot \exp \left(-\left|\frac{vs}{v_s}\right|^\alpha\right)$$
DISTRIBUTED LUGRE MODEL RESULTS

**Distributed LuGre Model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubber longitudinal lumped stiffness (a)</td>
<td>101.54</td>
</tr>
<tr>
<td>Viscous relative damping (m2)</td>
<td>0.0018</td>
</tr>
<tr>
<td>Radius of Wheel (r)</td>
<td>0.219</td>
</tr>
<tr>
<td>Stribeck relative velocity (vs)</td>
<td>6.57</td>
</tr>
<tr>
<td>Normalized Coulomb friction (µc)</td>
<td>0.6</td>
</tr>
<tr>
<td>Normalized static friction (µs)</td>
<td>1.55</td>
</tr>
<tr>
<td>Patch Length (L)</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**Graphs:**

- **Distributed LuGre model with uniform force distribution (driving case):**
  - Three lines for different speeds: v = 9 m/s, v = 18 m/s, v = 30 m/s.

- **Distributed LuGre model with uniform force distribution (braking case):**
  - Four lines for different θ values: θ = 1, θ = 0.6, θ = 0.4, θ = 0.2.

- **Distributed LuGre model with uniform force distribution (braking case) for v = 20 m/s:**
  - Three lines for different θ values: θ = 1, θ = 0.6, θ = 0.4.
CAD MODEL OF THE VEHICLE
THANK YOU