DIRECTIONAL STABILITY OF ARTICULATED VEHICLE

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OBJECTIVES

• Introduction

• Stability of single vehicle

• Stability of one articulated vehicle

• Results
INTRODUCTION

- Articulated Vehicle?
  A vehicle which has a permanent or semi-permanent pivot joint in its construction, allowing the vehicle to turn more sharply.

- Directional stability?
  Stability of a moving body or vehicle about an axis which is perpendicular to its direction of motion.

  Eg: Vehicle during yaw motion.
Factors influencing the stability?

- Steering characteristics
  - Under steering. \((\alpha_1 > \alpha_2)\)
  - Neutral steering. \((\alpha_1 = \alpha_2)\)
  - Over steering. \((\alpha_1 < \alpha_2)\)

Those are depends on front \((\alpha_1)\) and rear \((\alpha_2)\) tyre slip angles.
CAD MODEL
STABILITY OF SINGLE VEHICLE

Let us consider vehicle is moving with forward velocity $u$, then Equation of motion can be written as,

$$
\begin{bmatrix}
m_1 & 0 \\
0 & I_1
\end{bmatrix}
\begin{bmatrix}
\dot{v}_1 \\
\dot{r}_1
\end{bmatrix} = -\frac{1}{u}
\begin{bmatrix}
C & C_{s_1} + m_1u^2 \\
C_{s_1} & C_{q_1^2}
\end{bmatrix}
\begin{bmatrix}
v_1 \\
r_1
\end{bmatrix} + 
\begin{bmatrix}
C_1 \\
a_1C_1
\end{bmatrix} \delta_1
$$

$v_1 = \text{lateral velocity at centre of gravity}$

$r_1 = \text{Yaw velocity}$
\[ C = C_1 + C_2 \]

\[ C_{s_1} = a_1 C_1 - b_1 C_2 \]

\[ C_{q_1}^2 = a_1^2 C_1 + b_1^2 C_2 \]

\[ I_1 = m_1 k_1^2 \]

with,

\( C \): the total cornering stiffness,

\( s_1 \): the distance along the longitudinal axis of the vehicle from the centre of gravity to the neutral steer point S. The neutral steer point is the point on the vehicle where an external side force does not impose a yaw angle to the vehicle. This means that \( s_1 = 0 \) corresponds to a neutrally steered vehicle, \( s_1 < 0 \) to an under steered vehicle and \( s_1 > 0 \) to an over steered vehicle,

\( q_1 \): the length which corresponds to an average moment arm,

\( k_1 \): the radius of gyration.
By solving the above differential equation, we get the characteristic equation of a single vehicle is given by:

\[ c_0 \lambda^2 + c_1 \lambda + c_2 = 0 \]

with,
\[ c_0 = 1 \]
\[ c_1 = \frac{k_1^2 + q_1^2}{m_1 k_1^2 u} \]
\[ c_2 = \frac{C}{m_1 k_1^2 u^2} \left( \frac{C (q_1^2 - s_1^2)}{m_1} - s_1 u^2 \right), \text{since } q_1^2 - s_1^2 = l_1^2 \frac{C_1 C_2}{C} \]

By Hurwitz criterion, stability of vehicle can be studied, Therefore for stability,

\[ c_0 > 0: \text{always fulfilled} \]
\[ c_1 > 0: \text{always fulfilled} \]
\[ s_1 < 0: \text{always fulfilled} \]
\[ s_1 = 0: u < \infty \implies \text{always fulfilled} \]
\[ c_2 > 0: \left\{ \begin{array}{l}
   s_1 > 0: u < V_{crit} = \sqrt{\frac{C (q_1^2 - s_1^2)}{m_1 s_1}} \\
\end{array} \right. \]
\[ H_1 > 0: \text{always fulfilled} \]
Stability of one articulated vehicle

