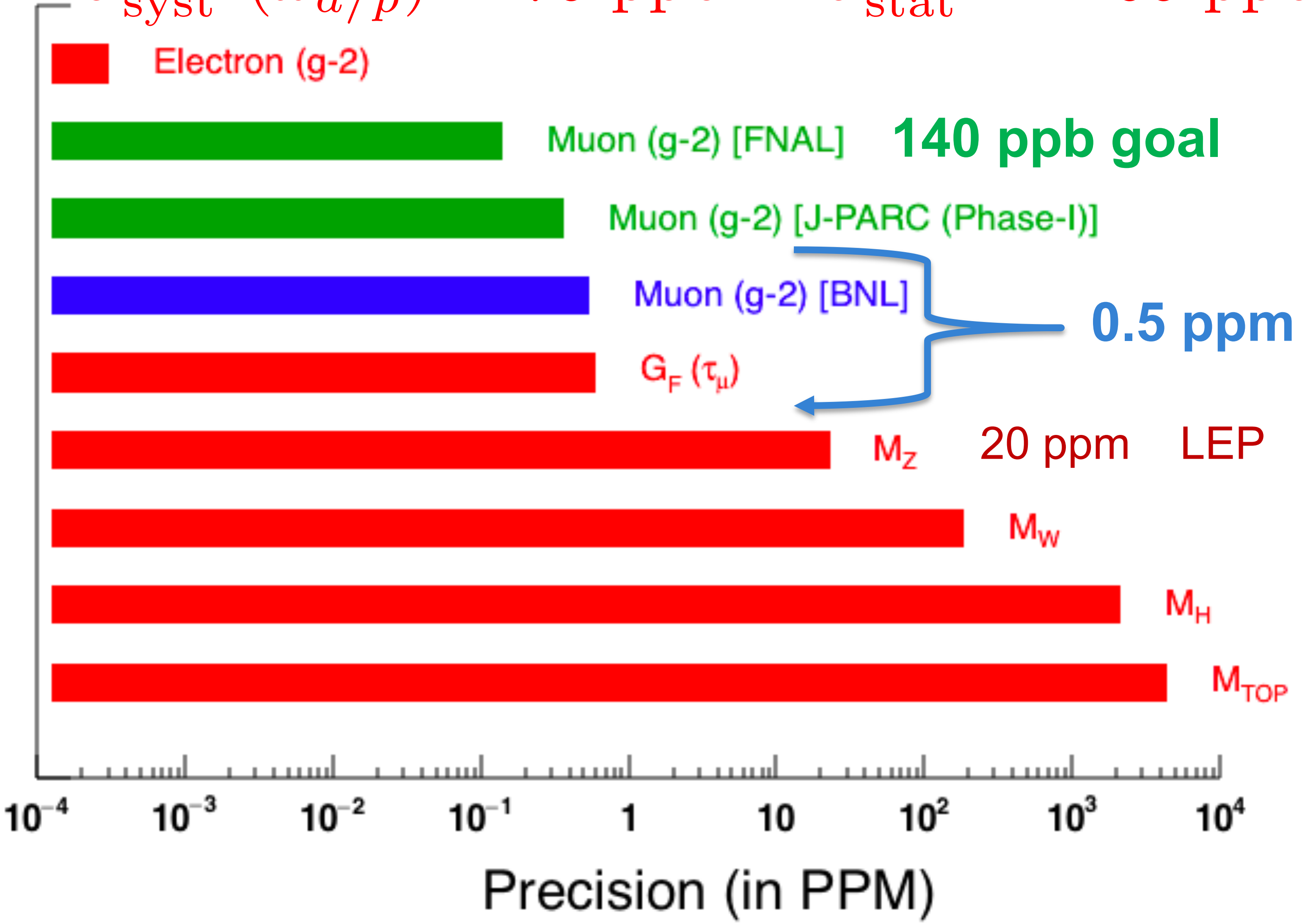
(Muon g-2): Status and future outlook

— Lee Roberts, Boston University

Precision in Particle Physics: Fermilab a_μ goal: 140 ppb

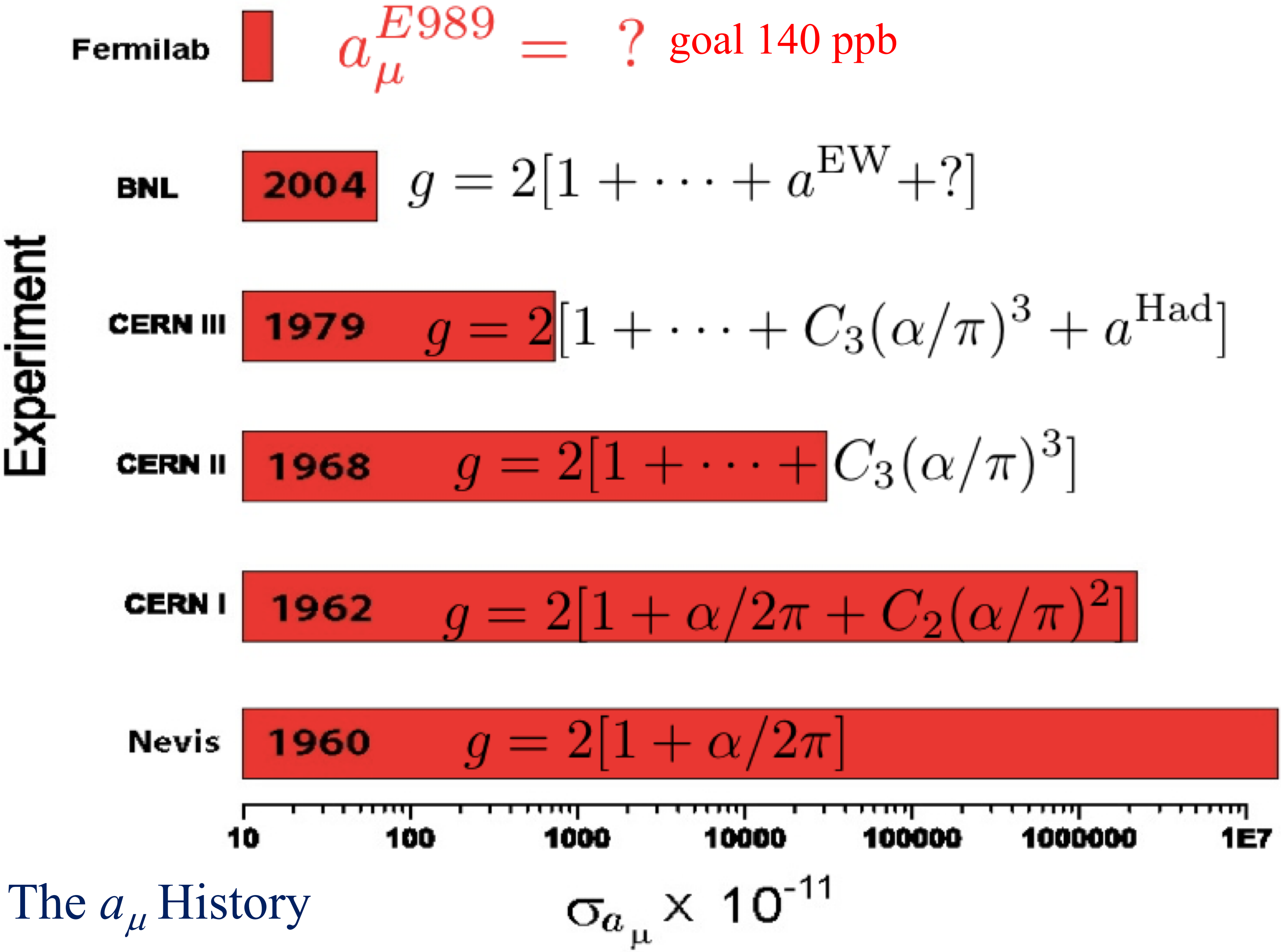
$$\sigma^{\mathrm{E989}}_{\mathrm{syst}}(\omega_{a/p}) = 70\ \mathrm{ppb} : \quad \sigma^{\mathrm{E989}}_{\mathrm{stat}} = 100\ \mathrm{ppb}$$



140 ppb goal

0.5 ppm

20 ppm LEP



$a_\mu^{E989} = \;?$ goal 140 ppb

$g = 2[1 + \cdots + a^{\mathrm{EW}} + ?]$

$g = 2[1 + \cdots + C_3(\alpha/\pi)^3 + a^{\mathrm{Had}}]$

$g = 2[1 + \cdots + C_3(\alpha/\pi)^3]$

$g = 2[1 + \alpha/2\pi + C_2(\alpha/\pi)^2]$

$g = 2[1 + \alpha/2\pi]$

The a_μ History

The Fermilab Site showing the Muon Campus

We use the old p-bar debuncher ring (“ our Delivery Ring”) as a 2 km pion decay line.

Muon g-2 Collaboration ~200 Collaborators

US Universities

- Boston
- Cornell
- Illinois
- James Madison
- Kentucky
- Massachusetts
- Michigan
- Michigan State
- Mississippi
- Northern Illinois
- Regis
- Texas
- Virginia
- Washington

National Labs

- Argonne
- Brookhaven
- Fermilab

Italy

- INFN
  - LNF Frascati,
  - Naples
  - Pisa SNS INFN
  - Roma 2
  - Trieste
  - Lecce
- Udine
- Naples
- Trieste
- Rjeka
- Molise
- SNS Pisa

China:

- Shanghai

Germany:

- Dresden (thy)
- Mainz

England

Cockroft Institute
Lancaster
Liverpool
Manchester
University College London

Korea

KAIST
IBS

Russia:

Dubna
Novosibirsk

In preparing this talk I have borrowed from a number of colleagues:
David Flay, David Hertzog and especially from James Mott.

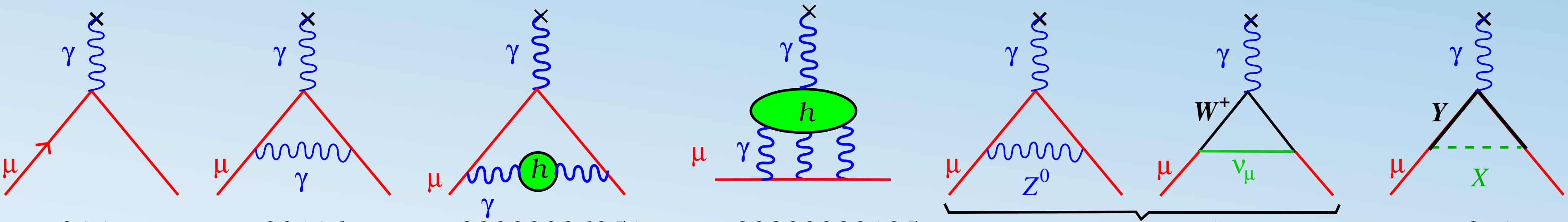
BOSTON UNIVERSITY

Outline

- Introduction
- Overview of the experimental approach
- Details of the magnetic field measurement
- Details of the muon spin rotation measurement
- Summary and outlook

a_μ : Radiative Corrections, known and unknown

- Both Standard Model, and perhaps BSM particles can contribute



$g = 2(\,1 \quad + .00116\ldots \quad + .00000006951\ldots + .00000000105\ldots \quad + .000000001536 \qquad +? \quad)$

QED hadronic electroweak

History:

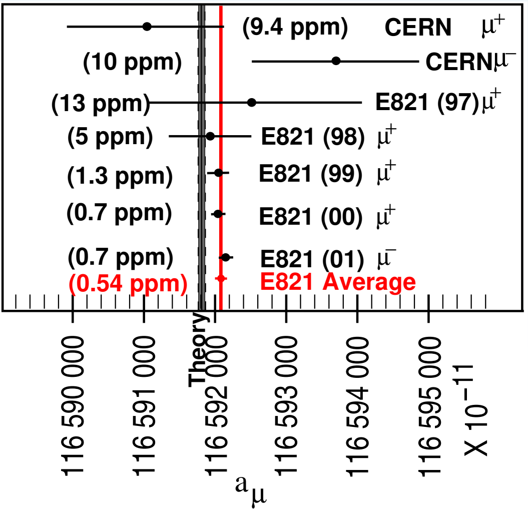
$$a_\mu^{exp} = 116\,592\,089(54)_{st}(33)_{sy}(63)_{tot} \times 10^{-11}$$

$$a_\mu^{SM} = 116\,591\,810(43) \times 10^{-11}$$

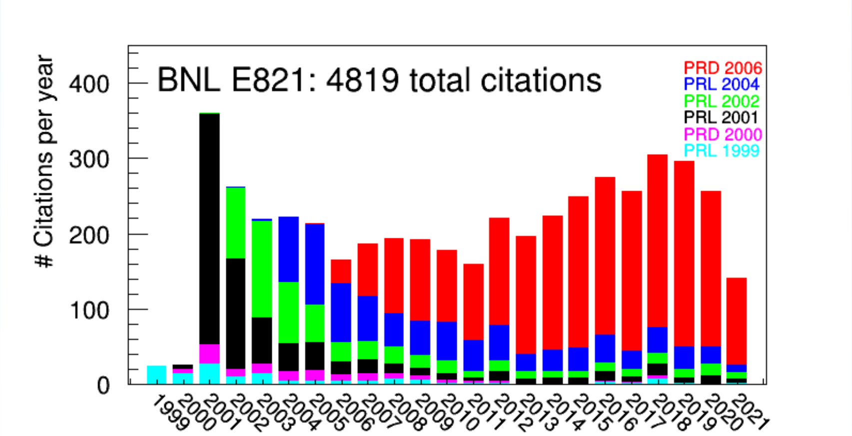
International g-2 theory initiative value

$$\Delta = 279(76) \times 10^{-11} \;\; 3.7\sigma$$

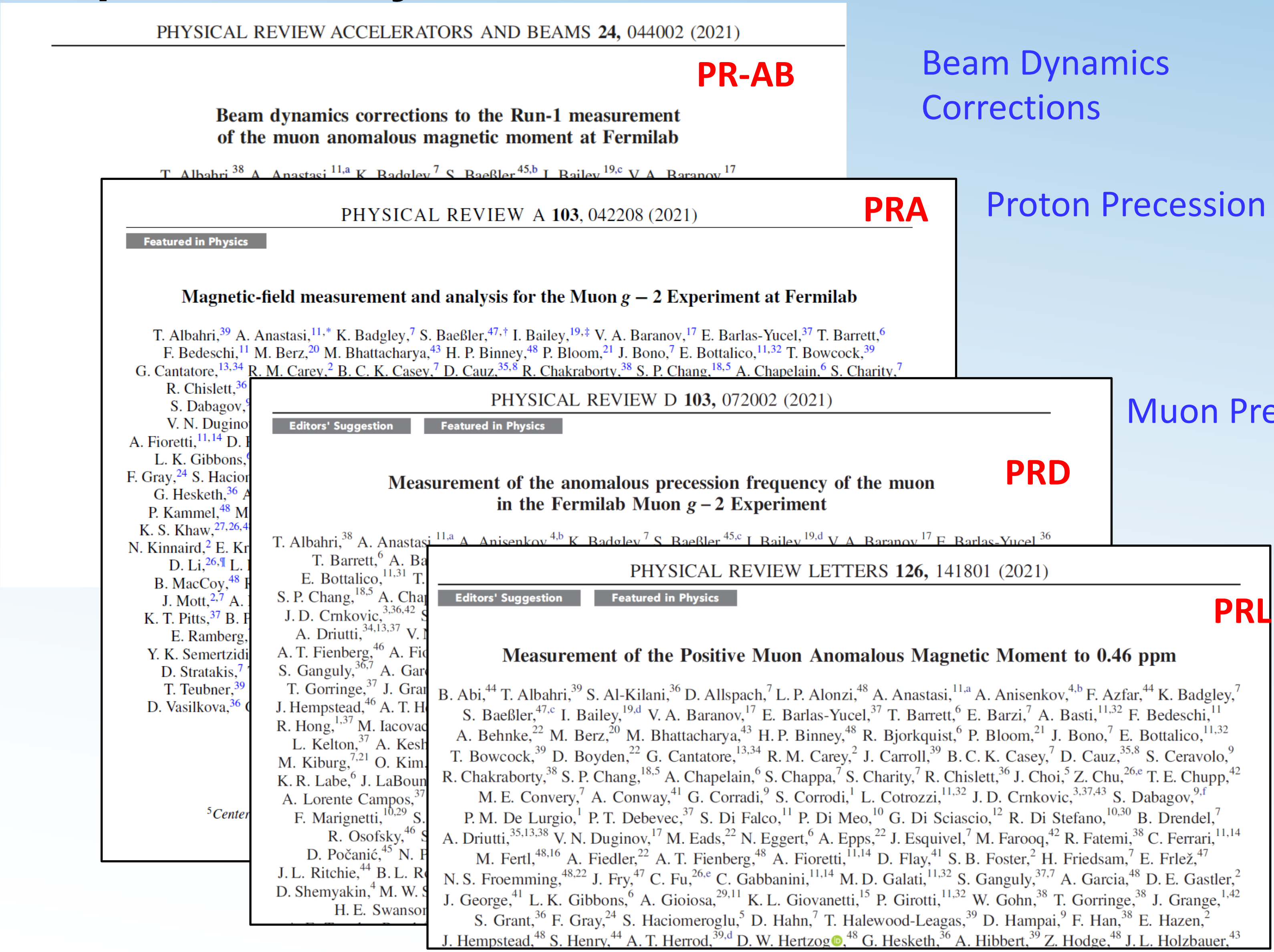
BNL E821 and CERN a_μ



The tension with the SM has been noticed for some time.



