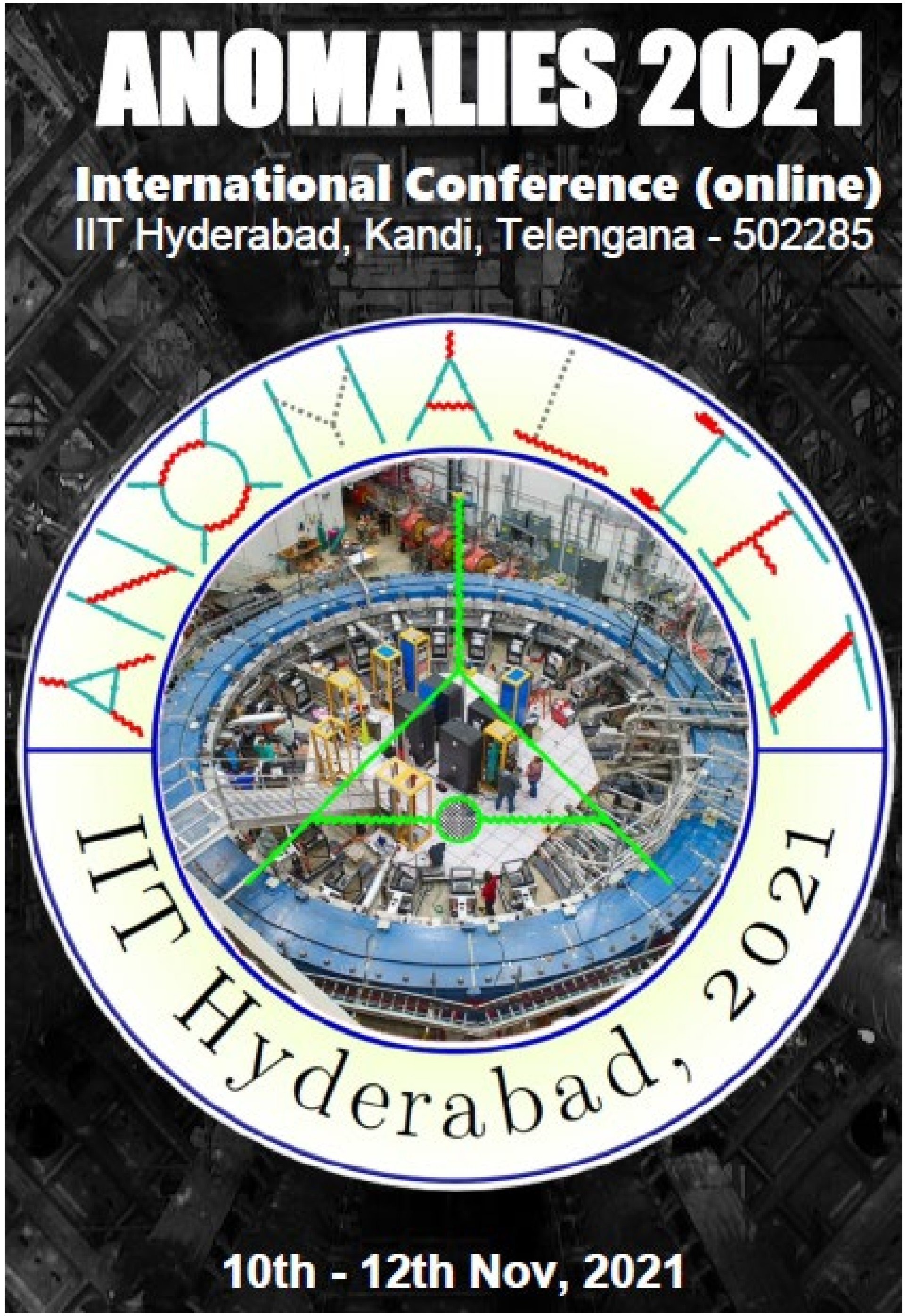

Scalar dark matter condensed by thermal fermions

Based on 2109.07423

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Outline

- Ultralight scalar field as a **cold dark matter** candidate.

$$\frac{n_\phi}{s} \sim 10^{19} \frac{10^{-20}\mathrm{eV}}{m_\phi}$$

- Its population can be in the form of **coherent oscillation** driven by misaligned initial amplitude in a flat potential.
- Classic example: QCD axion $\sim f_a$ relaxed by instanton potential.
- New idea: **Coherent state** driven by feeble coupling to **thermal fermions**.