There are two cases,
1) $e_1 < 0$, Hitch point is located in front of the rear axle of the towing vehicle.
   Ex: Tractor-semitrailer
2) $e_1 > 0$, Hitch point is located behind the rear axle of the towing vehicle.
   Ex: Truck-centre axle trailer
Equation of motion can be written as,

\[
\begin{bmatrix}
m_{1} + m_{2} & -m_{2}(h_{1} + a_{2}) & -m_{2}a_{2} & 0 \\
-m_{2}h_{1} & I_{1} + m_{2}h_{1}(h_{1} + a_{2}) & m_{2}h_{1}a_{2} & 0 \\
-m_{2}a_{2} & I_{2} + m_{2}a_{2}(h_{1} + a_{2}) & I_{2} + m_{2}a_{2}^{2} & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{v}_1 \\
\dot{r}_1 \\
\phi \\
\end{bmatrix}
= -\frac{1}{u}
\]

\[
\begin{bmatrix}
C + C_{3} \\
C_{s1} - C_{3}h_{1} \\
-C_{3}l_{2} \\
0 \\
\end{bmatrix}
\begin{bmatrix}
C_{s1} - C_{3}(h_{1} + l_{2}) + (m_{1} + m_{2})u^{2} \\
C_{q1}^{2} + C_{3}h_{1}(h_{1} + l_{2}) - m_{2}h_{1}u^{2} \\
C_{3}l_{2}(h_{1} + l_{2}) - m_{2}a_{2}u^{2} \\
0 \\
\end{bmatrix}
\begin{bmatrix}
-v_1 \\
-C_{3}l_{2} \\
-C_{3}u \\
-u \\
\end{bmatrix}
\begin{bmatrix}
\dot{v}_1 \\
\dot{r}_1 \\
\phi \\
\end{bmatrix}
+ \begin{bmatrix}
C_{1} \\
C_{1} \\
a_{1}C_{1} \\
0 \\
\end{bmatrix}\delta_{1},
\]

where,

\(v_1 = \text{lateral velocity at centre of gravity}\)

\(r_1 = \text{Yaw velocity}\)

\(\phi = \text{Relative articulation angle between the trailer and towing vehicle}\)

\(\theta = \beta + \phi\)

By solving above differential equation we arrive at the characteristic equation as:

\[c_0\lambda^4 + c_1\lambda^3 + c_2\lambda^2 + c_3\lambda + c_4 = 0\]
Where,

\[
\begin{align*}
  c_0 &= m_1 m_2 u \left[ m_1 k_1^2 (k_2^2 + a_2^2) + m_2 k_2^2 (k_1^2 + h_1^2) \right] \\
  c_1 &= m_1 m_2 \left[ C_3 (h_2^2 + k_2^2) (k_1^2 + h_1^2) + C (a_2^2 + k_2^2) (k_1^2 + q_1^2) \right] \\
  &\quad + m_1^2 C_3 k_1^2 l_2^2 + m_2^2 C k_2^2 (q_1^2 + h_1^2 + 2 h_1 s_1) \\
  c_2 &= \frac{1}{u} \left[ u^2 \left\{ m_1 C_3 l_2 \left( m_1 k_1^2 + m_2 \frac{b_2}{l_2} (k_1^2 + h_1^2) \right) - m_2 C (s_1 + h_1) \left( m_2 k_2^2 + m_1 \frac{s_1}{s_1 + h_1} (k_2^2 + a_2^2) \right) \right] \\
  &\quad + m_1 C_3 \frac{l_2^2}{u^2} (k_1^2 + q_1^2) + m_2 C \left\{ C_3 (q_1^2 + h_1^2 + 2 h_1 s_1) (k_2^2 + b_2^2) + C (q_1^2 - s_1^2) (k_2^2 + a_2^2) \right\} \right] \\
  c_3 &= \frac{C C_3 l_2^2}{u^2} \left[ u^2 \left\{ m_1 (q_1^2 + k_1^2 - s_1 l_2) + m_2 \frac{b_2}{l_2} (q_1^2 + h_1^2 + 2 h_1 s_1) - m_2 \frac{1}{l_2} (s_1 + h_1) (b_2^2 + k_2^2) \right\} \\
  &\quad + C l_2 (q_1^2 - s_1^2) \right] \\
  c_4 &= \frac{C C_3 l_2}{u} \left[ - u^2 \left\{ m_2 \frac{b_2}{l_2} (s_1 + h_1) + m_1 s_1 \right\} + C (q_1^2 - s_1^2) \right]
\end{align*}
\]