The results that I will talk about were published in 4 papers on April 7th … day of result release



PHYSICAL REVIEW ACCELERATORS AND BEAMS **24,** 044002 (2021)

PR-AB

Beam dynamics corrections to the Run-1 measurement of the muon anomalous magnetic moment at Fermilab

T. Albahri,[38] A. Anastasi,[11,a] K. Badgley,[7] S. Baeßler,[45,b] I. Bailey,[19,c] V. A. Baranov,[17]

PHYSICAL REVIEW A **103**, 042208 (2021)

PRA

Featured in Physics

Magnetic-field measurement and analysis for the Muon $g-2$ Experiment at Fermilab

T. Albahri,[39] A. Anastasi,[11,*] K. Badgley,[7] S. Baeßler,[47,†] I. Bailey,[19,‡] V. A. Baranov,[17] E. Barlas-Yucel,[37] T. Barrett,[6]
F. Bedeschi,[11] M. Berz,[20] M. Bhattacharya,[43] H. P. Binney,[48] P. Bloom,[21] J. Bono,[7] E. Bottalico,[11,32] T. Bowcock,[39]
G. Cantatore,[13,34] R. M. Carey,[2] B. C. K. Casey,[7] D. Cauz,[35,8] R. Chakraborty,[38] S. P. Chang,[18,5] A. Chapelain,[6] S. Charity,[7]
R. Chislett,[36]
A. Fioretti,[11,14]
F. Gray,[24]
G. Hesketh,[36]
P. Kammel,[48]
N. Kinnaird,[2]
D. Li,[26,¶]
B. MacCoy,[48]
J. Mott,[2,7]
K. T. Pitts,[37]
T. Teubner,[39]
D. Vasilkova,[36]

PHYSICAL REVIEW D **103,** 072002 (2021)

Editors' Suggestion Featured in Physics

PRD

Measurement of the anomalous precession frequency of the muon in the Fermilab Muon $g-2$ Experiment

T. Albahri,[38] A. Anastasi,[11,a] A. Anisenkov,[4,b] K. Badgley,[7] S. Baeßler,[45,c] I. Bailey,[19,d] V. A. Baranov,[17] E. Barlas-Yucel,[36]
T. Barrett,[6]
E. Bottalico,[11,31]
S. P. Chang,[18,5]
J. D. Crnkovic,[3,36,42]
A. Driutti,[34,13,37]
A. T. Fienberg,[46]
S. Ganguly,[36,7]
T. Gorringe,[37]
J. Hempstead,[46]
R. Hong,[1,37]
L. Kelton,[37]
M. Kiburg,[7,21]
K. R. Labe,[6]
A. Lorente Campos,[37]
F. Marignetti,[10,29]
R. Osofsky,[46]
D. Počanić,[45]
J. L. Ritchie,[44]
D. Shemyakin,[4]

PHYSICAL REVIEW LETTERS **126,** 141801 (2021)

Editors' Suggestion Featured in Physics

PRL

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm

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S. Baeßler,[47,c] I. Bailey,[19,d] V. A. Baranov,[17] E. Barlas-Yucel,[37] T. Barrett,[6] E. Barzi,[7] A. Basti,[11,32] F. Bedeschi,[11]
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M. E. Convery,[7] A. Conway,[41] G. Corradi,[9] S. Corrodi,[1] L. Cotrozzi,[11,32] J. D. Crnkovic,[3,37,43] S. Dabagov,[9,f]
P. M. De Lurgio,[1] P. T. Debevec,[37] S. Di Falco,[11] P. Di Meo,[10] G. Di Sciascio,[12] R. Di Stefano,[10,30] B. Drendel,[7]
A. Driutti,[35,13,38] V. N. Duginov,[17] M. Eads,[22] N. Eggert,[6] A. Epps,[22] J. Esquivel,[7] M. Farooq,[42] R. Fatemi,[38] C. Ferrari,[11,14]
M. Fertl,[48,16] A. Fiedler,[22] A. T. Fienberg,[48] A. Fioretti,[11,14] D. Flay,[41] S. B. Foster,[2] H. Friedsam,[7] E. Frlež,[47]
N. S. Froemming,[48,22] J. Fry,[47] C. Fu,[26,e] C. Gabbanini,[11,14] M. D. Galati,[11,32] S. Ganguly,[37,7] A. Garcia,[48] D. E. Gastler,[2]
J. George,[41] L. K. Gibbons,[6] A. Gioiosa,[29,11] K. L. Giovanetti,[15] P. Girotti,[11,32] W. Gohn,[38] T. Gorringe,[38] J. Grange,[1,42]
S. Grant,[36] F. Gray,[24] S. Haciomeroglu,[5] D. Hahn,[7] T. Halewood-Leagas,[39] D. Hampai,[9] F. Han,[38] E. Hazen,[2]
J. Hempstead,[48] S. Henry,[44] A. T. Herrod,[39,d] D. W. Hertzog,[48] G. Hesketh,[36] A. Hibbert,[39] Z. Hodge,[48] J. L. Holzbauer,[43]

Beam Dynamics Corrections

Proton Precession

Muon Precession

384 Citations to date.

Magnetic Dipole Moments

Magnetic Dipole in a B field:

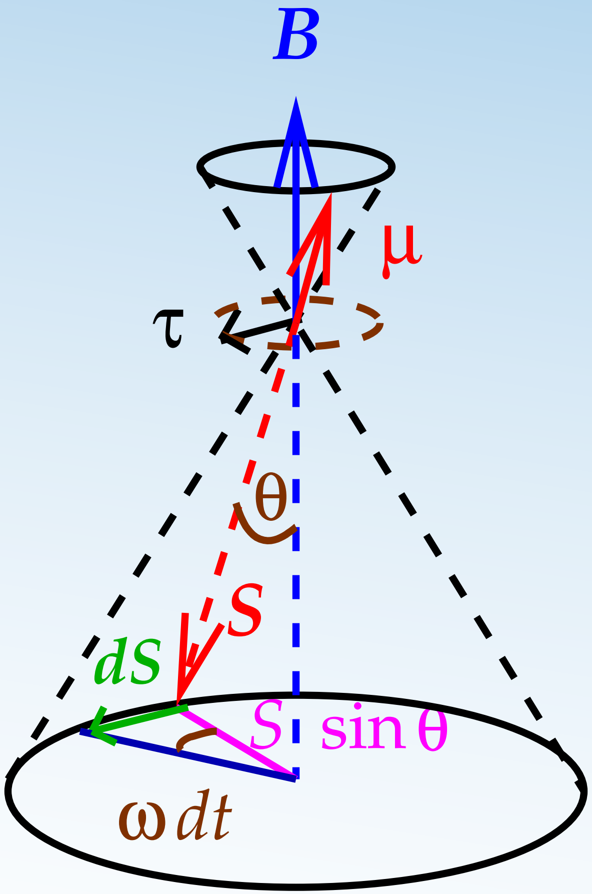
$$\text{Torque}: \tau = \vec{\mu} \times \vec{B}$$

$$\text{Energy}: H = -\vec{\mu} \cdot \vec{B}$$

μ_S for a particle with spin:

$$\vec{\mu}_s = g_s \frac{q}{2m} \vec{s} \quad g_s = 2(1+a)$$

Larmor precession



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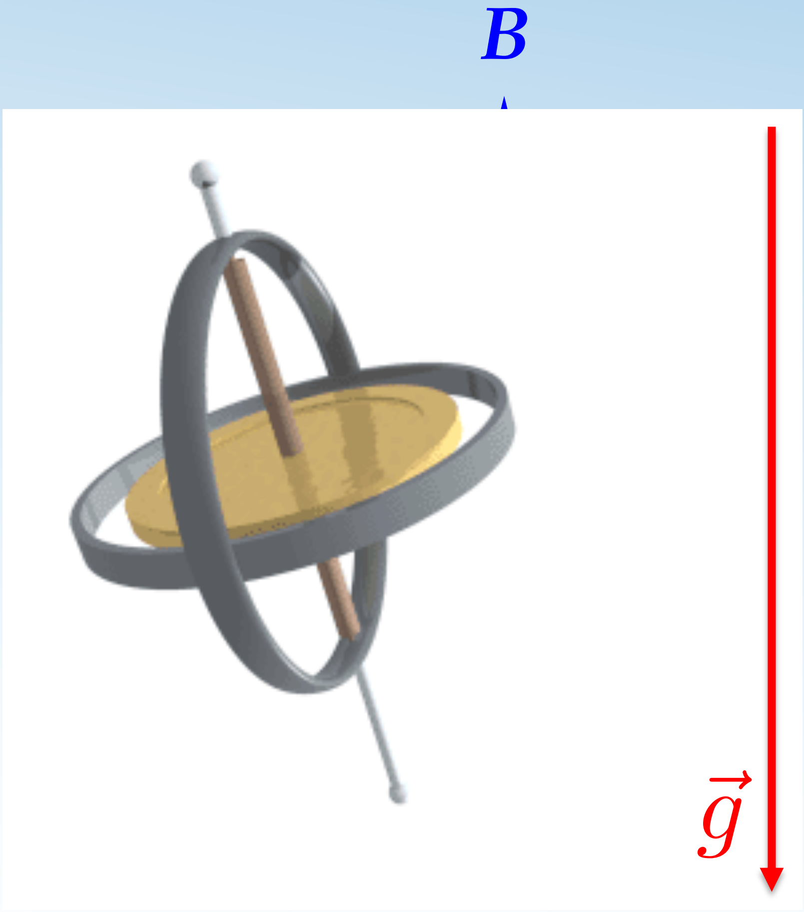
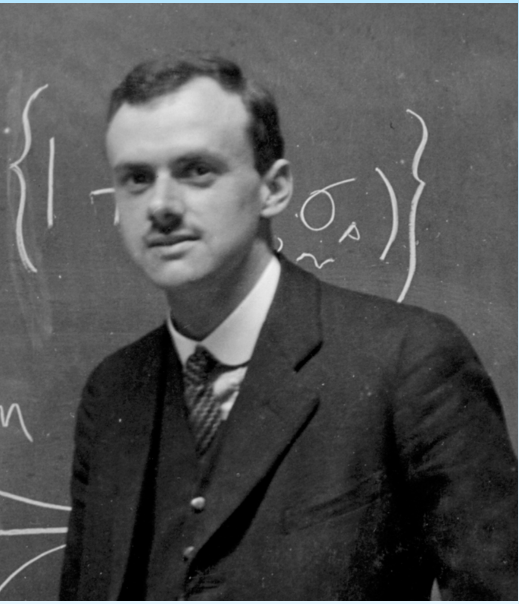
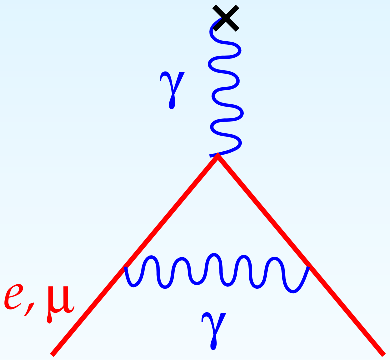


The next big step



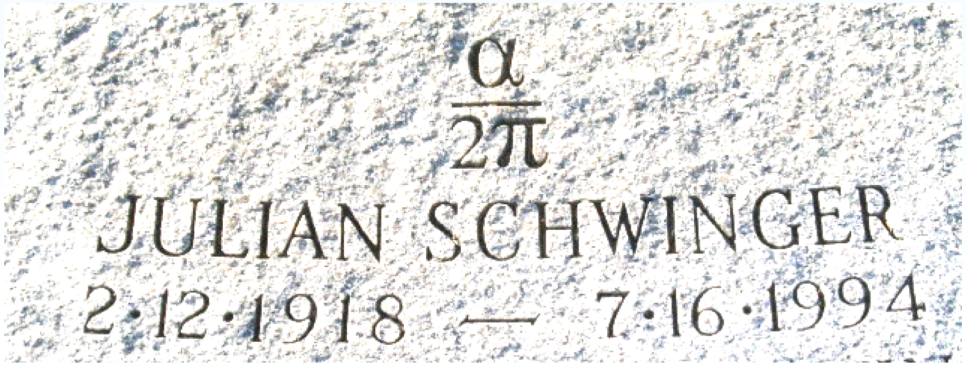
$$i(\partial_\mu - ieA_\mu(x))\gamma^\mu\psi(x) = m\psi(x)$$

$$g = 2\left(1+\frac{\alpha}{2\pi}\right)+\cdots\Big)$$



The very first radiative correction in quantum electrodynamics (QED).

It's mass independent!



Dirac Eqn.

$$i\hbar\frac{\partial\psi}{\partial t} = \left\{\vec{\alpha}\cdot\left(c\frac{\hbar}{i}\nabla - e\vec{A}\right) + \beta c^2 m + eA_0\right\}\psi$$

$$\langle\ell(p')|J_\mu(0)|\ell(p)\rangle = \bar{u}(p')\left\{\boxed{\gamma_\mu F_1(q^2)} + \boxed{\frac{i}{2m_\ell}\sigma_{\mu\nu}q^\nu F_2(q^2)} + \boxed{\gamma_5\sigma_{\mu\nu}q^\nu F_3(q^2)}\right\}u(p)$$

$\text{Dirac}: \quad F_1(0) = 1$

$F_2(0) = a_\ell$ the magnetic anomaly

Chiral changing, flavor conserving interaction.

$F_3(0) = d_\ell$ the electric dipole moment

$\cancel{P}$ $\cancel{T}$ and thus $\cancel{CP}$

At Fermilab we measured a_μ to 0.46 ppm. The new world average is 0.35 ppm (the original BNL goal).

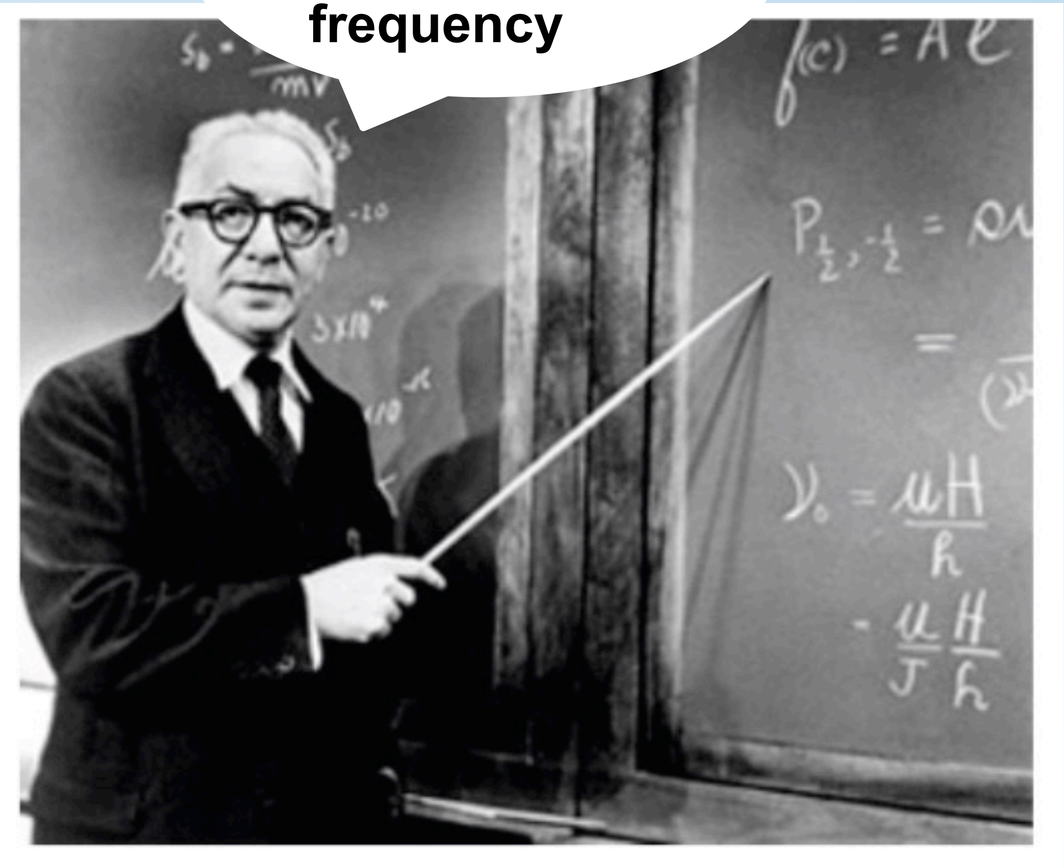
At BNL: $|d_\mu| < 1.8 \times 10^{-19}\ e\,\text{cm}$

Measuring Lepton Magnetic Dipole Moments

$$\vec{\mu} = g\left(\frac{q}{2m}\right)\vec{s}$$

$$\boxed{\begin{aligned}\sigma^{a_\mu}_{\mathrm{BNL}} &= 540\,\mathrm{ppb}\\ \sigma^{a_\mu}_{\mathrm{FNAL}} &= 140\,\mathrm{ppb}\end{aligned}}$$

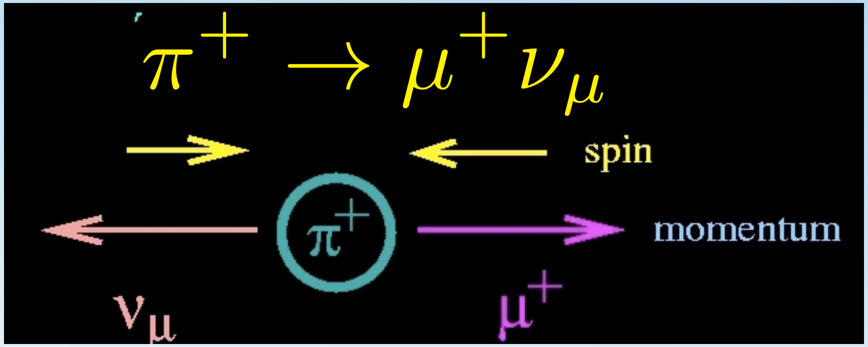
I.I. Rabi

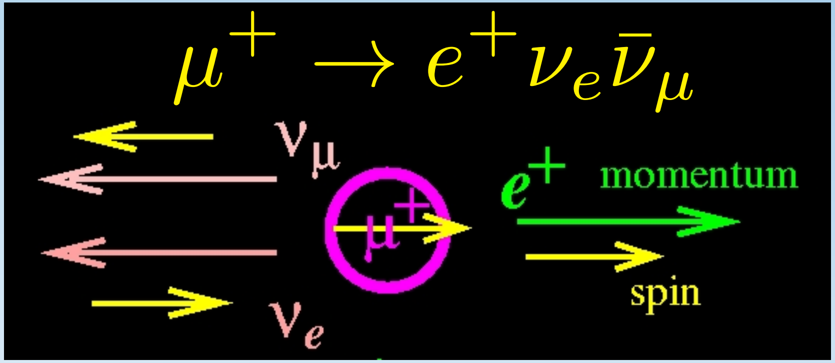


Never measure anything but frequency

How do we measure the muon frequency? Parity violation!