Coherent oscillation as DM

- Scalar field in a coherent state=a classical field oscillation.

$$\hat{\phi}(x) = \int_k [a_k e^{-ik\cdot x} + a_k^{+} e^{+ik\cdot x}]$$

$$\langle\hat{\phi}(x)\rangle_c = \phi_0 e^{-ik_0\cdot x} + \phi_0^{+}\, e^{+ik_0\cdot x}$$

$$\text{with } \; k_0 \approx (m_\phi, m_\phi \vec{v})$$

- Free field in the FRW universe:

$$\langle\hat{\phi}(x)\rangle_c = \phi(t)$$

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + m_\phi^2 \phi(t) = 0$$

$$H \gg m_\phi \; (m_\phi t \ll 1): \quad \phi = \phi_i; \dot{\phi} = 0$$

$$H \ll m_\phi \; (m_\phi t \gg 1): \quad \phi \sim \phi_i \frac{\sin\left(m_\phi t + \frac{\pi}{8}\right)}{(m_\phi t)^{3/4}}$$

- Cold DM density: $\rho_\phi \sim \dfrac{m_\phi^2 \phi_i^2}{(m_\phi t)^{3/2}} = \rho_{DM} \Rightarrow \phi_i \sim 0.01\, M_p \left(\dfrac{10^{-20}\text{eV}}{m_\phi}\right)^{1/4}$

$$T = c_t \sqrt{M_P/2t}$$

$$c_t = 1.74/g_*^{1/4}$$

Scalar field in thermal background

- Scalar field interacting with thermal fermions:

$$\mathcal{L}' = y_\phi \, \hat{\phi} \, (\bar{f}_R f_L + \bar{f}_L f_R)$$

$$\rightarrow V_{T,\mathrm{eff}}(\phi) = -\frac{g_f}{2\pi^2} T^4 J_F\left(\frac{(m_f + y_\phi \phi)^2}{T^2}\right)$$

Dolan+Jackiw, Weinberg, 1974

- Leading thermal effect in cosmological evolution:

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + \left(m_\phi^2 + m_T^2\right)\phi(t) \approx \frac{\partial}{\partial\phi}\langle\mathcal{L}'\rangle_T$$

$$m_T^2 = \frac{g_f}{24} y_\phi^2 T^2, \quad \langle\mathcal{L}'\rangle_T = y_\phi \phi \frac{g_f m_f T^2}{24}$$

$$g_f = 4N_c \;\; (2) \;\; \text{for} \;\; f = q, l \; (\nu)$$

Esteban+Salvado, 2101.05804

Batell+Ghalsasi, 2109.04476

General features

- Evolution from $T_{ew} \approx 100$ GeV down to $T_{eq} \approx 0.8$ eV.

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \left(1 + \frac{x_1}{x}\right)\tilde{\phi}(x) = \frac{x_S}{x}$$

$$x_1 \equiv y_\phi^2 \frac{c_t^2 g_f}{48}\frac{M_P}{m_\phi}, \qquad x_S \equiv y_\phi \frac{c_t^2 g_f}{48}\frac{m_f}{m_\phi}$$

- Notation: $x = m_\phi t,\ \tilde{\phi} = \phi/M_P,$

$$m_{20} \equiv m_\phi/10^{-20}\text{eV}$$

$$x_{ew} \approx 10^{-15} m_{20} \qquad x_{eq} \approx 2\times\ 10^7 m_{20}$$

$$x_f \approx 5\times 10^{-4} m_{20}\left(\frac{m_e}{m_f}\right)^2$$

$$x_1 \approx 5\times 10^{46}\ g_f y_\phi^2 m_{20}^{-1} \qquad x_S \approx 10^{24} g_f y_\phi\ m_{20}^{-1}\left(\frac{m_f}{m_e}\right)$$

