Status and Outlook of Lattice Calculations for the Muon (g-2) Anomaly

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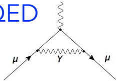
November 12, 2021 - Anomalies 2021

Contributions from known particles: The Standard Model

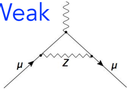
$$a_\mu(\mathrm{SM}) = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{Weak}) + a_\mu(\mathrm{Hadronic})$$

QED



+... $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

Weak

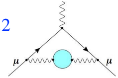


+... $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic...

...Vacuum Polarization (HVP)

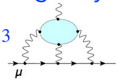
α^2



+... $6845\,(40) \times 10^{-11}$ [0.6%] 0.37 ppm

...Light-by-Light (HLbL)

α^3

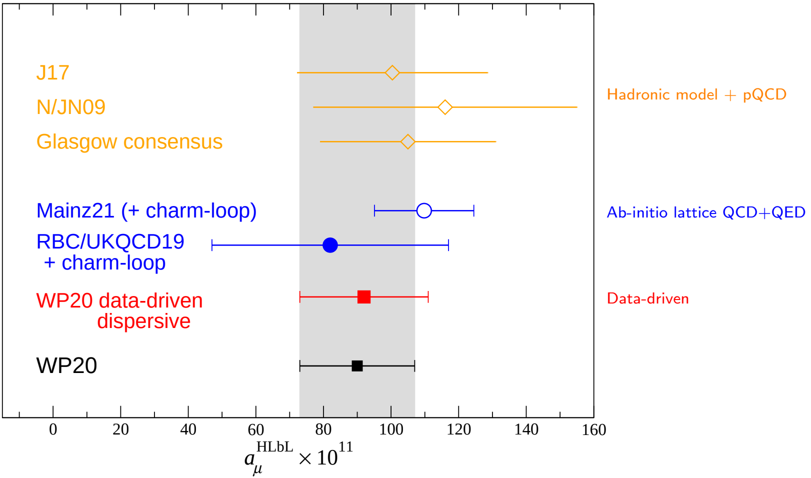


+... $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

Numbers from Theory Initiative Whitepaper

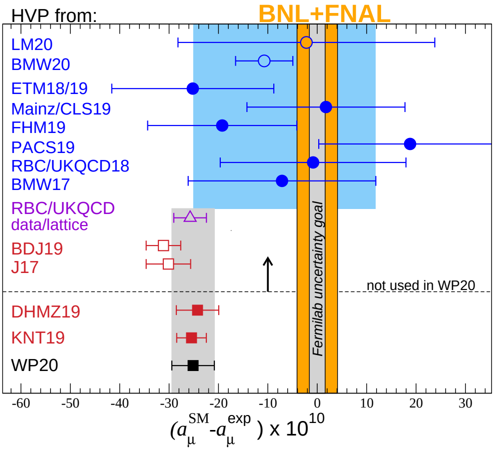
Uncertainty dominated by hadronic contributions

Status of hadronic light-by-light contribution



Systematically improvable methods are maturing; uncertainty to a_μ controlled at 0.15ppm; cross-checks detailed in Theory Initiative whitepaper

Status and impact of hadronic vacuum polarization contribution



Ab-initio lattice QCD(+QED) calculations are maturing

Difficult problem: scales from $2m_\pi$ to several GeV enter; cross-checks needed at high precision

Hybrid window method restricts scales that enter from lattice/dispersive data

Dispersive, $e^+e^- \to$ hadrons (20+ years of experiments)

Now first published lattice result with sub-percent precision available (BMW20), cross-checks are crucial to establish or refute high-precision lattice methodology (same situation as for HLbL)

Summary of HVP status:

- Decades of e^+e^- dispersive results suggest a strong tension (4.2σ)
- A single lattice result (BMW20) suggests only minimal tension (1.5σ)

How can we move forward in our understanding? Main topic of this talk.

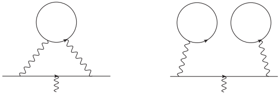
Two main questions:

- Consistency of BMW20 lattice result with previously know lattice results
- Consistency of lattice results with R-ratio

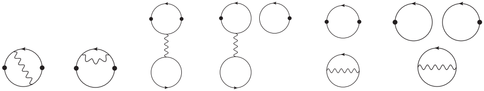
Consistency of BMW20 lattice result with previously know lattice results

Diagrams

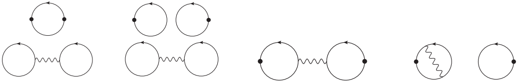
Isospin limit



QED corrections

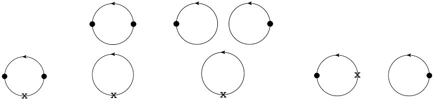


(a) V (b) S (c) T (d) T_d (e) D1 (f) $D1_d$



(g) D2 (h) $D2_d$ (i) F (j) D3

Strong isospin breaking



(a) M (b) R (c) R_d (d) O

Overview of individual contributions

Diagrams – Isospin limit

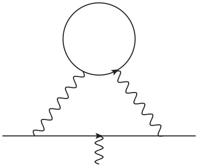
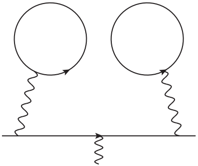
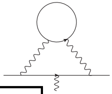
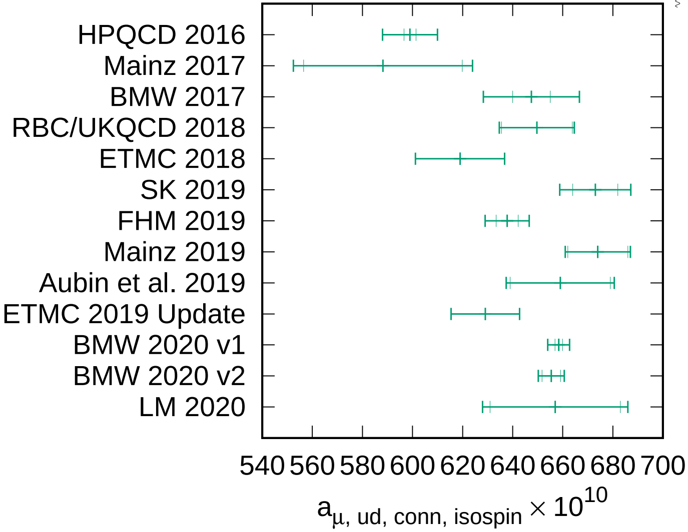



FIG. 1. Quark-connected (left) and quark-disconnected (right) diagram for the calculation of $a_\mu^{\mathrm{HVP\ LO}}$. We do not draw gluons but consider each diagram to represent all orders in QCD.