- We need polarized muons: **neutrinos are only left-handed**

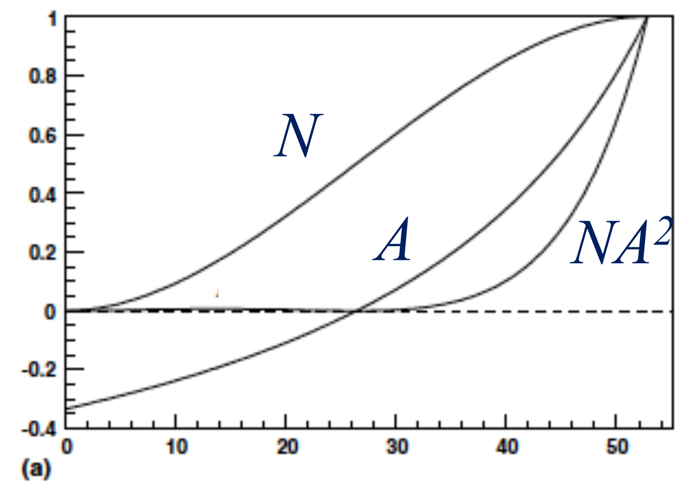




At rest: 100% polarization. In beam: very forward and very backward μ are polarized

At rest: highest energy e^+ are along μ spin.

Integral N, A, NA^2



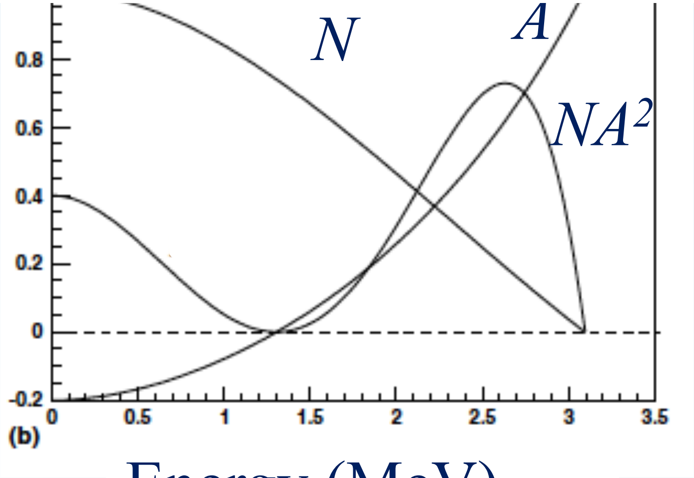
Energy (MeV)

At rest

N = Number

A= Asymmetry

FOM: NA^2



Energy (MeV)

In flight

Spin Motion in a Magnetic Field

$$\text{Larmor}: \quad \vec{\omega}_L = g\left(\frac{q}{2m}\right)\vec{B}$$

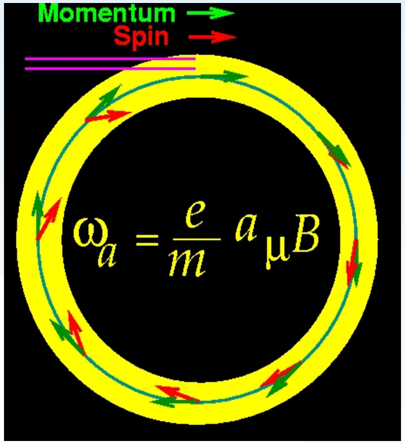
Particle: $q = \pm|e|$ moving in a magnetic field:

momentum turns with ω_{C}, spin turns with ω_{S}

$$\omega_C = -\frac{qB}{m\gamma}; \qquad \omega_S = -g\frac{q}{2m}B - (1-\gamma)\frac{qB}{\gamma m}$$

Spin turns relative to the momentum with ω_{a}

$$\omega_a = \omega_S - \omega_C = -\left(\frac{g-2}{2}\right)\frac{q}{m}B = -\boxed{a_\mu}\frac{q}{m}B$$



If $g = 2$, the spin will follow the momentum

We use a Uniform dipole magnetic field.
Provide vertical focusing with electrostatic quadrupoles →
Weak Focusing Betatron, but is a **Penning Trap** configuration.

Full spin equation with *E* and *B*:

$$\vec{\omega}_C = -\frac{q}{m}\left[\frac{\vec{B}}{\gamma} - \frac{\gamma}{\gamma^2-1}\left(\frac{\vec{\beta}\times\vec{E}}{c}\right)\right]$$

motional magnetic field

Cyclotron Frequency

$$\vec{\omega}_S = -\frac{q}{m}\left[\left(\frac{g}{2}-1+\frac{1}{\gamma}\right)\vec{B} - \left(\frac{g}{2}-1\right)\frac{\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(\frac{g}{2}-\frac{\gamma}{\gamma+1}\right)\left(\frac{\vec{\beta}\times\vec{E}}{c}\right)\right]$$

Thomas Equation: L. H. Thomas, Philos. Mag. 3, 1 (1927)

J.D. Jackson 3rd edition, p. 564.

note that : $\frac{g}{2} - 1 = a$

$$\omega_S - \omega_C =$$

$$\omega_a \cong -\frac{q}{m}\left[a_\mu\vec{B} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta} - \cancelto{0}{\left(a_\mu - \frac{1}{\gamma^2-1}\right)}\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

vertical betatron pitching motion

The “magic” $\gamma_m = 29.3$

We measure two frequencies: ω_a and $\omega_p'(B)$

$$\boxed{\vec{\omega}_a} = -\frac{q}{m}\left[a\boxed{\vec{B}} - \cancelto{\simeq 0}{\left(a - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}}\right]$$

$$\text{with } \gamma_{\text{magic}} = 29.3, \ p_{\text{magic}} = 3.09 \text{ GeV/c}$$

B is expressed in terms of the Larmor frequency of a <u>shielded</u> proton, *averaged over the muon distribution.*

Blind Analysis: ω_a ω_p

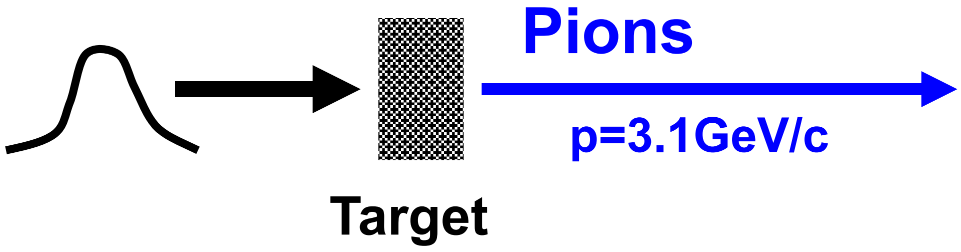
- When analyzing data, you must make choices. If you know which way these choices move your answer, you can bias the result.
- How do we remove this bias? E989 blinding scheme
  - The absolute clock frequency is blinded with a ± 30 ppm offset!
  - We had two independent teams analyzing ω_p
  - We had six independent people analyzing ω_a
  - Initially the separate analyzers are blind to each other.
  - For a reality check, once things are relatively mature, analyzers can relatively unblind to each other to see how it's going.
- Only when everything is complete, do we get the true clock frequency from people outside of the collaboration and "open the box".
- We opened the box on 25 February. Everyone present signed and embargo agreement.
- Nobody leaked the answer!!!!

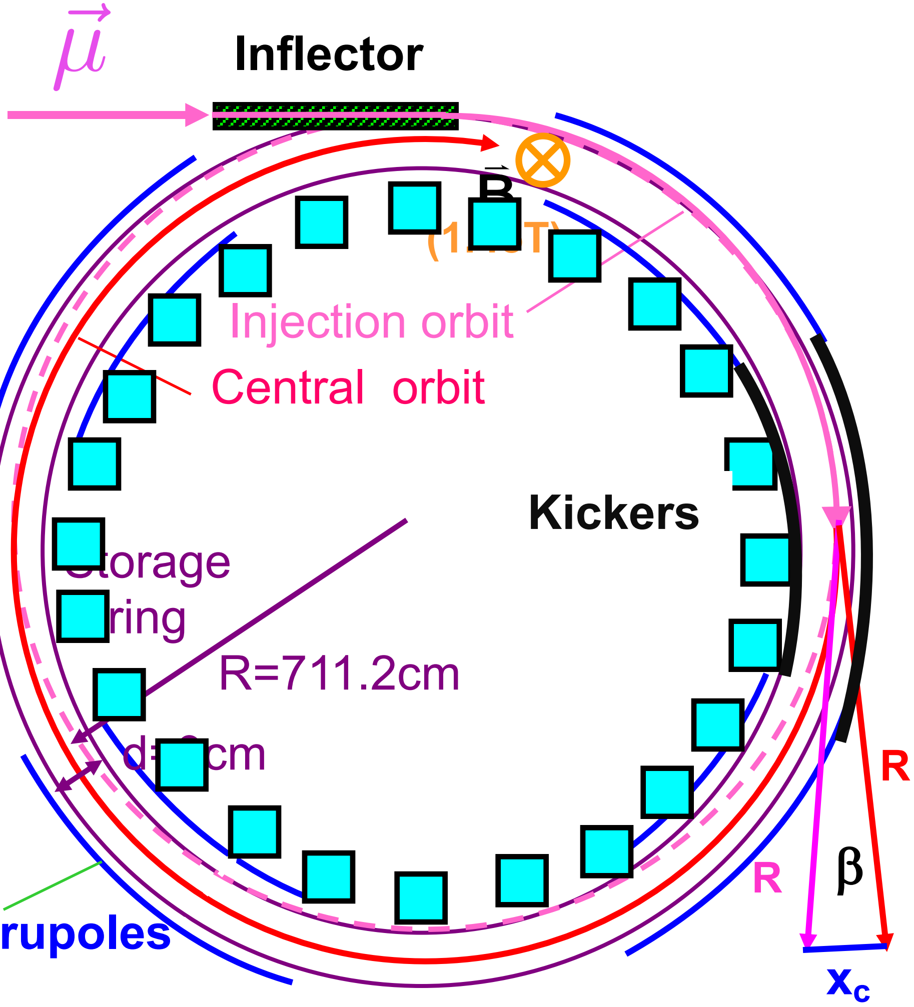
Experimental Technique

Muon lifetime $\gamma\tau_\mu$ = 64.4 μs
(g-2) period τ_a = 4.37 μs
Cyclotron period τ_C = 149 ns

narrow bunch of protons

$x_c \approx$ 77 mm
$\beta \approx$ 10 mrad
B·dl ≈ 0.1 Tm





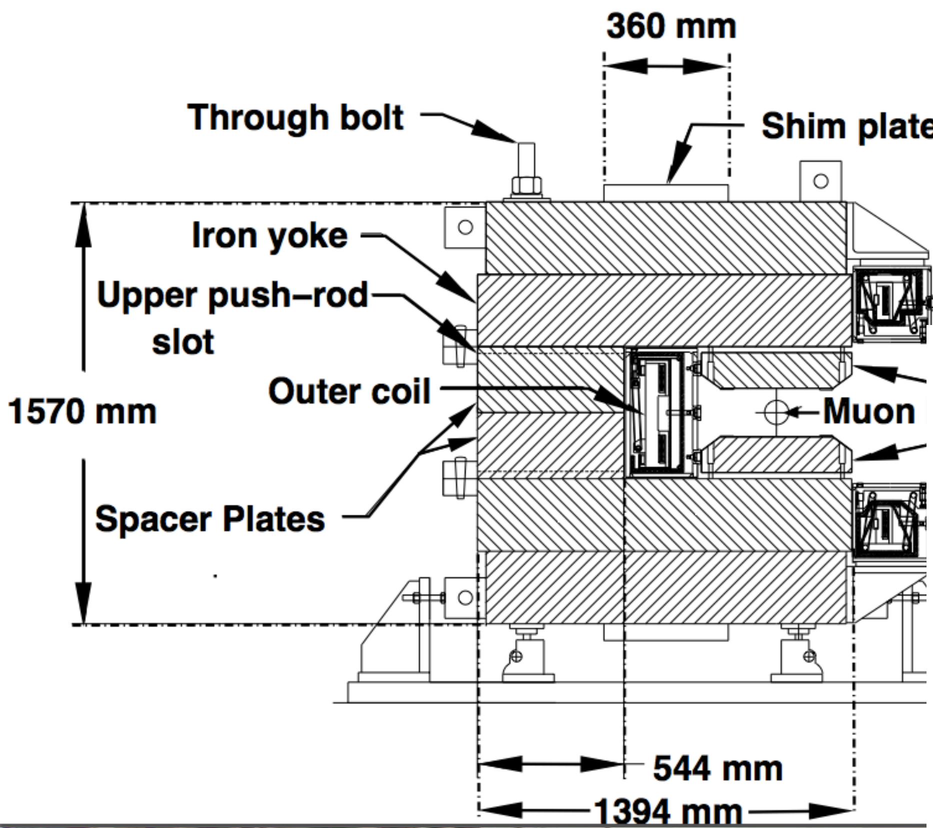
- Muon storage ring – weak focusing betatron
- Muon polarization
- Injection & kicking
- Focus with electric quads
- 24 electron calorimeters (E, t)

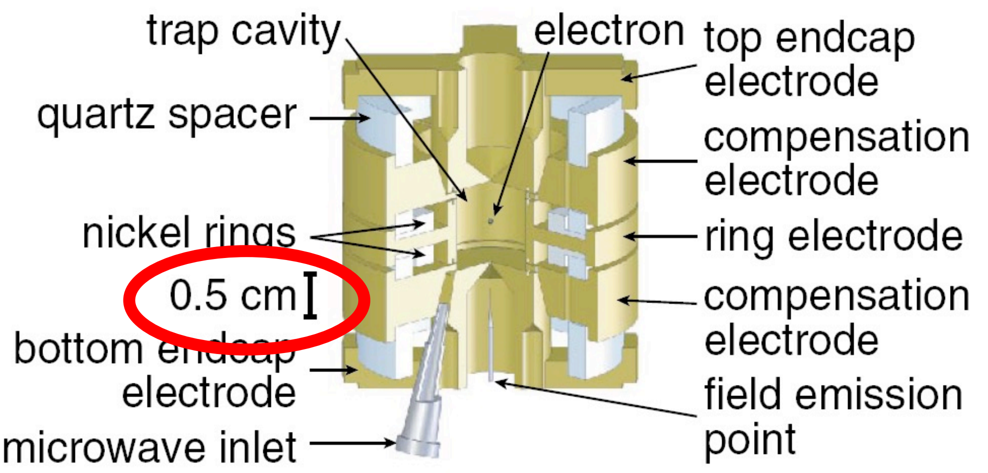
$$\vec{\omega}_a = -\frac{Qe}{m} a_\mu \vec{B}$$

Electric Quadrupoles

The E821 Muon storage ring magnet at Brookhaven Lab circa 1996. Was moved to Fermilab for E989

The E821 Muon storage ring magnet at Brookhaven Lab circa 1996. Was moved to Fermilab for E989

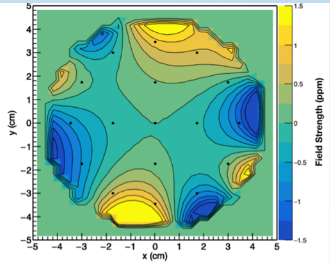




The Harvard electron penning trap

What do we need to measure?