- Solution to get c_f or

  the resulting DM density:

$$\rho_\phi(x) \approx \frac{c_f^2 x_S^2}{\pi}\frac{m_\phi^2 M_P^2}{x^{3/2}}$$

$$\rho_\phi = \rho_{DM} \Rightarrow c_f^2 x_S^2 \approx 2.5\times 10^{-4} m_{20}^{-1/2}$$

Asymptotic solutions in different regimes

- Vanishing initial condition at x_{ew}: $\phi = 0, \phi' = 0$.
- Nontrivial evolution from x_{ew} to x_f for $f = q, l$, and then free evolution down to x_{eq}.
- Evolution from x_{ew} to x_{eq} for $f = \nu$.

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \left(1 + \frac{x_1}{x}\right)\tilde{\phi}(x) = \frac{x_S}{x}$$

For $f = q, l$: I) $x_1 < x_{ew} < x_f$

II) $x_{ew} < x_f < x_1$

III) $x_{ew} < x_1 < x_f$

For $f = \nu$: $x_{ew} < x_1 < x_{eq}$

I) $x_1 < x_{ew} < x_f$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) \approx \frac{x_S}{x} \quad \text{for} \quad x_{ew} \ll x \ll x_f$$
$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = 0 \quad \text{for} \quad x_f \ll x\,.$$

$$\rho_\phi(x) \approx \frac{C_1^2 + C_2^2}{\pi}\frac{m_\phi^2 M_P^2}{x^{3/2}}$$

$$\frac{C_1}{x_S} = \frac{\pi}{2^{\frac{1}{4}}\Gamma(\frac{3}{4})}G_1(x_f) - \frac{2^{\frac{5}{4}}\Gamma(\frac{3}{4})}{3}G_2(x_f)$$
$$\frac{C_2}{x_S} = \frac{2^{\frac{5}{4}}\Gamma(\frac{3}{4})}{3}G_2(x_f)$$

$$c_f^2 \approx \begin{cases} 4\sqrt{2}\,\Gamma\left(\frac{5}{4}\right)^2 x_f^2 & \text{for} \quad x_f \ll 1, \\ \frac{\pi\Gamma(\frac{3}{4})^2}{2\sqrt{2}} & \text{for} \quad x_f \gg 1. \end{cases}$$

$$y_\phi \approx 3.6 \times 10^{-24}\frac{m_{20}^{-\frac{1}{4}}}{N_c}\frac{m_f}{m_e} \quad \text{with} \quad \frac{550}{N_c^{\frac{2}{5}}}\left(\frac{m_f}{m_e}\right)^{\frac{4}{5}} \ll m_{20} \ll 2 \times 10^3\left(\frac{m_f}{m_e}\right)^2 \quad \text{for} \quad x_f \ll 1,$$
$$y_\phi \approx 3 \times 10^{-27}\frac{m_{20}^{\frac{3}{4}}}{N_c}\frac{m_e}{m_f} \quad \text{with} \quad m_{20} \gg \text{Max}\left[\frac{2 \times 10^{16}}{N_c^2}\left(\frac{m_e}{m_f}\right)^4,\ 2 \times 10^3\left(\frac{m_f}{m_e}\right)^2\right] \quad \text{for} \quad x_f \gg 1.$$

(*) Neglecting Freeze-In:

$$m_\phi \ll 46 \text{ eV } (5.7 \text{ MeV}) \quad \text{for} \quad f = e\ (b),$$
$$y_\phi \ll 5.3\,(1.4) \times 10^{-11}.$$