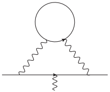
Up, down; isospin symmetric limit; $m_\pi = m_\pi^0$

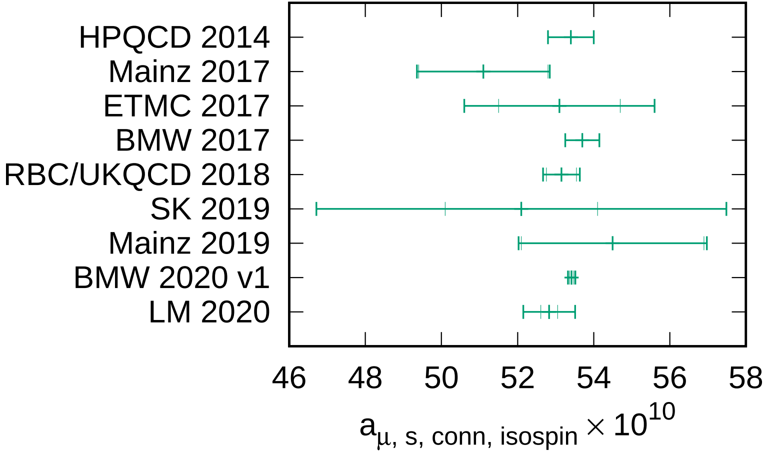




Some tensions to be understood

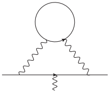
Strange

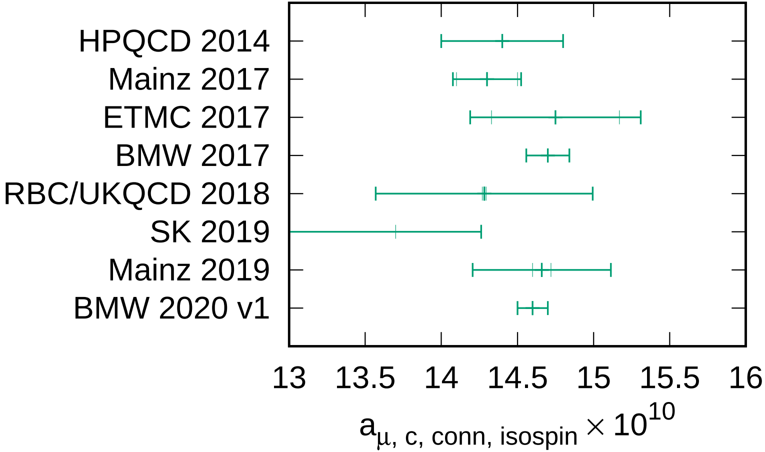


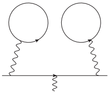


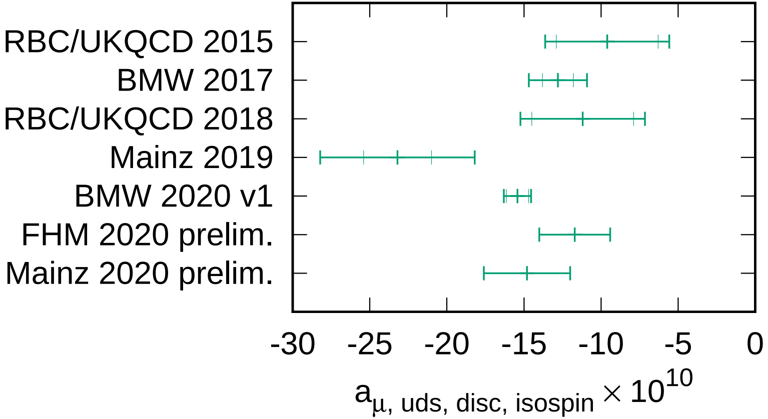
HPQCD 2014
Mainz 2017
ETMC 2017
BMW 2017
RBC/UKQCD 2018
SK 2019
Mainz 2019
BMW 2020 v1
LM 2020
46
48
50
52
54
56
58
$a_{\mu,\text{ s, conn, isospin}} \times 10^{10}$

Charm

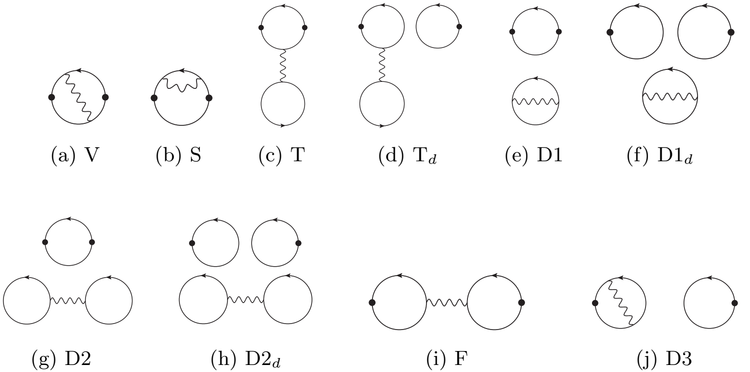



HPQCD 2014
Mainz 2017
ETMC 2017
BMW 2017
RBC/UKQCD 2018
SK 2019
Mainz 2019
BMW 2020 v1
13
13.5
14
14.5
15
15.5
16
$a_{\mu,\,c,\,conn,\,isospin} \times 10^{10}$