- The magnetic field over the 9 cm diameter, 44 m long toroid where the muon beam is stored.

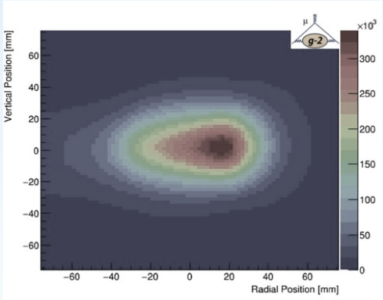


$$\langle B\rangle_{\mathrm{azimuth}}$$

$$\omega'_p$$

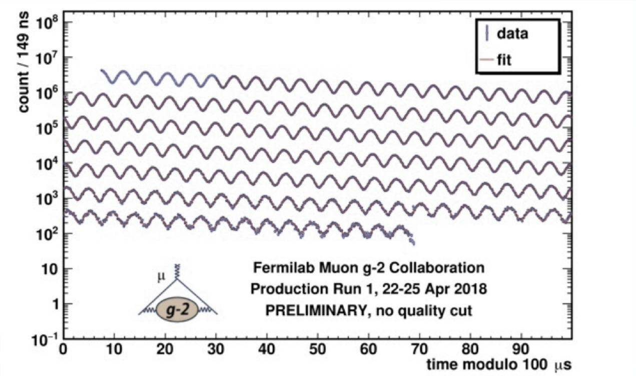
$$a_\mu \propto \frac{\omega_a}{\langle \omega'_p \times M_\mu \rangle}$$

The distribution of muons, which must be folded with the magnetic field to determine the average field experienced by the muons.



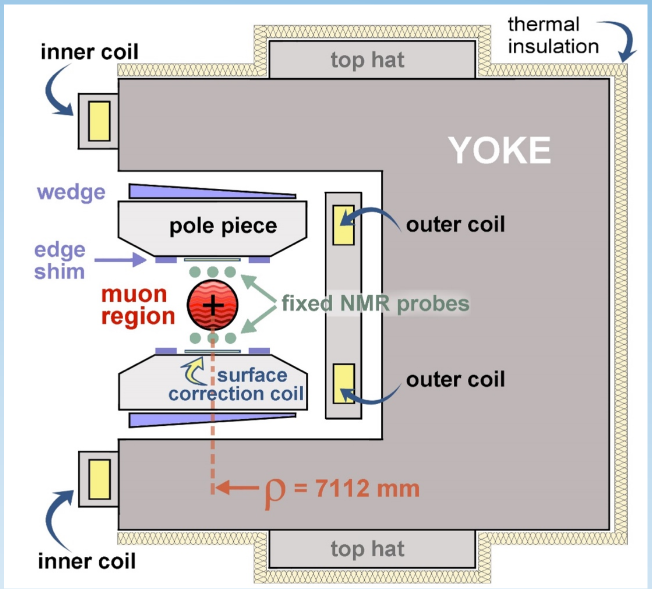
$$M_\mu(\vec{r})$$

The muon spin rotation frequency

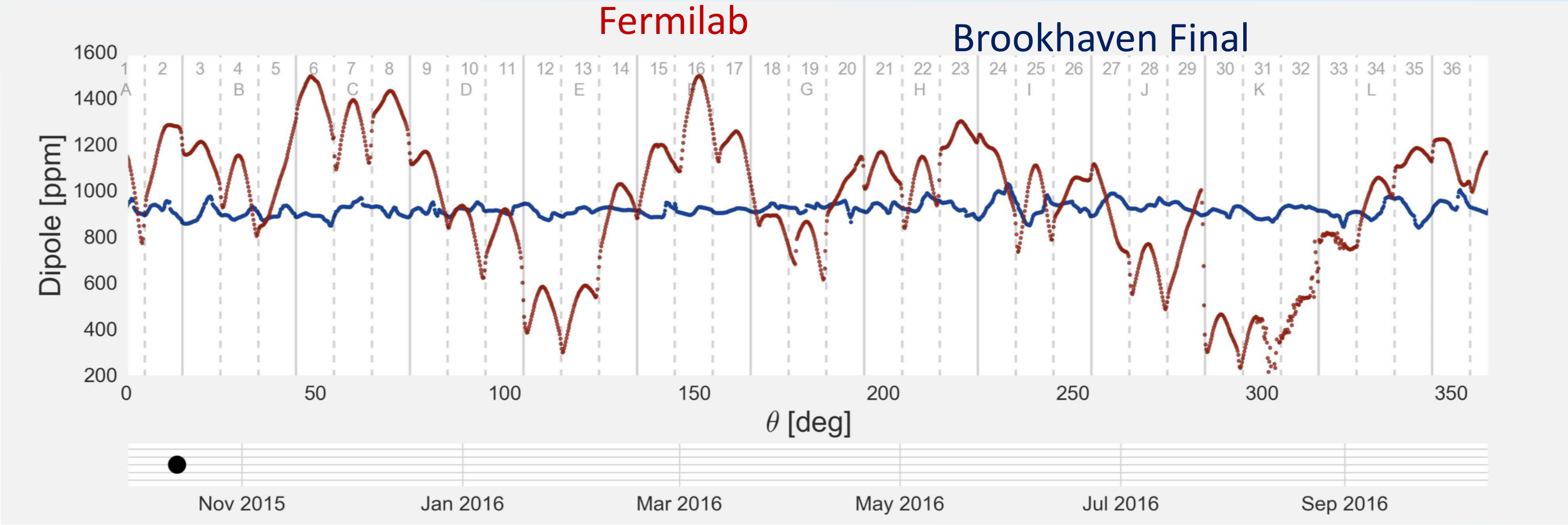


$$\omega_a$$

Magnet Design & Shimming

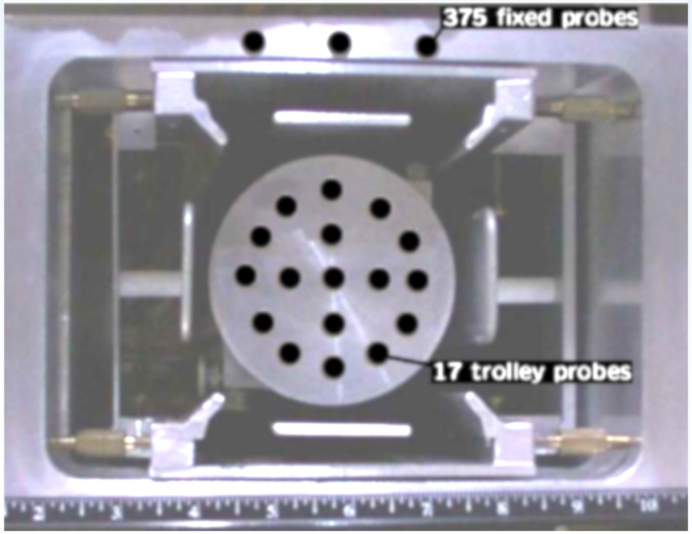


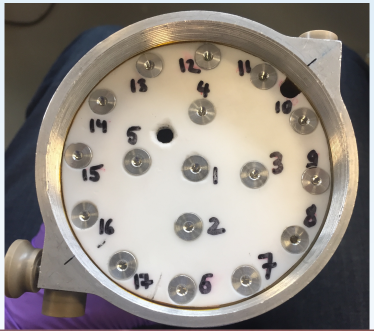
- 14.2 m diameter “C”-shape magnet with 1.45 T vertical field
- **Shimming** campaign from **2015-16** resulted in very uniform field
- **14 ppm RMS** across full azimuth & 3 × better than at BNL
- Beam confined to a 9 cm diameter 44 m circumference toroid.



We map the magnetic B field with the NMR Trolley

- We have a field mapping trolley that is pulled by cables around the ring to map the field.
- It is parked in a garage inside of the vacuum chamber while muon spin precession data are being collected.
- During data collection we monitor B with 378 “fixed probes” at 72 azimuthal locations

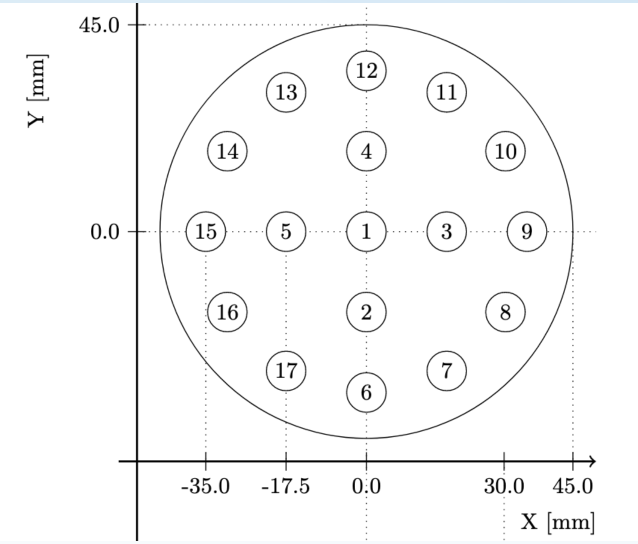




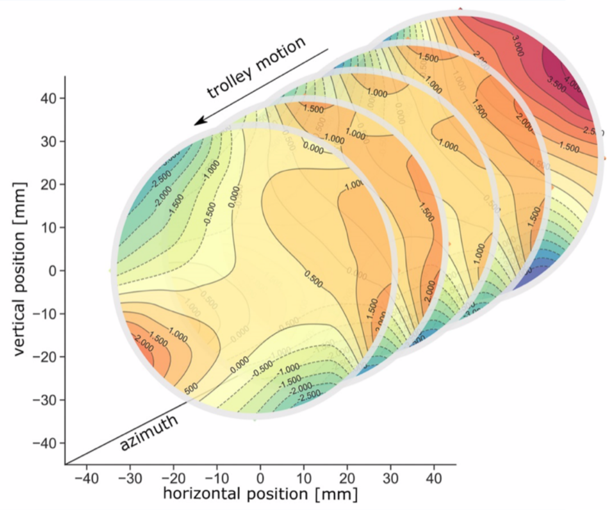
The NMR trolley in a vacuum cha

Measuring the Field: NMR Trolley

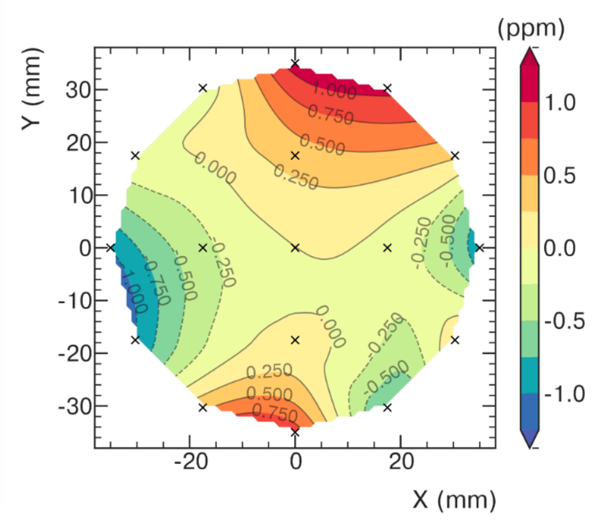
- **In-vacuum trolley** drives around the ring every ~3 days to 9000 azimuthal locations
- Measure B-field from **shielded proton precession** frequency ω'_p (comagnetometer)



17 NMR probes
Optimal arrangement
for multipole analysis



2D field maps
at ~9000 points
around ring

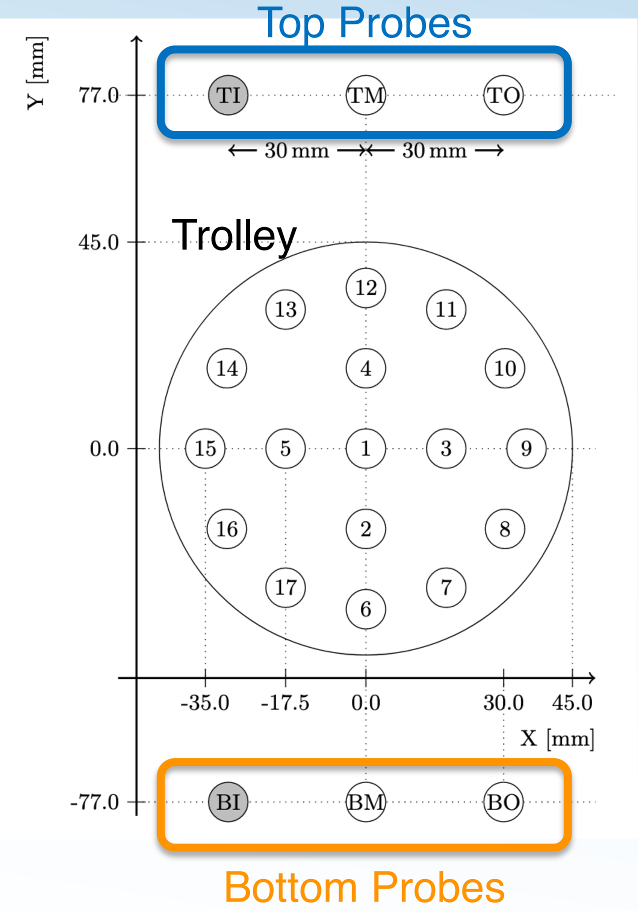


Typical Azimuthal Average

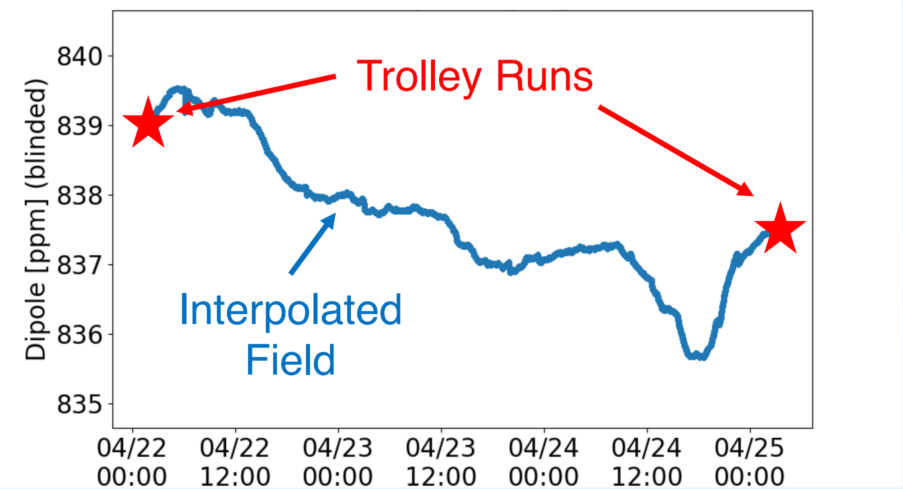
Variation < 1 ppm

Tracking the Field: Fixed Probes

- Can’t keep trolley in the ring while the muons are present
- Instead, monitor field during e^+ data-taking with 378 fixed probes



Example from one location
for one trolley run pair



Use fixed probe data to interpolate field between trolley runs, using a Brownian bridge model[1].

[1] Random walk with fixed endpoints.