II) $x_{ew} < x_f < x_1$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \frac{x_1}{x}\tilde{\phi}(x) \approx \frac{x_S}{x} \quad \text{for} \quad x_{ew} \ll x \ll x_f$$
$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) = 0 \quad \text{for} \quad x_f \ll x.$$

$$\frac{C_1}{x_S} = -\frac{\pi x_f^{\frac{1}{4}}}{4x_1}\left[2x_f Y_{\frac{5}{4}}(x_f) - Y_{\frac{1}{4}}(x_f)\cos(2\sqrt{x_1 x_f}) + \sqrt{\frac{x_f}{x_1}}Y_{-\frac{3}{4}}(x_f)\sin(2\sqrt{x_1 x_f})\right],$$
$$\frac{C_2}{x_S} = \frac{\pi x_f^{\frac{1}{4}}}{4x_1}\left[2x_f J_{\frac{5}{4}}(x_f) - J_{\frac{1}{4}}(x_f)\cos(2\sqrt{x_1 x_f}) + \sqrt{\frac{x_f}{x_1}}J_{-\frac{3}{4}}(x_f)\sin(2\sqrt{x_1 x_f})\right].$$

$$c_f^2 \approx \begin{cases} 4\sqrt{2}\,\Gamma\left(\frac{5}{4}\right)^2 x_f^2 & \text{for} \quad x_f \ll x_1 \ll 1, \\ \frac{\pi x_f^3}{2x_1^2} & \text{for} \quad 1 \ll x_f \ll x_1. \end{cases}$$

$$y_\phi \approx 3.6 \times 10^{-24} \frac{m_{20}^{-\frac{1}{4}}}{N_c}\frac{m_f}{m_e} \quad \text{with} \quad m_{20} \ll \frac{31}{N_c^{\frac{2}{5}}}\left(\frac{m_f}{m_e}\right)^{\frac{8}{5}} \quad \text{for} \quad x_f \ll x_1 \ll 1,$$
$$y_\phi \approx 5.3 \times 10^{-24} m_{20}\left(\frac{m_e}{m_f}\right)^{\frac{1}{2}} \quad \text{with} \quad m_{20} \gg 2 \times 10^3 \left(\frac{m_f}{m_e}\right)^2 \quad \text{for} \quad 1 \ll x_f \ll x_1.$$

(*) BBN constraint:

$$m_\phi \ll 10^{-6}\,(2.5 \times 10^{-3})\ \text{eV} \quad \text{for} \quad f = e\ (b),$$
$$y_\phi \ll 5.7 \times 10^{-10}\,(1.5 \times 10^{-8}).$$

III) $x_{ew} < x_1 < x_f$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \frac{x_1}{x}\tilde{\phi}(x) \approx \frac{x_S}{x} \quad \text{for} \quad x_{ew} \ll x \ll x_1$$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) \approx \frac{x_S}{x} \quad \text{for} \quad x_1 \ll x \ll x_f$$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) \approx 0 \quad \text{for} \quad x_f \ll x.$$

(i) $x_1 \ll x_f \ll 1$:

$$y_\phi \approx 3.6 \times 10^{-24} \frac{m_{20}^{-\frac{1}{4}}}{N_c} \frac{m_f}{m_e} \quad \text{with} \quad \frac{28}{N_c^{\frac{2}{5}}} \left(\frac{m_f}{m_e}\right)^{\frac{8}{5}} \ll m_{20} \ll \text{Min}\left[2 \times 10^3 \left(\frac{m_f}{m_e}\right)^2, \frac{6 \times 10^5}{N_c^{\frac{2}{5}}} \left(\frac{m_f}{m_e}\right)^{\frac{4}{5}}\right]$$

(ii) $x_1 \ll 1 \ll x_f$:

$$y_\phi \approx 3 \times 10^{-27} \frac{m_{20}^{\frac{3}{4}}}{N_c} \frac{m_e}{m_f} \quad \text{with} \quad 2 \times 10^3 \left(\frac{m_f}{m_e}\right)^2 \ll m_{20} \ll \text{Min}\left[3.2 \times 10^{11} N_c^2 \left(\frac{m_f}{m_e}\right)^4, \frac{2.1 \times 10^{16}}{N_c^2} \left(\frac{m_e}{m_f}\right)^4\right]$$