\[
\begin{align*}
  Cs_2 &= l_1^* C_1 + e_1 C_2 \\
  s_2 &= h_1 + s_1 \\
  C q_2^2 &= l_1^* C_1 + e_1 C_2 \\
  q_2 &= q_1^2 - s_1^2 + s_2^2 \\
  &= q_1^2 + h_1^2 + 2 h_1 s_1
\end{align*}
\]

\( l_1^* = \) Distance between the front axle and hitch point
By Hurwitz criterion, stability of vehicle can be satisfied under the conditions of,

\[
\begin{align*}
    &c_0 > 0 : \text{always fulfilled} \\
    &c_1 > 0 : \text{always fulfilled} \\
    &c_2 > 0 : \begin{cases} 
        &D_{c_2} < 0 : \text{always fulfilled} \\
        &D_{c_2} = 0 : u < \infty \Rightarrow \text{always fulfilled} \\
        &D_{c_2} > 0 : u < V_{\text{crit,}c_2} \\
        &D_{c_3} < 0 : \text{always fulfilled} \\
        &D_{c_3} = 0 : u < \infty \Rightarrow \text{always fulfilled} \\
        &D_{c_3} > 0 : u < V_{\text{crit,}c_3} \\
        &D_{c_4} < 0 : \text{always fulfilled} \\
        &D_{c_4} = 0 : u < \infty \Rightarrow \text{always fulfilled} \\
        &D_{c_4} > 0 : u < V_{\text{crit,}c_4} \\
    \end{cases} \\
    &H_1 > 0 : \text{always fulfilled} \\
    &H_3 > 0 : c_1 c_2 c_3 - c_0 c_3^2 - c_4 c_1^2 > 0
\end{align*}
\]

With,

\[
\begin{align*}
    V_{\text{crit,}c_2} &= \frac{m_1 C C_3 l_2^3 (k_1^2 + q_1^2) + m_2 C \left\{ C_2 q_2^2 (k_2^2 + b_2^2) + C (q_1^2 - s_1^2) (k_2^2 + a_2^2) \right\}}{\sqrt{-m_1 C l_2 \left\{ m_1 k_1^2 + m_2 b_2^2 / l_2 (k_1^2 + h_1^2) \right\} + m_2 C s_2 \left\{ m_2 k_2^2 + m_1 b_1^2 s_1^2 / l_2 (k_2^2 + a_2^2) \right\}}} \\
    V_{\text{crit,}c_3} &= \sqrt{C (q_1^2 - s_1^2)} \frac{C l_2 (k_2^2 + b_2^2)}{-m_1 (q_1^2 + k_1^2 - s_1 l_2) - m_2 b_2^2 / l_2 q_2^2 + m_2 s_2^2 / l_2 (b_2^2 + k_2^2)} \\
    V_{\text{crit,}c_4} &= \sqrt{C (q_1^2 - s_1^2)} \frac{C l_2 (k_2^2 + b_2^2)}{m_2 b_2^2 s_2 + m_1 s_1}
\end{align*}
\]

where, \( D_{c_2}, D_{c_2}, D_{c_2} \) the denominators of the critical speeds of \( V_{\text{crit,}c_2}, V_{\text{crit,}c_3} \) and \( V_{\text{crit,}c_4} \) respectively.
RESULTS

• Observed stability of vehicle during understeer case through car-sim software within the radius of 100m.
Observed double lane change test at different speeds and concluded that at high velocity vehicle becomes unstable.
REFERENCES

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