The absolute calibration for B

$$a_\mu \propto \frac{\omega_a}{B}$$

Us

Others

$$a_\mu = \boxed{\frac{\omega_a}{\tilde{\omega}'_p(T_r)}}\ \boxed{\frac{\mu'_p(T_r)}{\mu_e(H)}\frac{\mu_e(H)}{\mu_e}\frac{m_\mu}{m_e}\frac{g_e}{2}}$$

ω_a: e$^+$ oscillation frequency

$\tilde{\omega}'_p(T_r)$: magnetic field from precession of protons in $H_2$0, weighted by muon distribution

Run 1 Result: 462 ppb

Final Goal total uncertainty:
140 ppb = 100 ppb (stat)
$\oplus$ 100 ppb (syst)

$\frac{\mu_e(H)}{\mu'_p(T)}$	Measured to 10.5 ppb accuracy at T = 34.7°C Metrologia **13**, 179 (1977)
$\frac{\mu_e}{\mu_e(H)}$	Bound-state QED (exact) Rev. Mod. Phys. **88** 035009 (2016)
$\frac{m_\mu}{m_e}$	Known to 22 ppb from muonium hyperfine splitting Phys. Rev. Lett. **82**, 711 (1999)
$\frac{g_e}{2}$	Measured to 0.28 ppt Phys. Rev. A **83**, 052122 (2011)

Total < 25 ppb

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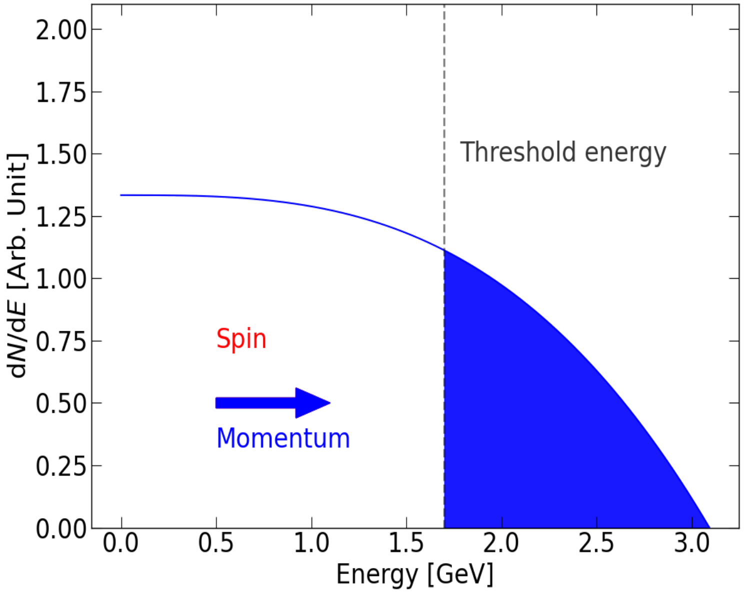
ω_a $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$

- The highest energy positrons are correlated with muon spin.
- Count $e+$ hitting calos above threshold; or weight the hits by the decay asymmetry $A(E)$.
- In beam, as the spin rotates forward and backward, the number of e^+ is modulated by ω_a

$$N(t) = N_0 \cos(\omega_a t + \phi)$$

$$\text{what if } \phi = \phi(t)?$$

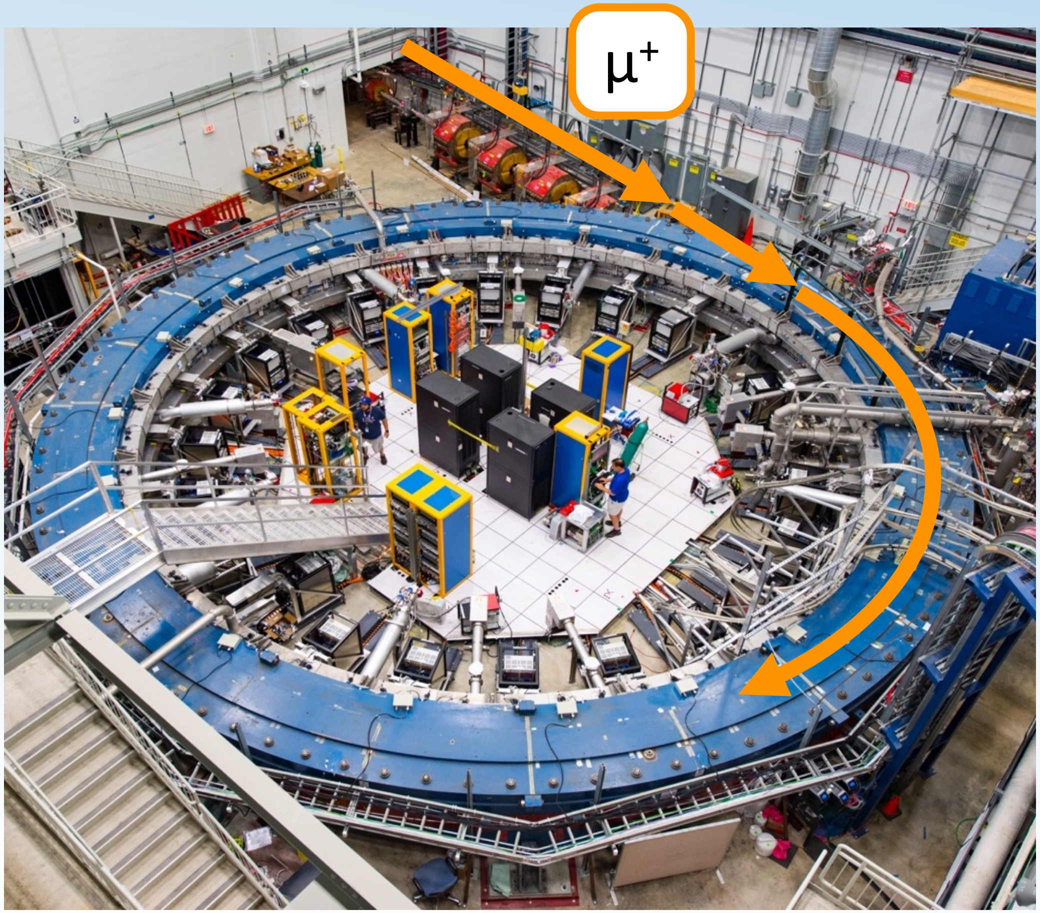
The e^+ energy oscillates with the spin

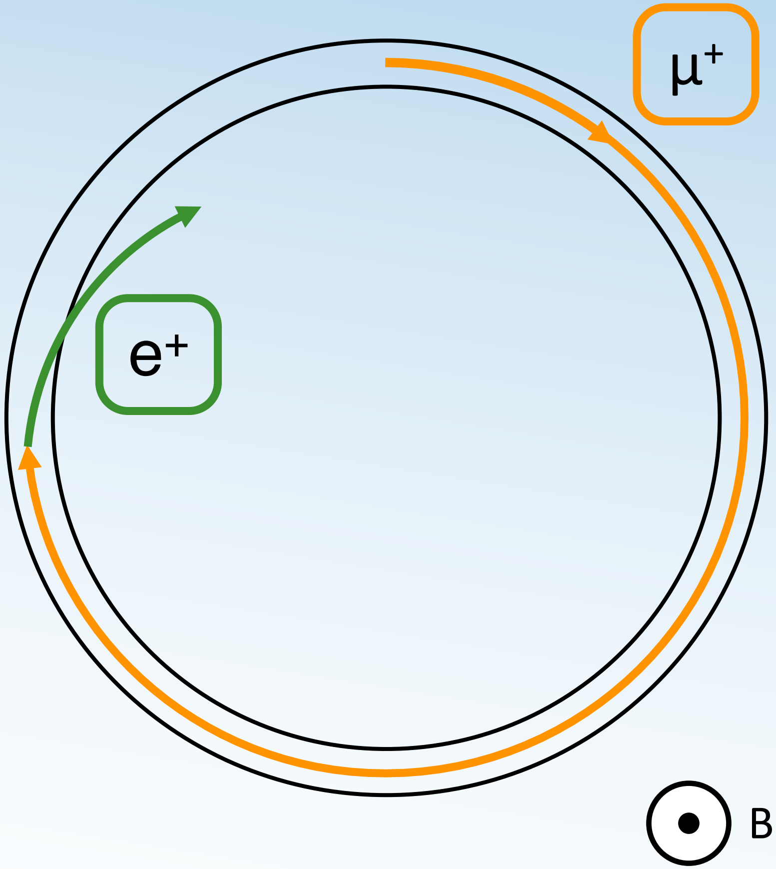


Real World Experiment: Positrons

$$\mu^+ \rightarrow e^+ + \bar{\nu}_e + \bar{\nu}_\mu$$

- Experiment measures decay e^+ which curl inwards since they have lower momentum

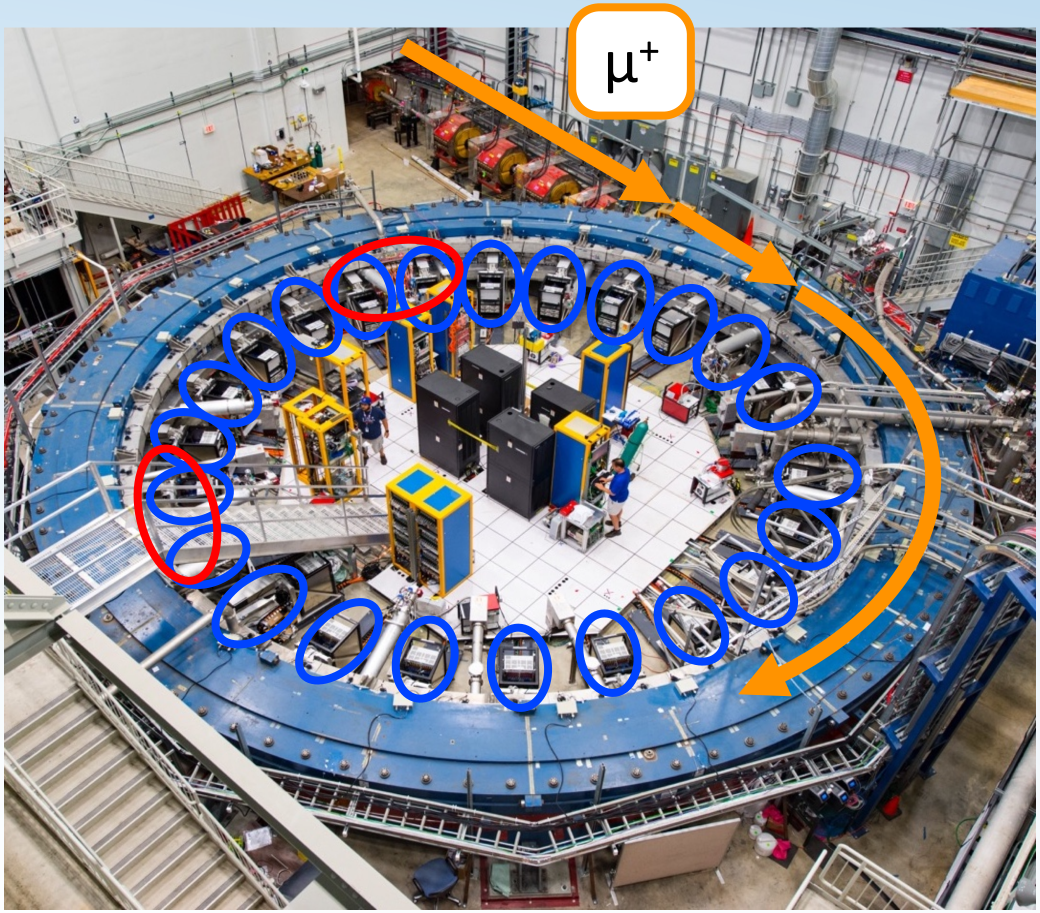


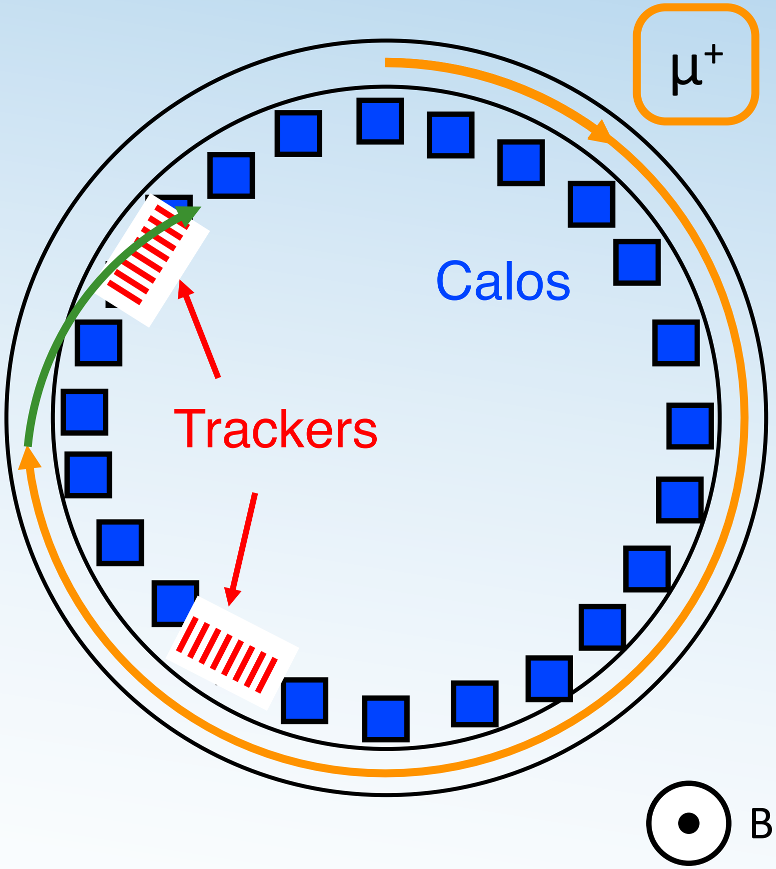


Real World Experiment: Detectors

- Decay e+ are measured by **24 calorimeters**

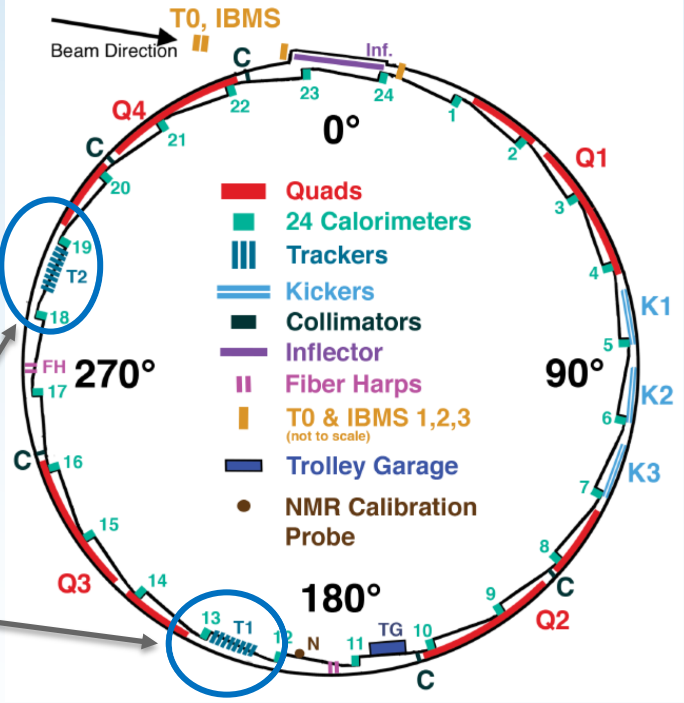
& **2 trackers**





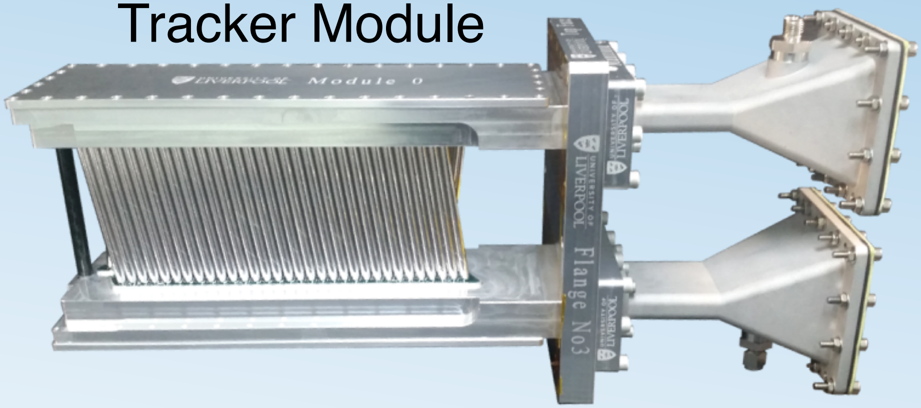
Muon Distribution M_μ

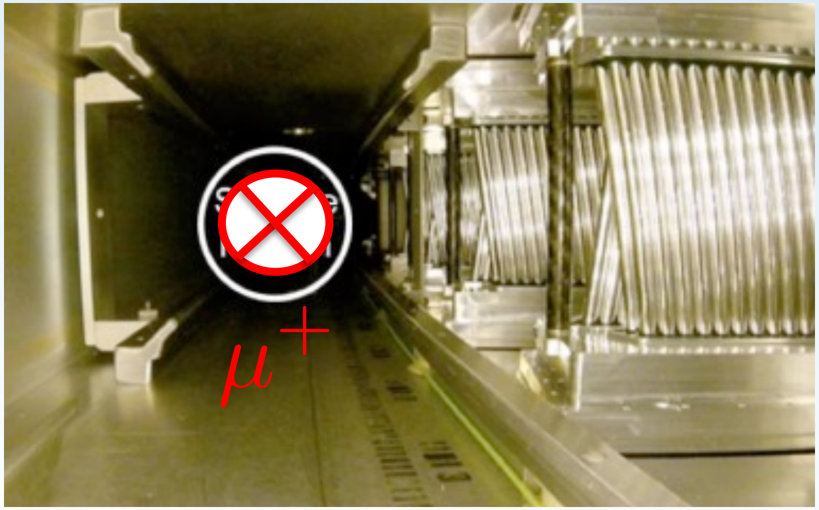
- Want the **field experienced by muons**, so need to know **where muons are** in the field map
- Measured with **two straw trackers** inside storage vacuum