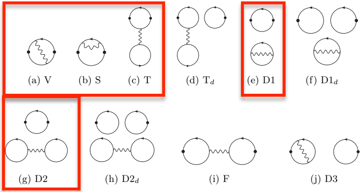


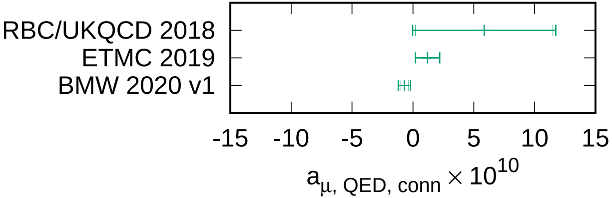

RBC/UKQCD 2015
BMW 2017
RBC/UKQCD 2018
Mainz 2019
BMW 2020 v1
FHM 2020 prelim.
Mainz 2020 prelim.
-30
-25
-20
-15
-10
-5
0
$a_{\mu,\text{ uds, disc, isospin}} \times 10^{10}$

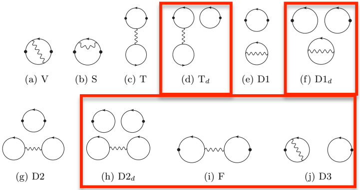
Diagrams – QED corrections

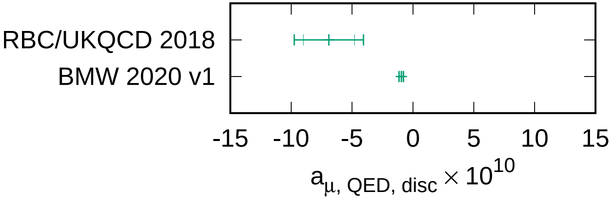


For diagram F we enforce exchange of gluons between the quark loops as otherwise a cut through a single photon line would be possible. This single-photon contribution is counted as part of the HVP NLO and not included for the HVP LO.


(a) V
(b) S
(c) T
(d) T_d
(e) D1
(f) $D1_d$
(g) D2
(h) $D2_d$
(i) F
(j) D3

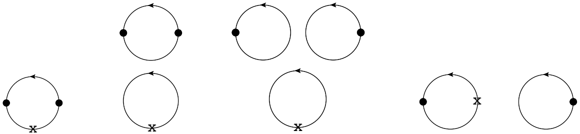

RBC/UKQCD 2018
ETMC 2019
BMW 2020 v1
-15
-10
-5
0
5
10
15
$a_{\mu,\,\mathrm{QED,\,conn}} \times 10^{10}$





Attention needed

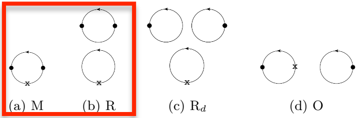
Diagrams – Strong isospin breaking

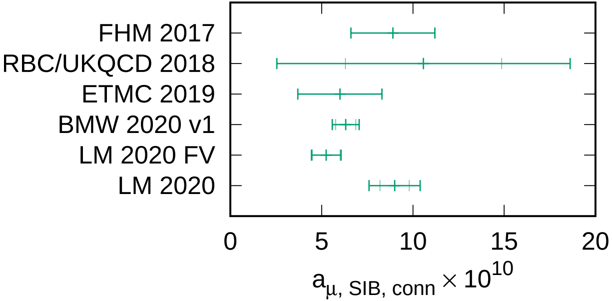


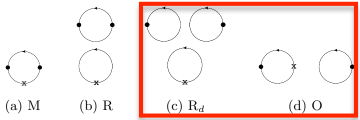
(a) M (b) R (c) R_d (d) O

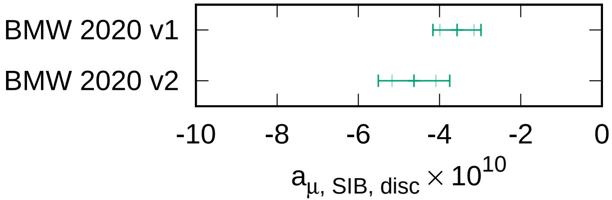
For the HVP R is negligible since $\Delta m_u \approx -\Delta m_d$ and O is SU(3) and $1/N_c$ suppressed.

Lehner, Meyer 2020: NLO PQChPT: FV effects in connected and disconnected cancel but are each significant $O(4 \times 10^{-10})$; PQChPT expects cancellation between connected and disconnected contribution $a_\mu^{\mathrm{SIB,\ conn.}} = -a_\mu^{\mathrm{SIB,\ disc.}} = 6.9 \times 10^{-10}$


(a) M
(b) R
(c) R_d
(d) O


FHM 2017
RBC/UKQCD 2018
ETMC 2019
BMW 2020 v1
LM 2020 FV
LM 2020
0
5
10
15
20
$a_{\mu,\ \mathrm{SIB,\ conn}} \times 10^{10}$


(a) M
(b) R
(c) R_d
(d) O


BMW 2020 v1
BMW 2020 v2
-10
-8
-6
-4
-2
0
$a_{\mu,\text{ SIB, disc}} \times 10^{10}$

Attention on light-quark isospin-symmetric contribution and QED disconnected contribution

Lattice QCD – Time-Moment Representation

Starting from the vector current $J_\mu(x) = i\sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu \Psi_f(x)$ we may write

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

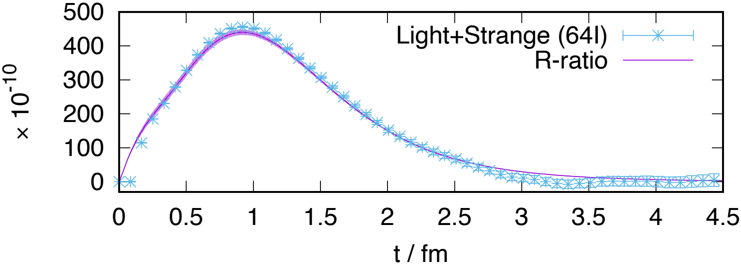
with

$$C(t) = \frac{1}{3}\sum_{\vec{x}}\sum_{j=0,1,2} \langle J_j(\vec{x},t) J_j(0)\rangle$$

and w_t capturing the photon and muon part of the HVP diagrams (Bernecker-Meyer 2011).

