Probability and Computing

Assignment Four

Deadline: 3 May 2020

1. Let $M = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.5 & 0.3 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}$ be the transition probability matrix of a Markov chain. Find the probability distribution vector after 4 time-steps, when the initial distribution is $(1/3, 1/3, 1/3)$.

2. Find $T_{i,j}$ (expected time from i to j) when $i \leq j$, for the random walk on a path on $\{0, 1, 2, \ldots, n\}$.

3. Consider a random walk on the Hamming cube with vertex set $[0, 1]^3$ that begins at $(0, 0, 0)$. Find the expected number of steps to reach $(1, 1, 1)$.

4. Find the stationary distribution for a random walk on $G = (V, E)$, where $V = \{v_1, v_2, \ldots, v_6\}$, and $E = \{\{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_1, v_5\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_5, v_6\}\}$.

5. A coloring of a graph is an assignment of a color to each of its vertices. A graph is properly k-colorable if thre is a coloring of the vertices with k colors such that no two adjacent vertices have the same color. Let G be a properly 3-colorable graph.

   (a) Show that there exists a coloring of the graph with two colors such that no triangle is monochromatic (i.e. no triangle has all three vertices having the same color).

   (b) Describe and analyze a randomized algorithm to find such a 2-coloring, given a properly 3-colorable graph G as input. Note that the 3-coloring itself is not given as input.