Trackers

Tracker Module

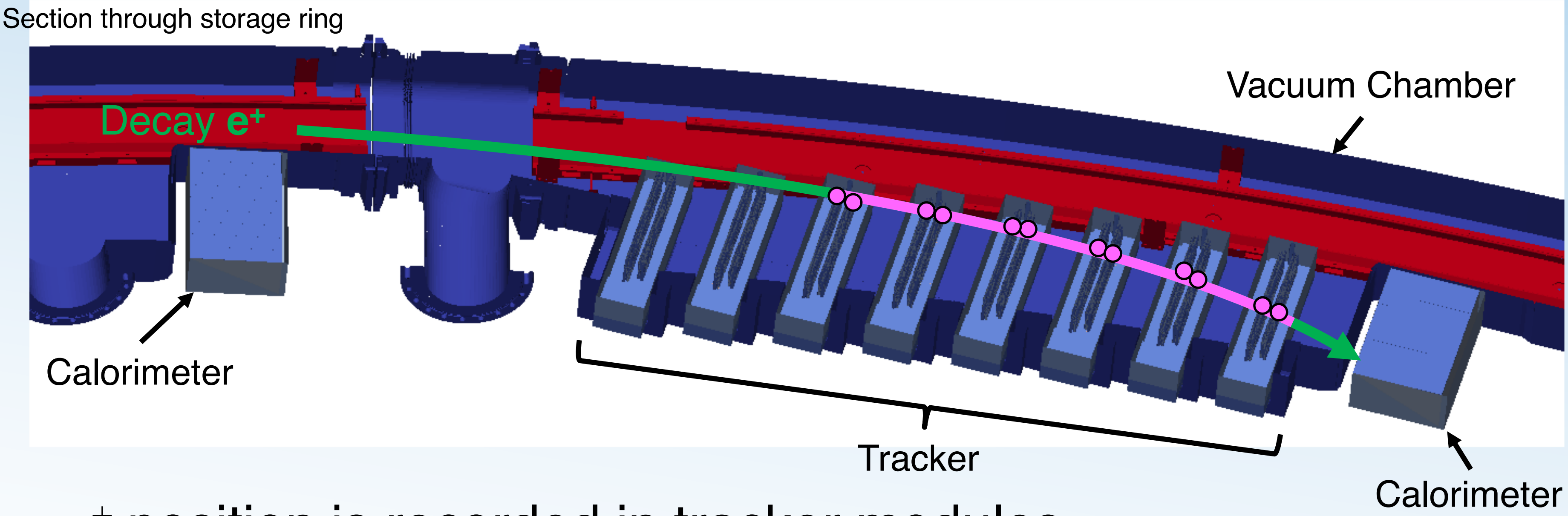




Muon's view of a tracker

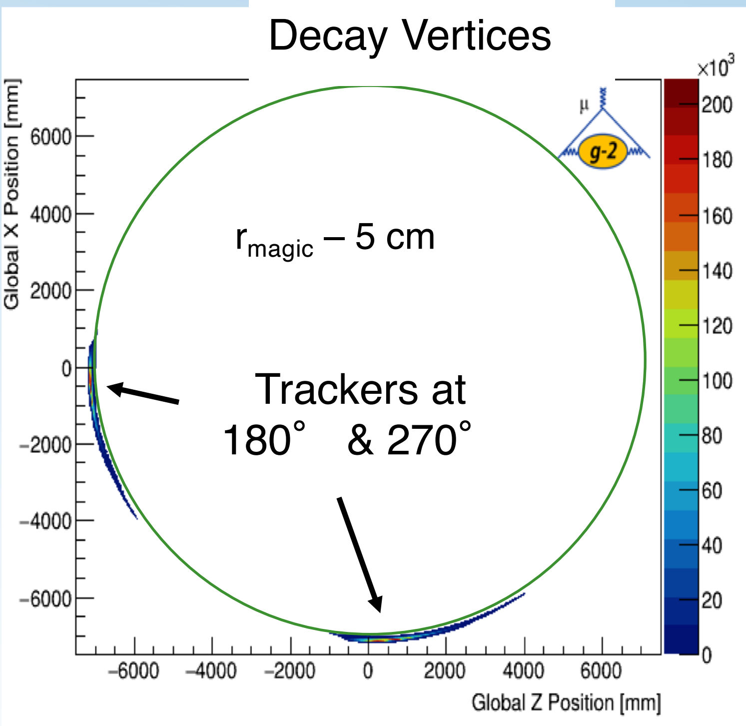
Tracker: Top view

- A muon decays to a positron which travels through tracker

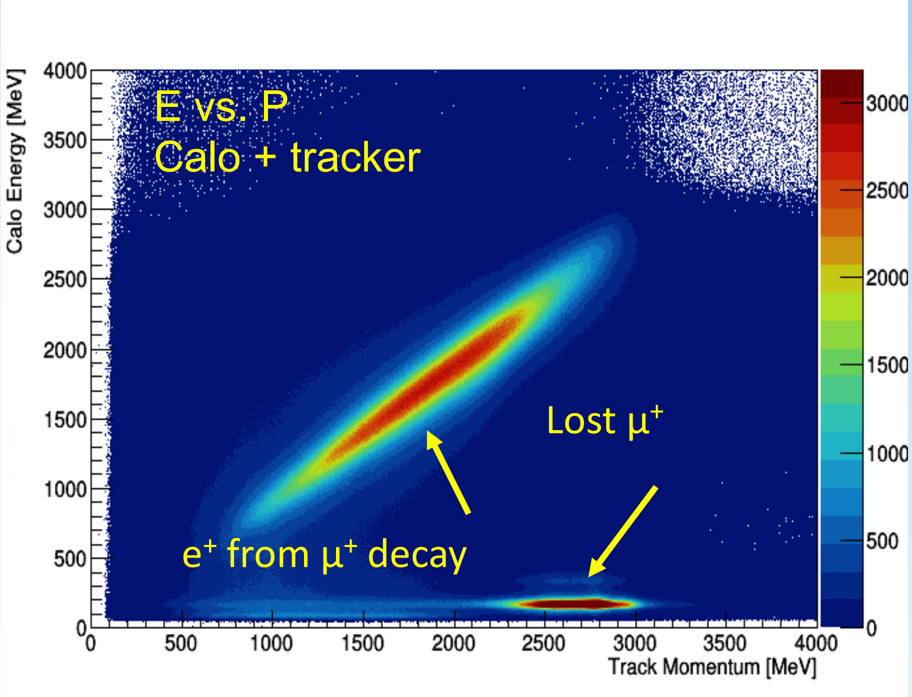


- e^+ position is recorded in tracker modules
- Hits are grouped and reconstructed into a track
- Track is extrapolated backwards to beam storage region

Run 1 Data from the Tracker



Top-down view of decay vertices



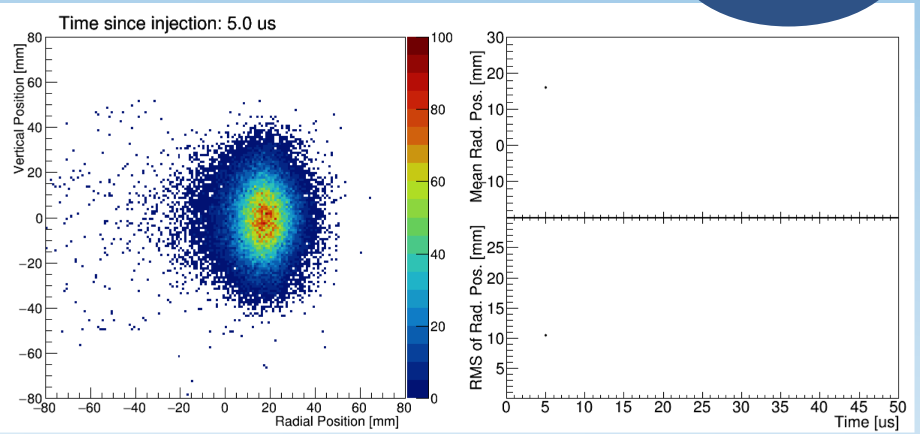
Extrapolate tracks back to point of tangency to get beam distribution:

Main Systematic Issues

- 3 main systematics for ω_a measurement
- Variety of mitigation strategies
- Well under control – total is **56 ppb**

Beam Oscillations

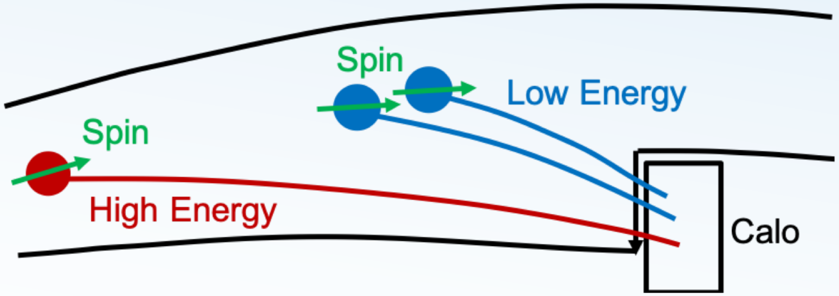
~40 ppb



Tracker data & beam dynamics

Pile Up

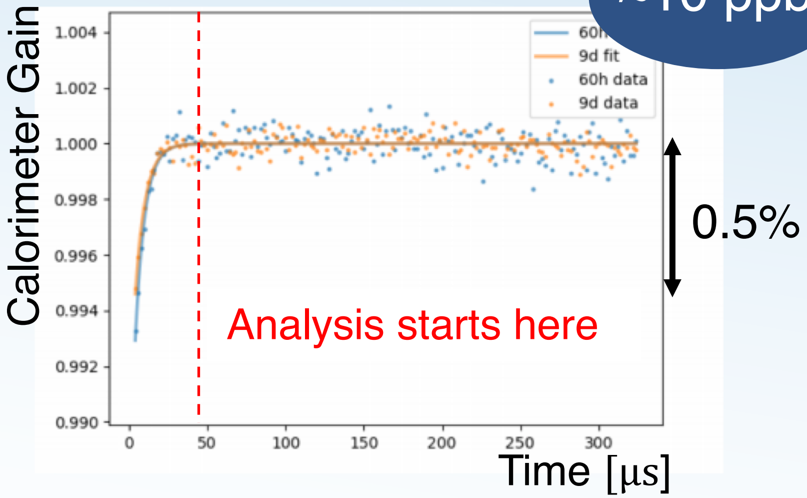
~30 ppb



Empirical correction using calo data

Gain Change

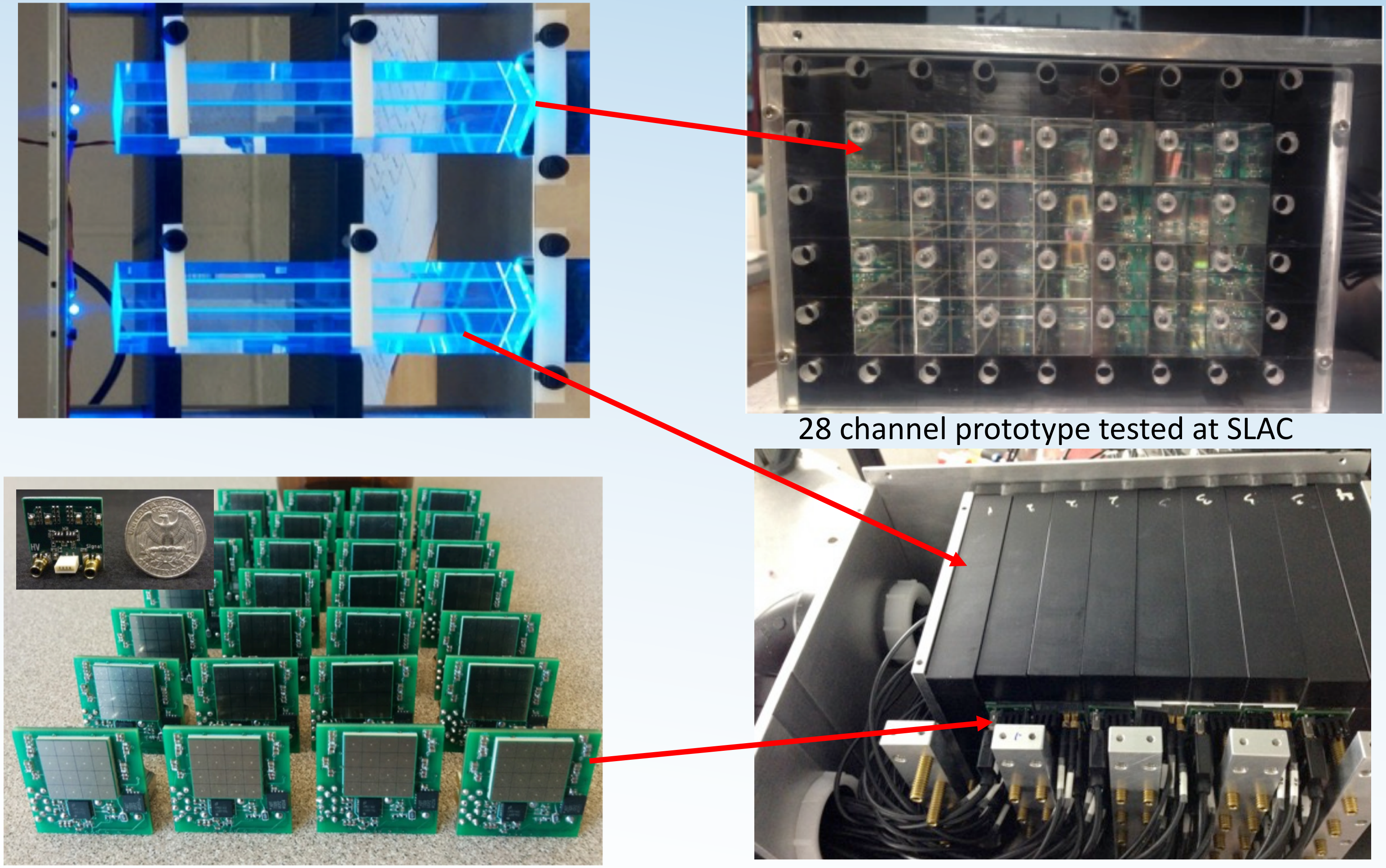
~10 ppb



Dedicated laser calibration system

Calorimeter Design

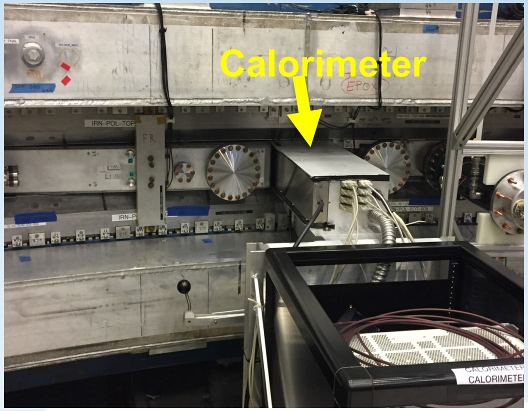
- Array of 54 PbF_2 crystals - 2.5 x 2.5 cm^2 x 14 cm ($15X_0$)
- Readout by SiPMs to continuous 800 MHz WFDs (1296 channels)



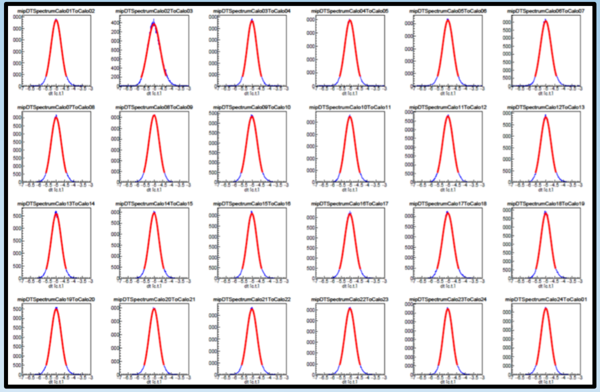
28 channel prototype tested at SLAC

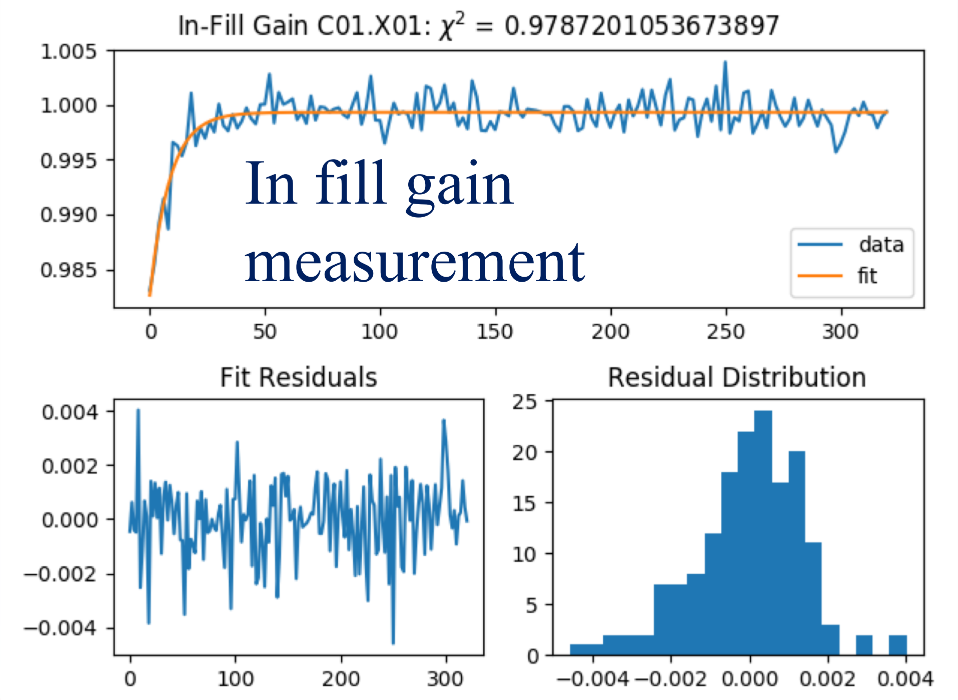
Detector Overview

- **Calorimeters** (x24) – E&t of decay positrons
  - 6 × 9 array of 2cm PbF2 crystals