The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, and charm quark contributions. The missing bottom quark contributions are computed in pQCD.

Lattice QCD – Example of correlation function $C(t)$ (RBC/UKQCD18)



Large discretization errors at short distance, large finite-volume errors and statistical errors at large distance

Window method (introduced in RBC/UKQCD 2018)

We therefore also consider a window method. Following Meyer-Bernecker 2011 and smearing over t to define the continuum limit we write

$$a_\mu = a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$$

with

$$a_\mu^{\rm SD} = \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)] \,,$$

$$a_\mu^{\rm W} = \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)] \,,$$

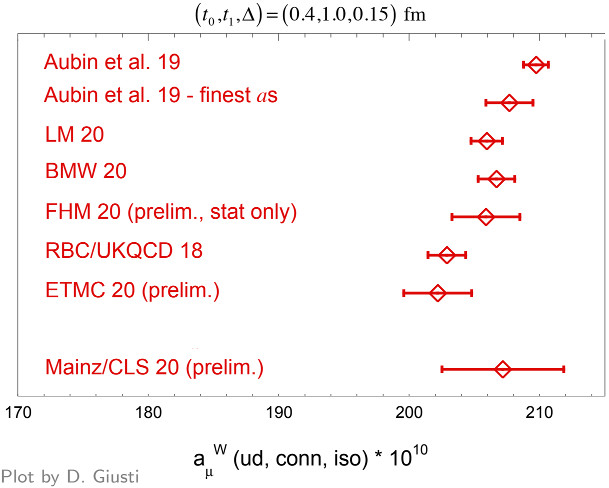
$$a_\mu^{\rm LD} = \sum_t C(t) w_t \Theta(t, t_1, \Delta) \,,$$

$$\Theta(t, t', \Delta) = [1 + \tanh [(t - t')/\Delta]] /2 \,.$$

All contributions are well-defined individually and can be computed from lattice or R-ratio via $C(t) = \frac{1}{12\pi^2} \int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2} \sigma(s, e^+e^- \to {\rm had})$.

$a_\mu^{\rm W}$ has small statistical and systematic errors on lattice!

Use these windows as a lattice internal cross-check



Plot by D. Giusti

Plot from recent theory initiative workshop (https://indico.cern.ch/event/956699/)

Status of consistency of lattice results

Significant difference between published high-precision LQCD results (BMW20 and RBC/UKQCD18) for window with $t_0 = 0.4\text{fm}$ and $t_1 = 1.0\text{fm}$:

$$a_{\mathrm{W}}^{\mathrm{BMW20}} = 207.3(1.4) \times 10^{-10}\,, \tag{1}$$

$$a_{\mathrm{W}}^{\mathrm{RBC/UKQCD18}} = 202.9(1.4)(0.4) \times 10^{-10} \tag{2}$$

and therefore there is a 2.2σ tension

$$a_{\mathrm{W}}^{\mathrm{BMW20}} - a_{\mathrm{W}}^{\mathrm{RBC/UKQCD18}} = 4.4(2.0) \times 10^{-10}\,. \tag{3}$$

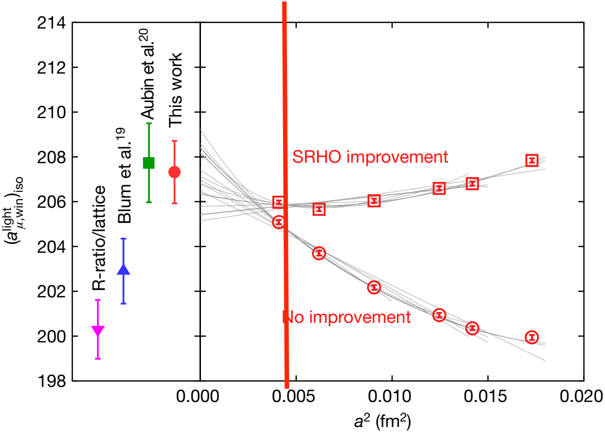
Scaled to the total a_μ^{HVP} this corresponds to 15×10^{-10} uncertainty on the lattice HVP compared to current 5.5×10^{-10} uncertainty of BMW20.

Urgently need new results for this and other windows. Update by RBC/UKQCD 2018 is in preparation. Hopefully available within two months. More groups to join. Important: different regulators!

Continuum extrapolation - What lattice spacing is fine enough?

Logarithmic corrections to a^n behavior: Husung, Marquard, Sommer Eur.Phys.J.C 80 (2020) 3, 200

BMW 20 - light quark window

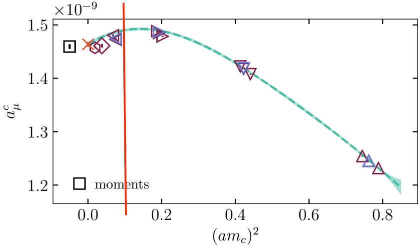


3.7σ tension between BMW20 and R-ratio for Window! Discuss in second part of talk.

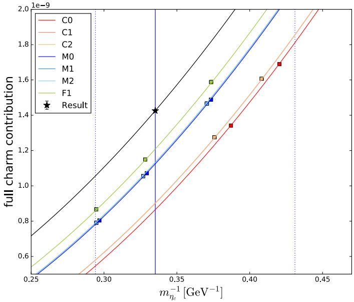
Red line for comparison with next slide

Continuum extrapolation - What lattice spacing is fine enough?

