Probability and Computing

Assignment Three

Deadline: 20 March 2020

1. In Morris' algorithm, let C_n be the value of the counter after seeing n elements in the stream. Show that $E[2^{C_n}] = n$ and $Var[2^{C_n}] = n(n-1)/2$.

2. Consider the Count-Min algorithm. How does the algorithm and the guarantee change, if, instead of inserting elements, each element of the stream is a pair of the form $(x_i, a(x_i))$, where $a(x_i)$ can be negative, and the goal is to estimate the total frequency of y, defined as the sum of all $a(y)$, the sum being over the occurrences of y in the stream.

3. Let $G = (V, E)$ be an undirected graph. Suppose that every vertex $v \in V$ is independently added to exactly one of three sets: A, B, C. Find the expected number of inter-part edges: $E[A, B] + E[B, C] + E[A, C]$, where $E[A, B] = \{\{u, v\} : u \in A, v \in B\}$, and $E[B, C], E[A, C]$ are defined analogously. Also describe an algorithm to find a partition with at least those many inter-part edges.