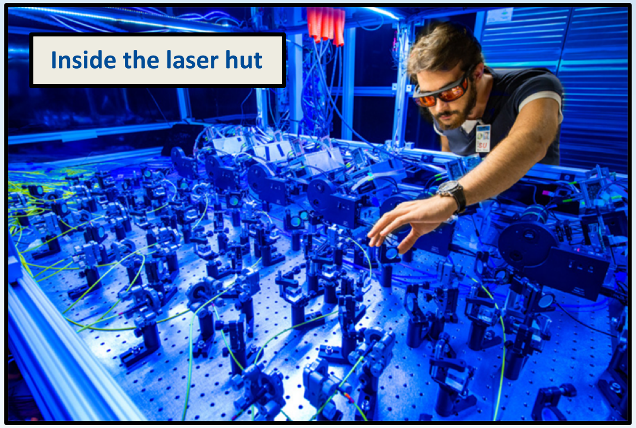


Relative time alignment to 5 ps using a laser system





In fill gain measurement

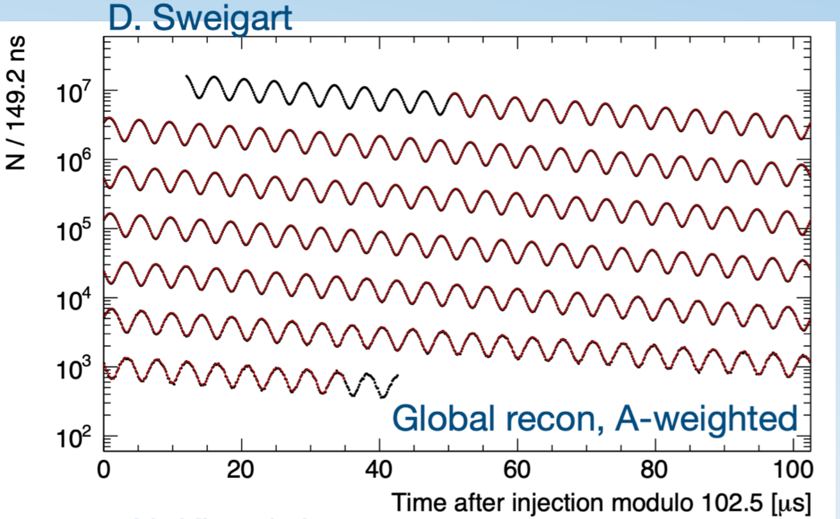


Inside the laser hut

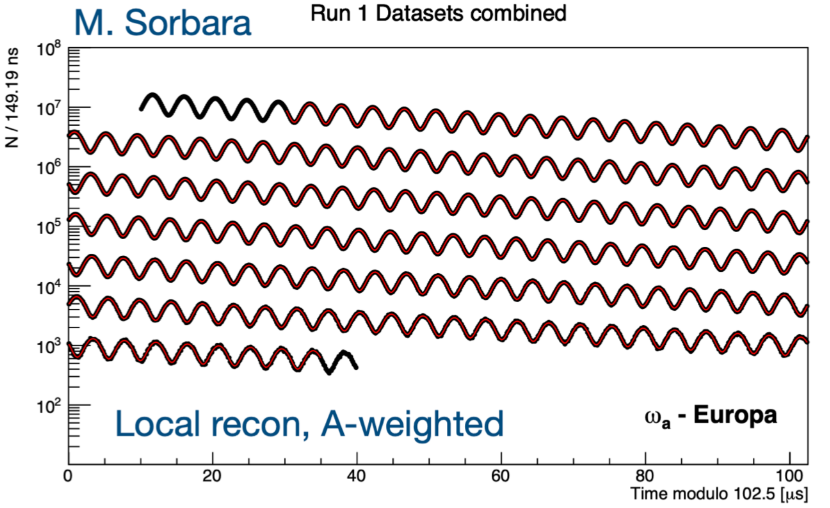
> 5 ns beam pileup separation

Data from the Calorimeters and tracker, $p_e > 1.8$ GeV/c

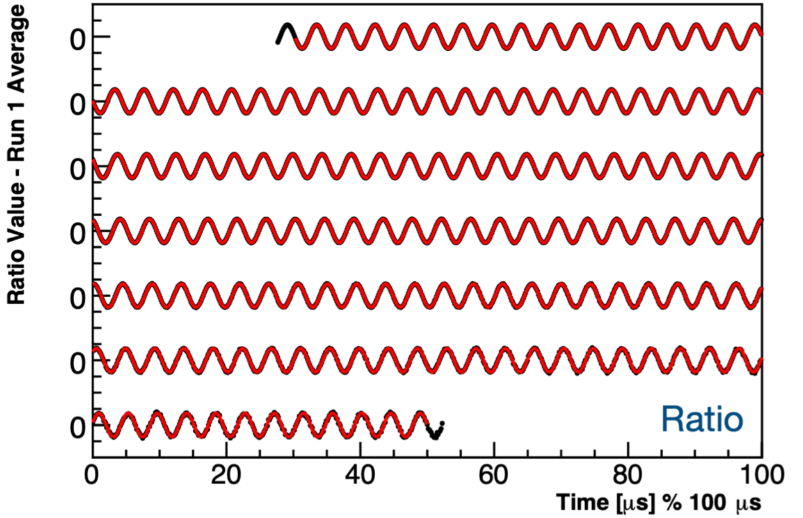
D. Sweigart



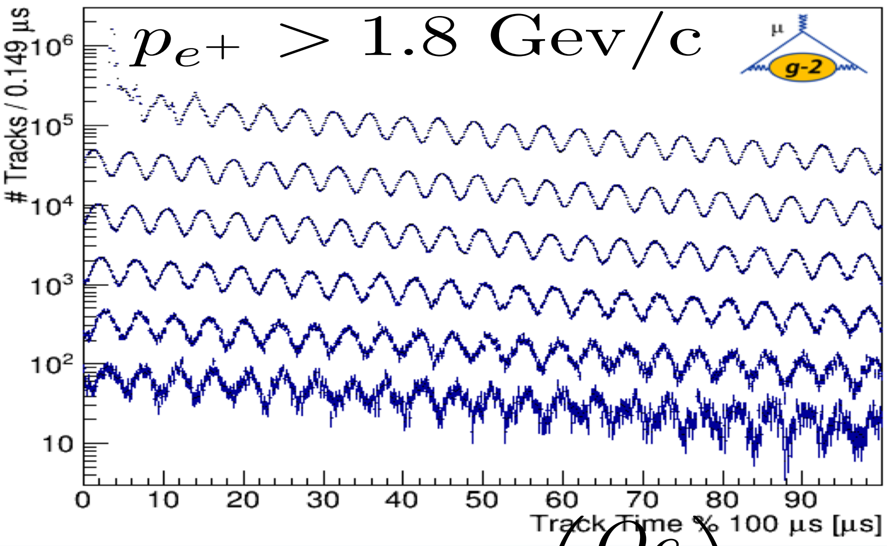
M. Sorbara



N. Kinnaird



J. Mott Tracker data



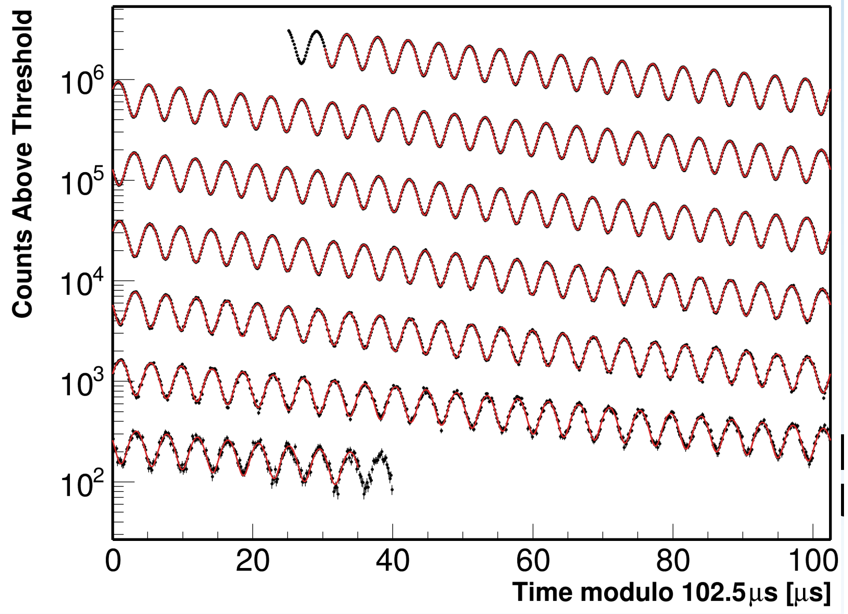
$$\omega_a = -a\left(\frac{Qe}{m}\right)B$$

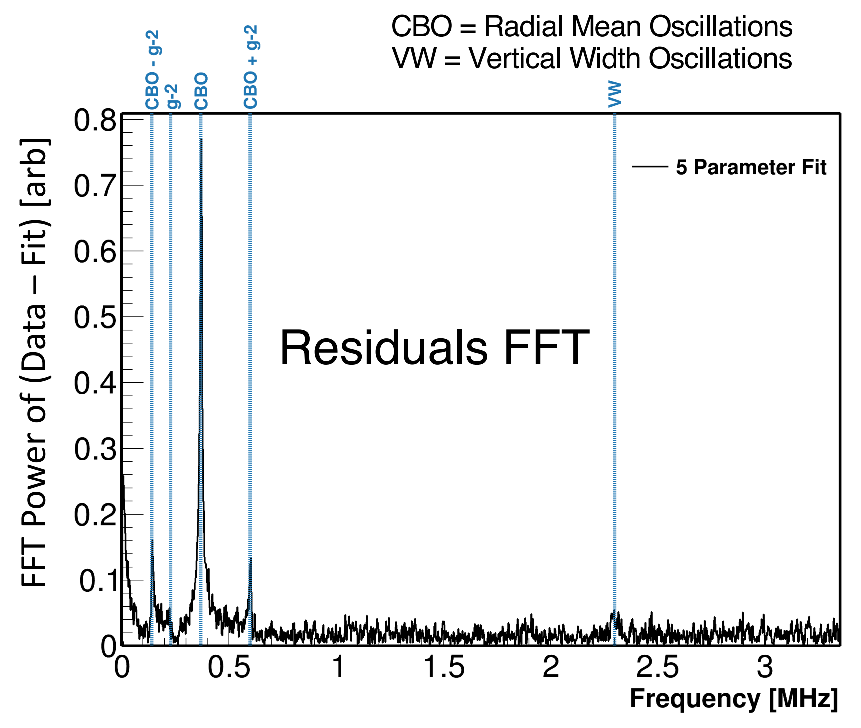
Simple fit: residuals

- Simplest form for fit is an exponentially decaying oscillation:

$$N_0 e^{-t/\tau}(1 + A\cos(\omega_a t + \phi))$$

χ^2 / ndf = 8191 / 4149





Beam **oscillations couple to acceptance** & change number of e⁺ detected with time, and **exponential isn't perfect.**

All of these frequencies in the residual are from well understood beam dynamics and other effects.

Fit with beam dynamics terms

- Add terms to fit function to deal with complications:

$$N_0 e^{-t/\tau}(1 + A\cos(\omega_a t + \phi))$$

$$\Downarrow$$

$$f(t) = N_0 e^{-t/\tau}\, \underline{\Lambda(t)}\, \underline{N_{cbo}(t)}\, \underline{N_{2cbo}(t)}\, (1 + \underline{A_{cbo}(t)} cos(\omega_a t + \underline{\phi_{cbo}(t)})$$

- Muons that are lost from storage ring before they decay:

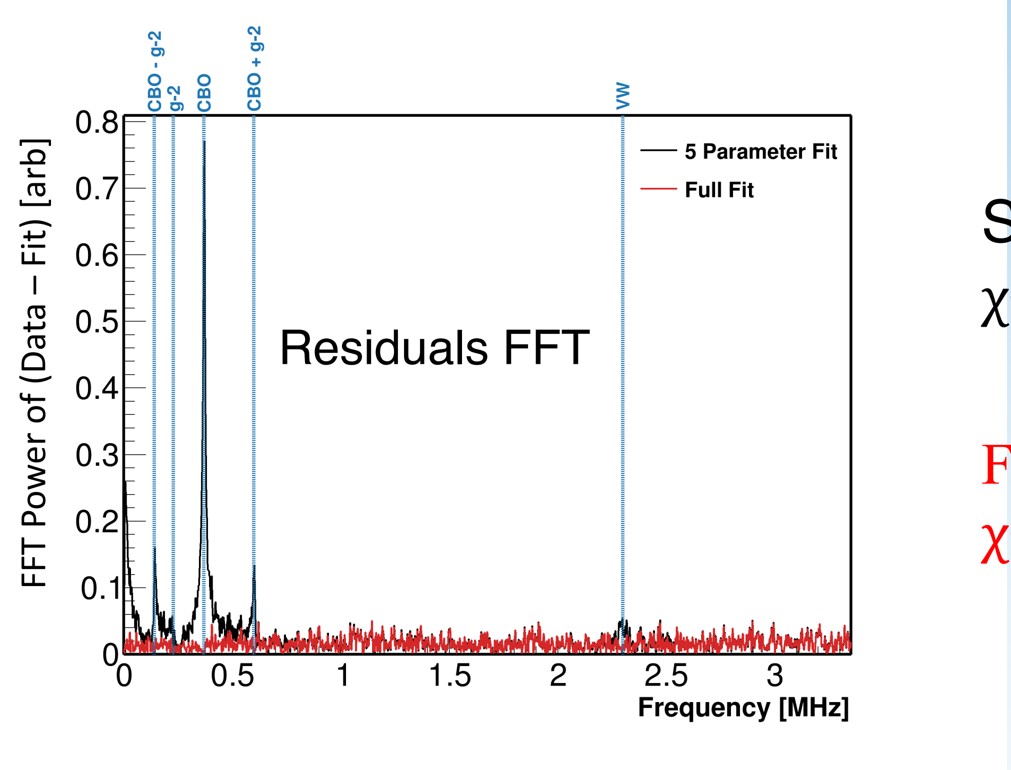
$$\Lambda(t) = 1 - \boldsymbol{\kappa_{loss}} \int_{t_0}^{t} L(t')e^{(t'/\tau)}dt'$$

- Beam oscillations that modulate decay rate:

e.g. $N_{cbo}(t) = (1 + \boldsymbol{A_{cbo-N}} \cdot e^{-t/\boldsymbol{\tau_{cbo}}} \cdot \cos(\omega_{cbo}(t) \cdot t + \boldsymbol{\phi_{cbo-N}}))$

Fit with beam dynamics terms: residuals

- Adding terms removes these extra frequencies:



Simple 5-parameter fit

χ^2 / ndf = 8191 / 4149

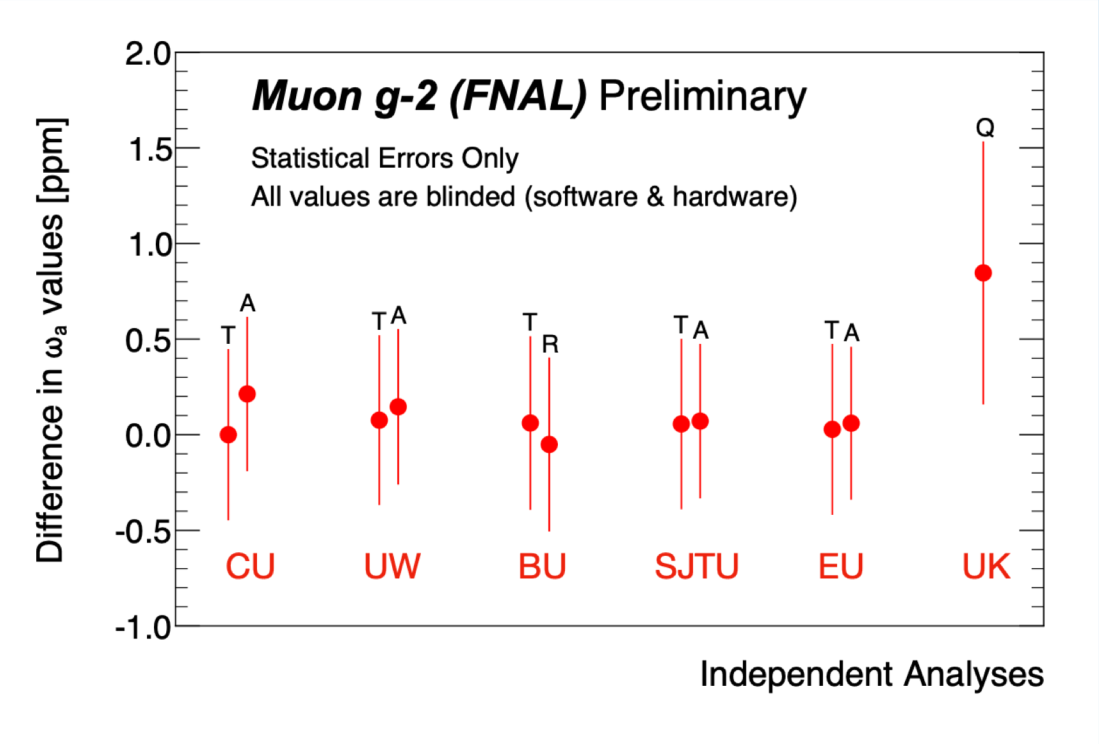
Fit with extra terms

χ^2 / ndf = 4005 / 4134

- Important to get it right: ω_a changes by 2.2 ppm if we don't include this effect.
- Good residuals and χ^2 are necessary, but not sufficient condition. Still need to deal with other systematic effects…

Spin Precession (ω_a): Relative Unblinding

- Independent analysers using different techniques get consistent results for ω_a



Run 1 Combined

Stat. error 434 ppb

- Hundreds of pages written on this in a number of Ph.D dissertations. Discussed in our ω_a paper in Phys. Rev. D
- All are consistent within expected variation, which is much smaller than statistical uncertainty.

