Section I: Problems of enumeration

1. In how many ways can the letters of NEWYORK be arranged without containing NEW or YORK as substrings?

2. Let $S = \{1, 2, \ldots, 10^8\}$. How many numbers in $S$ contain the digit 1 or the digit 7 but not both?

3. Find the number of subsets $S$ of $\{1, 2, 3, 4, 5, 6, 7\}$ such that the sum of the elements of $S$ is divisible by 7.

4. Find the number of ways to partition $mn$ people into $m$ groups of $n$ each.

5. How many $n$-letter strings can be formed using the letters $A$ and $B$ and not containing three consecutive As?

Section II: Pigeon-hole problems

1. Suppose that there are 50 points inside a circle of radius 1. Show that two of these points must be at a distance less than one from each other. Can you do this with a much smaller number of points than 50?

2. Assuming friendship to be a symmetric relation, show that in any group of people, there must be two with the same number of friends in that group.
Section III: Extremal problems

1. What is the maximum number of vectors in $\mathbb{R}^n$ that you can pick so that every pair of them make an obtuse angle?

2. What is the maximum number of elements in $\{1, 2, \ldots, n\}$ that you can pick, so that no two numbers sum to a third among the picked elements?