HPQCD 20 charm quark full a_μ arXiv:2005.01845



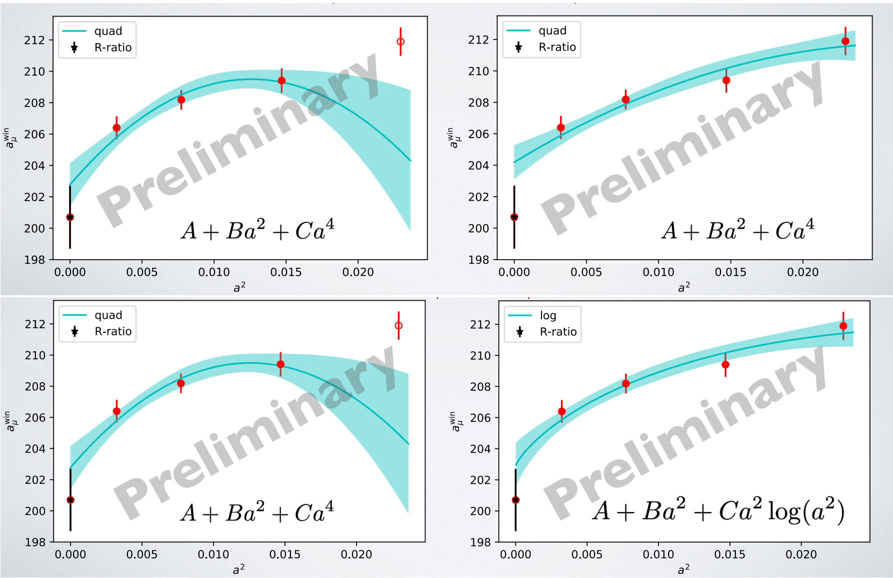
RBC 18 charm quark full a_μ



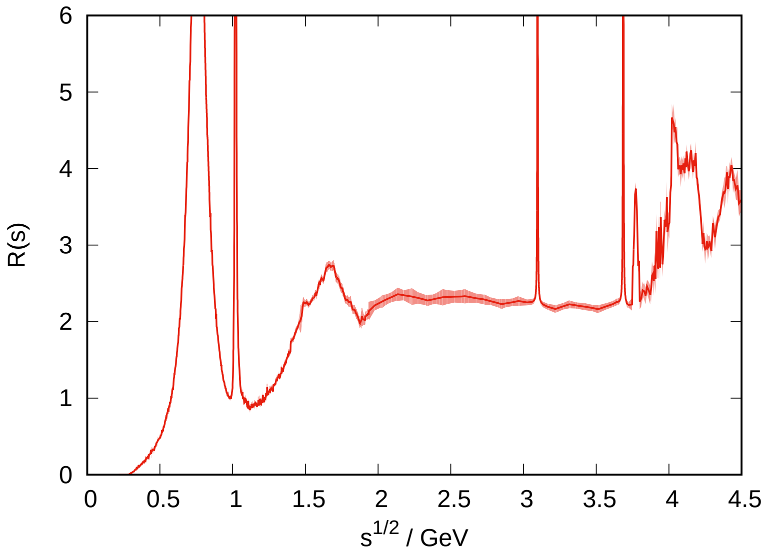
Finest lattice spacing in this extrapolation is green; approximately corresponds to red line in previous plots

Restricting to fixed lattice spacing range can lead to different discretization errors for different UV regulators; systematically independent calculations very desirable!

Aubin et al. 2021 preliminary

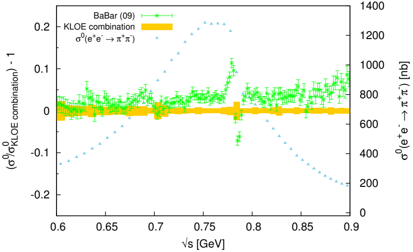


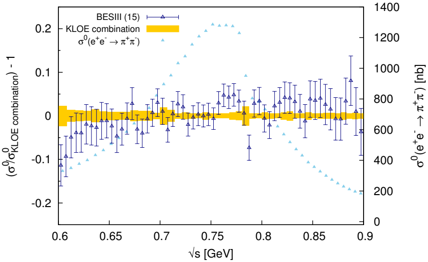
Consistency of lattice result with R-ratio

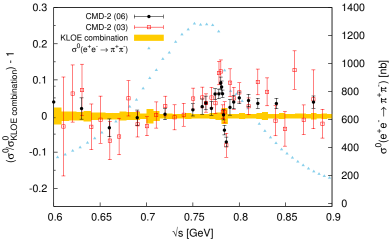


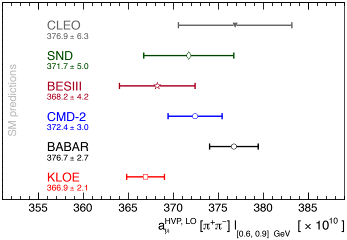
$$R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \mathrm{had})\,, \quad C(t) = \frac{1}{12\pi^2}\int_0^\infty d(\sqrt{s})R(s)se^{-\sqrt{s}t}$$

Tensions in input data, however, already taken into account in WP20 merger of KNT19 and DHMZ19:









What does tension in windows mean for R-ratio?

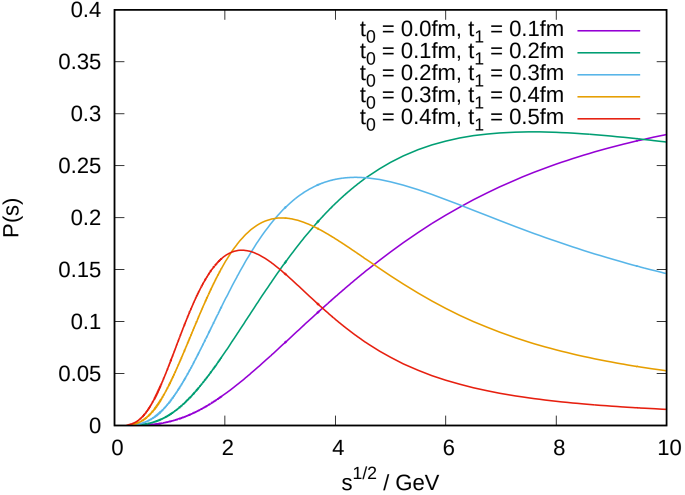
If there is a shift in R-ratio, it crucially depends on which energy to understand what the impact on $\Delta\alpha$ and EW precision physics is.