Real World Complications: Corrections

- We need to make six small corrections to get our final result:

Clock Blinding

$$\frac{\omega_a}{\langle \omega'_p \times M_\mu \rangle} = \frac{f_{clock}\, \omega_a^m}{f_{calib} \langle \omega'_p \times M_\mu \rangle^m} \frac{1 + \overbrace{C_e + C_p + C_{ml} + C_{pa}}^{\text{Beam Dynamics: E-field, Pitch, Muon Losses, Phase-Acceptance}}}{1 + \underbrace{B_k + B_q}_{\text{Transient Magnetic Fields: Kicker Eddy Current, Quad Vibrations}}}$$

Field Calibration

"3 ingredients"

$C_e = 489 \text{ ppb}, \; \delta_{C_e} = 53 \text{ ppb}$ $\quad C_p = 189 \text{ ppb}, \; \delta_{C_p} = 13 \text{ ppb}$

$C_{pa} = -158 \text{ ppb} \;\; \delta_{\text{pa}} = 75 \text{ ppb}$ $\quad B_q = -17 \text{ ppb}, \delta_{B_q} = 92 \text{ ppb}$

$B_k = -27 \text{ ppb}, \delta_{B_k} = 37 \text{ ppb}$

- Total correction is 544 ppb (c.f. stat. error of 434 ppb)
- We don't have same precision requirements on these corrections as we do on ω_a & ω'_p but we still evaluate them carefully…
- C_{pa} will disappear in Run 2-3, δB_q and δB_k will significantly improve.

Clock Blinding for a Blind Analysis

$$\frac{\omega_a}{\langle \omega'_p \times M_\mu \rangle} = \frac{\boxed{f_{clock}}\, \omega_a^m}{f_{calib} \langle \omega'_p \times M_\mu \rangle^m} \text{ x Corrections}$$

- f_{clock} is the frequency that our clock ticks relative to 40 MHz
  - Precision timepiece, stable at ppt level
- The exact clock frequency was kept secret from all collaborators

Greg Bock &
Joe Lykken

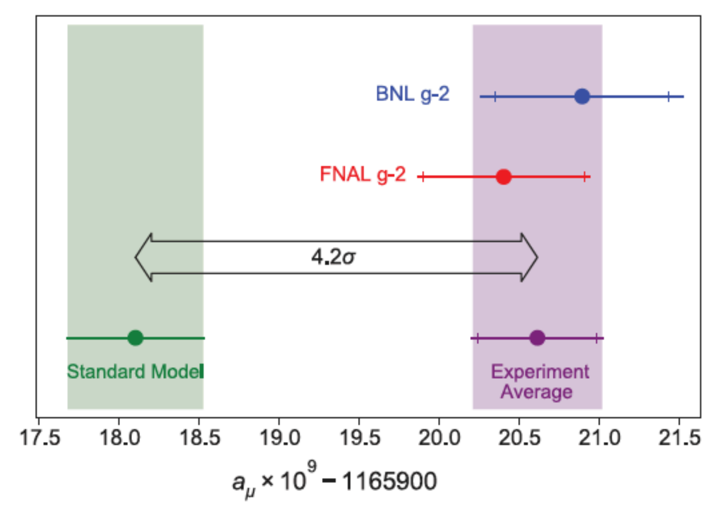
Two of FNAL Directorate set frequency to (40 - δ MHz) and check the clock each week

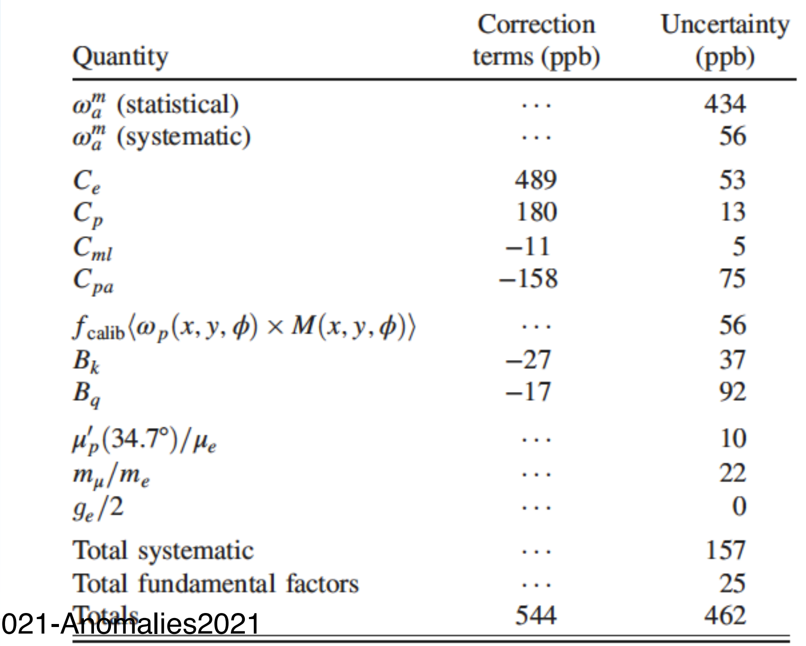
Clock was locked and value was kept secret until analysis completed

Feb 25th 2021: Virtual Unblinding

- Unanimous vote from all collaborators to unblind the value!

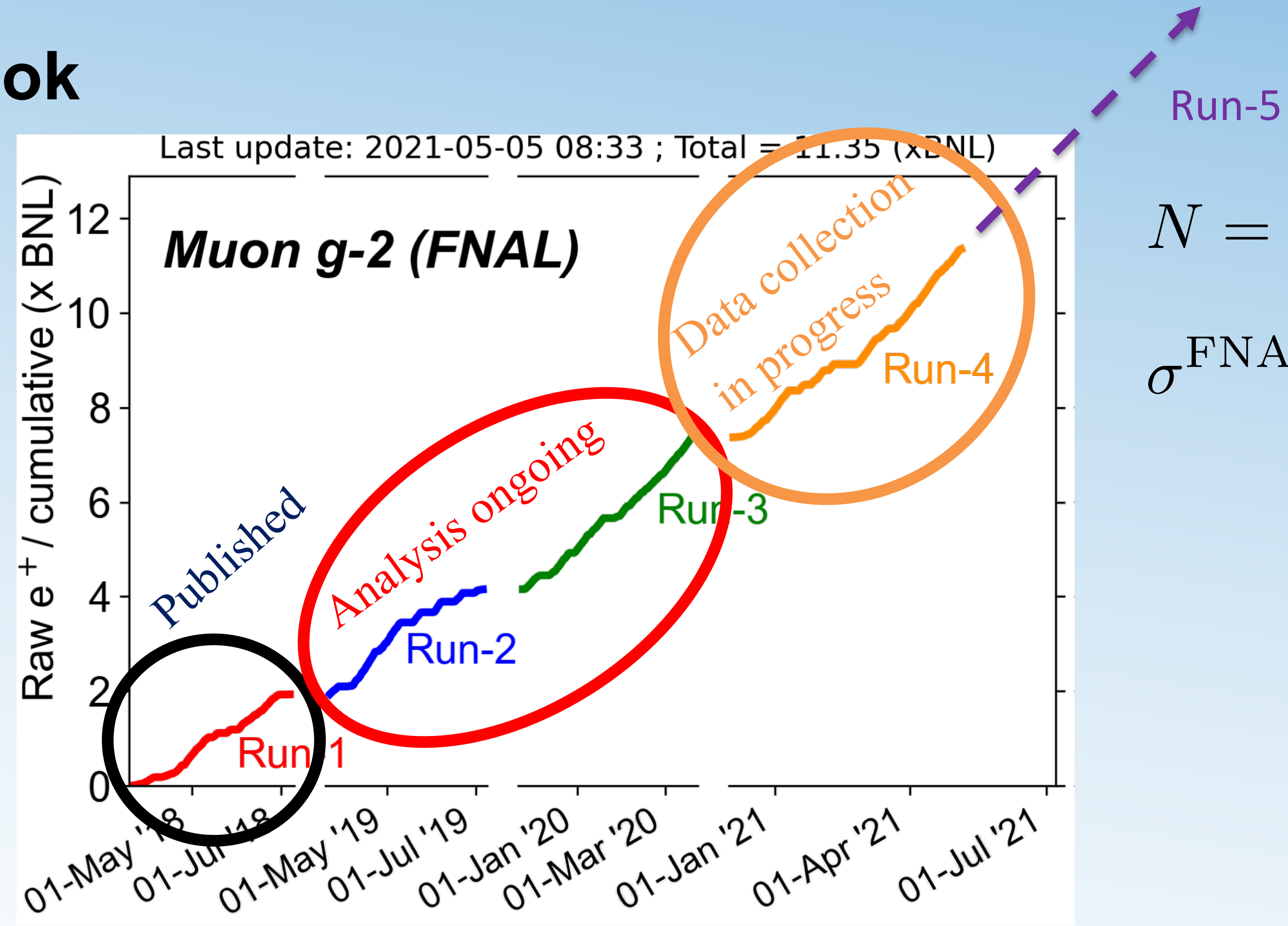
- 3 years after taking the data, secret envelopes were opened to reveal the hidden clock frequency and our first result…





Quantity	Correction terms (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	⋯	434
ω_a^m (systematic)	⋯	56
C_e	489	53
C_p	180	13
C_{ml}	−11	5
C_{pa}	−158	75
$f_{\text{calib}}\langle\omega_p(x,y,\phi)\times M(x,y,\phi)\rangle$	⋯	56
B_k	−27	37
B_q	−17	92
$\mu'_p(34.7^\circ)/\mu_e$	⋯	10
m_μ/m_e	⋯	22
$g_e/2$	⋯	0
Total systematic	⋯	157
Total fundamental factors	⋯	25
Totals	544	462

Outlook



$$N = \times 20\ \mathrm{BNL};$$

$$\sigma^{\mathrm{FNAL}} \simeq \frac{\sigma^{\mathrm{BNL}}}{4}$$

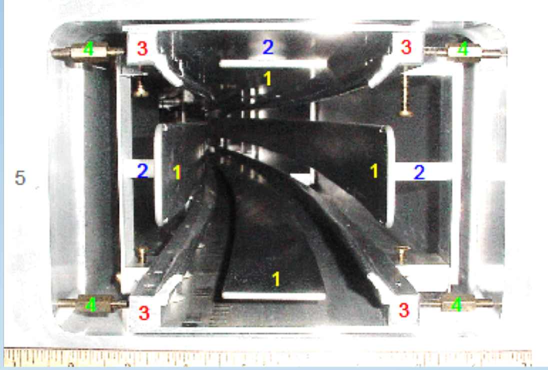
- Run 1 is only 6% of the final data set
- Run 2 and 3: Expect a factor of 2 × improvement in precision -18-months?
- Run 4 is in progress, expect to bring in 13 × BNL statistics; ½ σ BNL
- Run 5 next year should reach the design statistics of 20 × BNL, total uncertainty ↓ 4

Summary:

- We have measured a_μ to 460 ppb, and this result agrees well with BNL.
- We discovered several subtle new effects that we have corrected for.
- Do any of these change the BNL result?
  - **No!** None of these new effects are at a level that justifies any change to the previous result.
- There continues to be a tension with the Standard Model value.
- We are analyzing a significantly larger data set and hope to have a new result by summer or fall 2022.
- On the theory side, significant work is ongoing to understand the difference between the BMW lattice QCD result and the dispersive result.

Stay Tuned!

The Weak Focusing Betatron (Beam Dynamics)



- Uniform vertical magnetic field with Electric quadrupoles for vertical focusing.

field index *n*, where κ is the electric quadrupole gradient $\kappa \equiv \frac{dE_r}{dr}$ $n = \frac{\kappa R_0}{\beta B_0} \simeq 0.135$

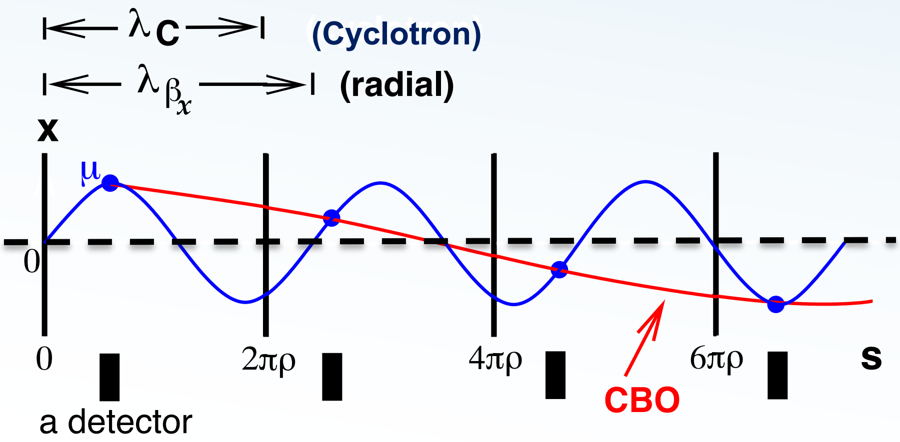
(SHM motion in *x* and *y* or in arc length *s*)

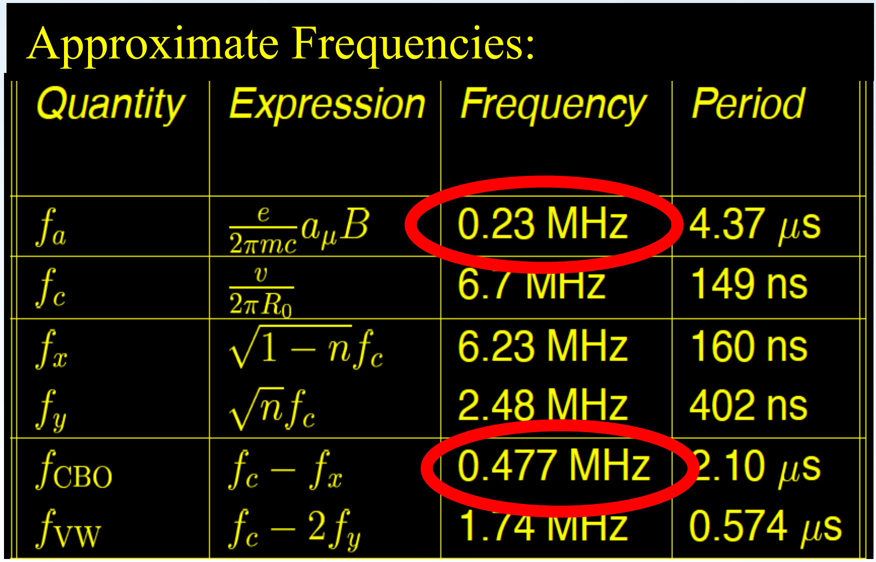
$$x = x_e + A_x \cos(\nu_x \frac{s}{R_0} + \delta_x) \quad \text{and} \quad y = A_y \cos(\nu_y \frac{s}{R_0} + \delta_y)$$

Horizontal and vertical tune: $\nu_x = \sqrt{1-n}$ and $\nu_y = \sqrt{n}$

- Betatron frequencies $f_y = f_C\sqrt{n} \simeq 0.37 f_C\,; \quad f_x = f_C\sqrt{1-n} \simeq 0.929 f_C$

$$f_{CBO} = f_C - f_x$$





Approximate Frequencies:

Quantity	Expression	Frequency	Period
f_a	$\frac{e}{2\pi mc} a_\mu B$	0.23 MHz	4.37 μs
f_c	$\frac{v}{2\pi R_0}$	6.7 MHz	149 ns
f_x	$\sqrt{1-n} f_c$	6.23 MHz	160 ns
f_y	$\sqrt{n} f_c$	2.48 MHz	402 ns
$f_{\rm CBO}$	$f_c - f_x$	0.477 MHz	2.10 μs
$f_{\rm VW}$	$f_c - 2f_y$	1.74 MHz	0.574 μs