(iii) $1 \ll x_1 \ll x_f$:

$$y_\phi \approx 1.9 \times 10^{-22} \frac{m_{20}^{\frac{1}{3}}}{N_c^{\frac{1}{6}}} \left(\frac{m_f}{m_e}\right)^{\frac{2}{3}} \quad \text{with } 1.2 \times 10^8 N_c^{\frac{5}{4}} \left(\frac{m_f}{m_e}\right)^{\frac{13}{4}} \ll m_{20} \ll 4.4 \times 10^{11} N_c^2 \left(\frac{m_f}{m_e}\right)^4 \quad \text{for } x_1^5 \ll x_f$$

$$y_\phi \approx 4.5 \times 10^{-28} \frac{m_{20}}{N_c} \left(\frac{m_e}{m_f}\right)^{\frac{3}{2}} \quad \text{with} \quad m_{20} \gg 4.9 \times 10^8 N_c^{\frac{5}{4}} \left(\frac{m_f}{m_e}\right)^{\frac{13}{4}} \quad \text{for } x_f \ll x_1^5.$$

(*) BBN+Feeze-In: $y_\phi \ll 5.7\,(1.2) \times 10^{-10}$, $\quad m_\phi \ll 1.3 \times 10^{-2}\,(6.1 \times 10^3)$ eV for $f = e\,(b)$

Neutrinos driving DM condensate

$x_{ew} < x_1 < x_{eq} < x_\nu$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \frac{x_1}{x}\tilde{\phi}(x) \approx \frac{x_S}{x} \quad \text{for} \quad x_{ew} \ll x \ll x_1$$

$$\tilde{\phi}''(x) + \frac{3}{2x}\tilde{\phi}'(x) + \tilde{\phi}(x) \approx \frac{x_S}{x} \quad \text{for} \quad x_1 \ll x$$

$$\frac{C_1}{x_S} = -\frac{\pi}{2^{\frac{1}{4}}\Gamma(\frac{3}{4})}G_1(x_1) + \frac{2^{\frac{5}{4}}\Gamma(\frac{3}{4})}{3}G_2(x_1) - \frac{\pi}{2}\frac{Y_{\frac{1}{4}}(x_1)\sin^2(x_1) - Y_{-\frac{3}{4}}(x_1)[x_1 - \frac{1}{2}\sin(2x_1)]}{x_1^{3/4}}$$

$$\frac{C_2}{x_S} = -\frac{2^{\frac{5}{4}}\Gamma(\frac{3}{4})}{6}G_2(x_1) + \frac{\pi}{2}\frac{J_{\frac{1}{4}}(x_1)\sin^2(x_1) - J_{-\frac{3}{4}}(x_1)[x_1 - \frac{1}{2}\sin(2x_1)]}{x^{3/4}}$$

$$c_\nu^2 = \frac{\pi\Gamma(\frac{3}{4})^2}{2\sqrt{2}} + \frac{2^{\frac{3}{4}}\pi^{\frac{3}{2}}C_1}{\Gamma(\frac{1}{4})x_S} + \frac{C_1^2 + C_2^2}{x_S^2} = \begin{matrix} 3.34 \quad \text{for } x_1 \gg 1 \\ 1.67 \quad \text{for } x_1 \ll 1 \end{matrix}$$

$$y_\phi \approx 4.2 \times 10^{-20} m_{20}^{3/4}\left(\frac{0.05\text{eV}}{m_\nu}\right) \quad \text{with} \quad m_{20} \gg 710\left(\frac{0.05\text{eV}}{m_\nu}\right)^4 \qquad (x_1 \gg 1)$$

(*) BBN: $y_\phi \lesssim 7 \times 10^{-6}$ $\quad m_\phi \lesssim 0.092\ \text{eV}\left(\frac{m_\nu}{0.05\ \text{eV}}\right)^{4/3}$

Conclusion

- A ultralight scalar boson can form a coherent DM state through its tiny little coupling to SM fermions.
- Considering the ϕqq or ϕll coupling, it works for wide ranges of $m_\phi \sim (10^{-22}, 10^{6})\mathrm{eV}$ and $y_\phi \sim (10^{-28}, 10^{-8})$.
- For the $\phi \nu\nu$ coupling, DM genesis requires $m_\phi \sim (10^{-17}, 10^{-1})\mathrm{eV}$ and $y_\phi \sim (10^{-17}, 10^{-5})$, and can occur **very late**, even at around T_{eq}.