Express Euclidean Windows in time-like region:

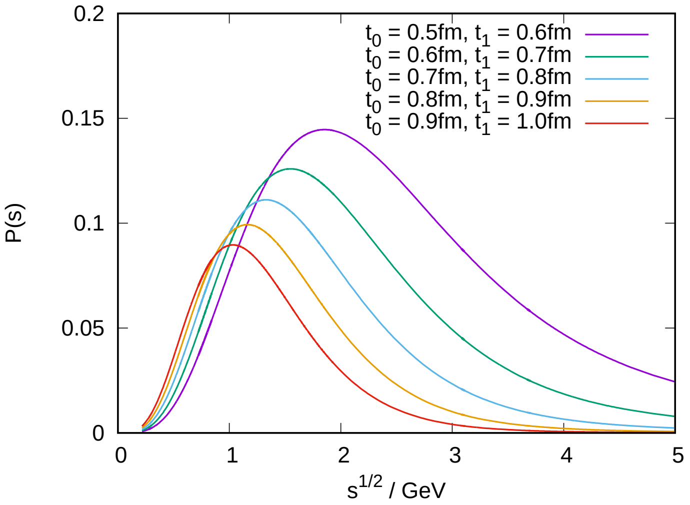
$$a_\mu = \int_0^\infty ds\, R(s)K(s) \tag{4}$$

and window

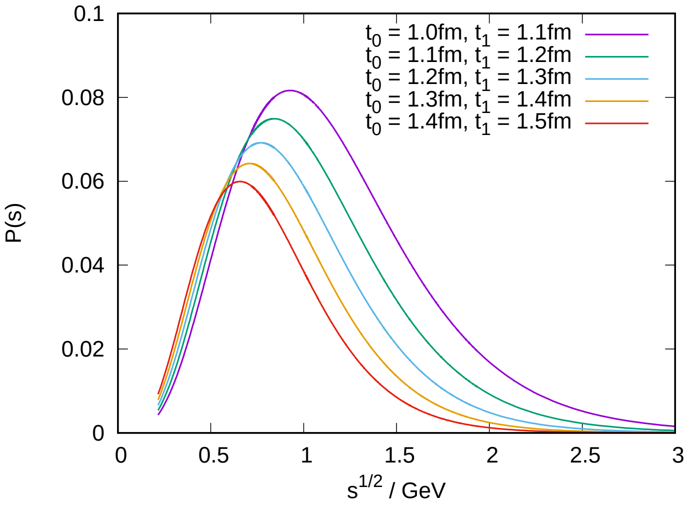
$$a_\mu^{\mathrm{W}} = \int_0^\infty ds\, R(s)K(s)P(s)\,. \tag{5}$$



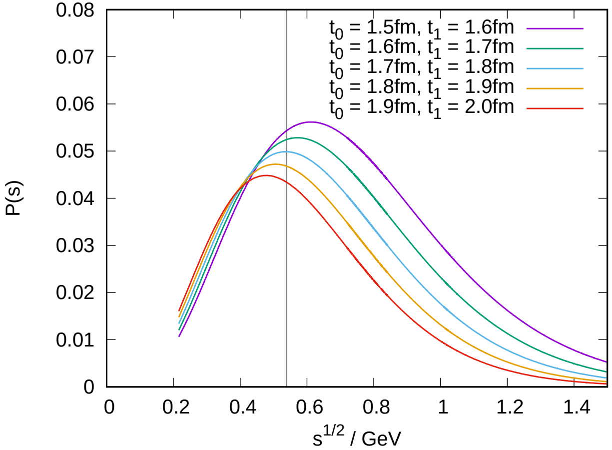
Study of windows for different t_0 and t_1 can give some energy resolution!



Study of windows for different t_0 and t_1 can give some energy resolution!

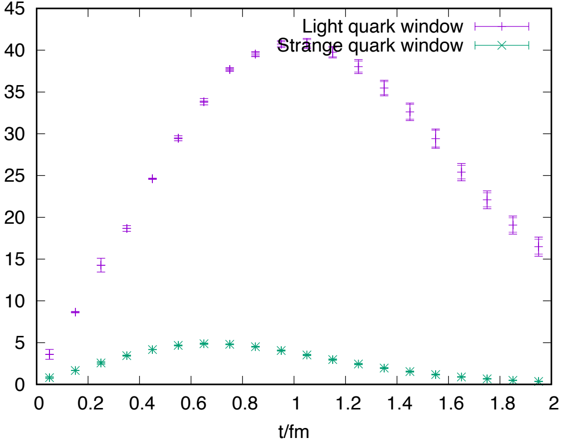


Study of windows for different t_0 and t_1 can give some energy resolution!



Below black line, we can use Lellouche-Lüscher-Meyer formalism to get $R(s)$ from lattice directly! Programs for this by Mainz and RBC/UKQCD.

First results for more windows already available - Lehner & Meyer 2020



Here: $t_0 = t,\ t_1 = t + 0.1\text{fm}$

No results for QED, SIB, and charm contribution yet available.

First results for more windows already available - Lehner & Meyer 2020

t_0/fm	t_1/fm	Δ/fm	$a_\mu^{\text{ud,conn.,isospin}} 10^{10}$	$a_\mu^{\text{s,conn.,isospin}} 10^{10}$
Total			657(26)(12)	52.83(22)(65)
0.0	0.1	0.15	3.60(00)(59)	0.81(00)(12)
0.1	0.2	0.15	8.649(03)(73)	1.666(01)(12)
0.2	0.3	0.15	14.27(01)(82)	2.57(00)(16)
0.3	0.4	0.15	18.67(02)(35)	3.448(05)(65)
0.4	0.5	0.15	24.617(35)(63)	4.170(07)(20)
0.5	0.6	0.15	29.47(06)(29)	4.666(10)(59)
0.6	0.7	0.15	33.85(10)(37)	4.866(13)(74)
0.7	0.8	0.15	37.71(14)(15)	4.799(16)(39)
0.8	0.9	0.15	39.55(20)(21)	4.505(17)(44)
0.9	1.0	0.15	40.77(27)(31)	4.058(19)(65)
1.0	1.1	0.15	40.86(44)(41)	3.527(19)(76)
1.1	1.2	0.15	39.81(54)(42)	2.973(19)(75)
1.2	1.3	0.15	38.10(65)(51)	2.441(18)(77)
1.3	1.4	0.15	35.54(77)(53)	1.955(17)(67)
1.4	1.5	0.15	32.70(88)(56)	1.534(15)(60)
1.5	1.6	0.15	29.50(100)(58)	1.181(13)(52)
1.6	1.7	0.15	25.51(81)(66)	0.894(12)(44)
1.7	1.8	0.15	22.20(85)(66)	0.667(10)(37)
1.8	1.9	0.15	19.18(86)(67)	0.491(08)(30)
1.9	2.0	0.15	16.59(89)(75)	0.357(07)(24)

0.0	0.2	0.15	12.25(00)(52)	2.48(00)(11)
0.2	0.4	0.15	32.95(03)(48)	6.02(01)(10)
0.4	0.6	0.15	54.08(10)(29)	8.837(18)(74)
0.6	0.8	0.15	71.55(24)(38)	9.666(29)(91)
0.8	1.0	0.15	80.33(47)(44)	8.56(04)(10)
0.3	1.0	0.15	224.6(0.8)(1.1)	30.51(08)(25)
0.3	1.3	0.15	343.1(2.6)(2.0)	39.45(13)(35)
0.3	1.6	0.15	441.0(5.1)(3.4)	44.12(17)(49)
0.4	1.0	0.15	205.97(79)(90)	27.06(08)(21)
0.4	1.3	0.15	324.6(2.6)(1.9)	36.01(13)(36)
0.4	1.6	0.15	422.4(5.1)(3.5)	40.68(17)(51)
0.4	1.0	0.05	216.5(0.8)(6.2)	27.9(0.1)(1.1)
0.4	1.0	0.1	209.80(77)(79)	27.70(08)(21)
0.4	1.0	0.2	202.10(82)(91)	26.24(08)(21)

More results expected by other collaborations soon!

What can we expect from LQCD in the coming years?

- More published results with high precision with different regulators for the standard window $t_0 = 0.4$fm, $t_1 = 1.0$fm, $\Delta = 0.15$fm. This will clarify the 2.2σ tension between BMW20 and RBC/UKQCD18 for this quantity.
- More results for different windows, which will give energy resolution to locate possible remaining tension with R-ratio in time-like energy. After this: any impact on $\Delta\alpha$ and EW precision physics?
- More results of complete high-precision HVP results from major lattice collaborations. RBC/UKQCD18 aims for end of this year.

Outlook

- Expect more lattice HVP calculations at few per-mille level precision which allows for proper scrutiny at high precision; For total a_μ as well as windows!
- Data-driven dispersive results will improve with expected experimental results from Belle II, BESIII, CMD-3, and SND
- MUonE at CERN will provide complementary measurements for the HVP
- Theory Initiative will publish updated SM predictions as experiment and theory improves; provides platform for cross-checks and establishing new methodology

Thank You!

Papers that directly enter the WP20 SM prediction

- T. Blum et al. Phys. Rev. Lett. **124**, no.13, 132002 (2020)
- A. Keshavarzi, D. Nomura and T. Teubner, Phys. Rev. D **101**, no.1, 014029 (2020)
- G. Colangelo, F. Hagelstein, M. Hoferichter, L. Laub and P. Stoffer, JHEP **03**, 101 (2020)
- J. Bijnens, N. Hermansson-Truedsson and A. Rodríguez-Sánchez, Phys. Lett. B **798**, 134994 (2019)
- M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur. Phys. J. C **80**, no.3, 241 (2020) [erratum: Eur. Phys. J. C **80**, no.5, 410 (2020)]
- M. Hoferichter, B. L. Hoid and B. Kubis, JHEP **08**, 137 (2019)
- A. Gérardin, H. B. Meyer and A. Nyffeler, Phys. Rev. D **100**, no.3, 034520 (2019)
- T. Aoyama, T. Kinoshita and M. Nio, Atoms **7**, no.1, 28 (2019)
- G. Colangelo, M. Hoferichter and P. Stoffer, JHEP **02**, 006 (2019)
- M. Hoferichter, B. L. Hoid, B. Kubis, S. Leupold and S. P. Schneider, JHEP **10**, 141 (2018)
- A. Keshavarzi, D. Nomura and T. Teubner, Phys. Rev. D **97**, no.11, 114025 (2018)
- M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur. Phys. J. C **77**, no.12, 827 (2017)
- G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, JHEP **04**, 161 (2017)
- P. Masjuan and P. Sanchez-Puertas, Phys. Rev. D **95**, no.5, 054026 (2017)
- G. Colangelo, M. Hoferichter, A. Nyffeler, M. Passera and P. Stoffer, Phys. Lett. B **735**, 90-91 (2014)
- A. Kurz, T. Liu, P. Marquard and M. Steinhauser, Phys. Lett. B **734**, 144-147 (2014)
- C. Gnendiger, D. Stöckinger and H. Stöckinger-Kim, Phys. Rev. D **88**, 053005 (2013)
- T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, Phys. Rev. Lett. **109**, 111808 (2012)
- K. Melnikov and A. Vainshtein, Phys. Rev. D **70**, 113006 (2004)
- A. Czarnecki, W. J. Marciano and A. Vainshtein, Phys. Rev. D **67**, 073006 (2003) [erratum: Phys. Rev. D **73**, 119901 (2006)]

Results in plots that have appeared after the WP deadline

- E. H. Chao, R. J. Hudspith, A. Gérardin, J. R. Green, H. B. Meyer and K. Ottnad, [arXiv:2104.02632 [hep-lat]].
- Borsanyi, S., Fodor, Z., Guenther, J.N. et al. Leading hadronic contribution to the muon magnetic moment from lattice QCD. Nature (2021)
- C. Lehner and A. S. Meyer, Phys. Rev. D **101**, 074515 (2020)

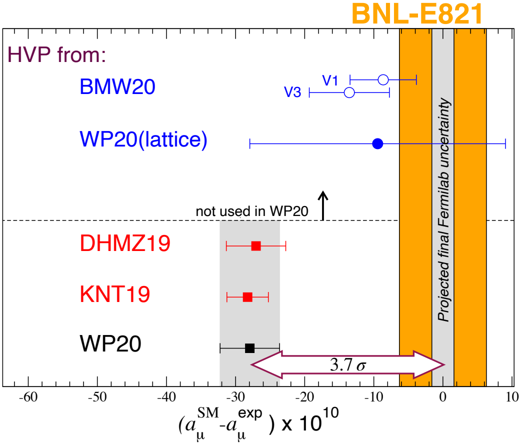
Backup

The anomalous magnetic moment of the muon in the Standard Model

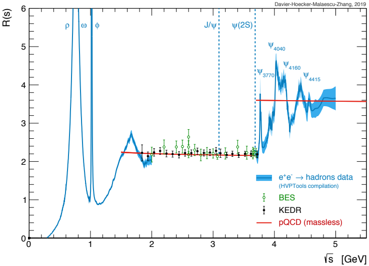
T. Aoyama[1,2,3], N. Asmussen[4], M. Benayoun[5], J. Bijnens[6], T. Blum[7,8], M. Bruno[9], I. Caprini[10], C. M. Carloni Calame[11], M. Cè[9,12,13], G. Colangelo[†14], F. Curciarello[15,16], H. Czyż[17], I. Danilkin[12], M. Davier[†18], C. T. H. Davies[19], M. Della Morte[20], S. I. Eidelman[†21,22], A. X. El-Khadra[†23,24], A. Gérardin[25], D. Giusti[26,27], M. Golterman[28], Steven Gottlieb[29], V. Gülpers[30], F. Hagelstein[14], M. Hayakawa[31,2], G. Herdoíza[32], D. W. Hertzog[33], A. Hoecker[34], M. Hoferichter[†14,35], B.-L. Hoid[36], R. J. Hudspith[12,13], F. Ignatov[21], T. Izubuchi[37,8], F. Jegerlehner[38], L. Jin[7,8], A. Keshavarzi[39], T. Kinoshita[40,41], B. Kubis[36], A. Kupich[21], A. Kupść[42,43], L. Laub[14], C. Lehner[†26,37], L. Lellouch[25], I. Logashenko[21], B. Malaescu[5], K. Maltman[44,45], M. K. Marinković[46,47], P. Masjuan[48,49], A. S. Meyer[37], H. B. Meyer[12,13], T. Mibe[†1], K. Miura[12,13,3], S. E. Müller[50], M. Nio[2,51], D. Nomura[52,53], A. Nyffeler[†12], V. Pascalutsa[12], M. Passera[54], E. Perez del Rio[55], S. Peris[48,49], A. Portelli[30], M. Procura[56], C. F. Redmer[12], B. L. Roberts[†57], P. Sánchez-Puertas[49], S. Serednyakov[21], B. Shwartz[21], S. Simula[27], D. Stöckinger[58], H. Stöckinger-Kim[58], P. Stoffer[59], T. Teubner[†60], R. Van de Water[24], M. Vanderhaeghen[12,13], G. Venanzoni[61], G. von Hippel[12], H. Wittig[12,13], Z. Zhang[18],

M. N. Achasov[21], A. Bashir[62], N. Cardoso[47], B. Chakraborty[63], E.-H. Chao[12], J. Charles[25], A. Crivellin[64,65], O. Deineka[12], A. Denig[12,13], C. DeTar[66], C. A. Dominguez[67], A. E. Dorokhov[68], V. P. Druzhinin[21], G. Eichmann[69,47], M. Fael[70], C. S. Fischer[71], E. Gámiz[72], Z. Gelzer[23], J. R. Green[9], S. Guellati-Khelifa[73], D. Hatton[19], N. Hermansson-Truedsson[14], S. Holz[36], B. Hörz[74], M. Knecht[25], J. Koponen[1], A. S. Kronfeld[24], J. Laiho[75], S. Leupold[42], P. B. Mackenzie[24], W. J. Marciano[37], C. McNeile[76], D. Mohler[12,13], J. Monnard[14], E. T. Neil[77], A. V. Nesterenko[68], K. Ottnad[12], V. Pauk[12], A. E. Radzhabov[78], E. de Rafael[25], K. Raya[79], A. Risch[12], A. Rodríguez-Sánchez[6], P. Roig[80], T. San José[12,13], E. P. Solodov[21], R. Sugar[81], K. Yu. Todyshev[21], A. Vainshtein[82], A. Vaquero Avilés-Casco[66], E. Weil[71], J. Wilhelm[12], R. Williams[71], A. S. Zhevlakov[78]

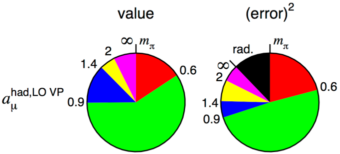
$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm Weak} + a_\mu^{\rm HVP} + a_\mu^{\rm HLbL} = 116591810\,(43) \times 10^{-11}$$

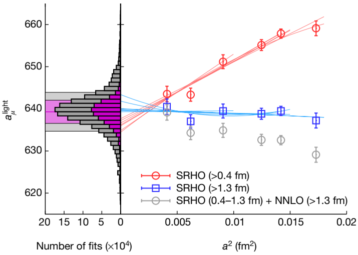


[M. Davier et al, arXiv:1908.00921]

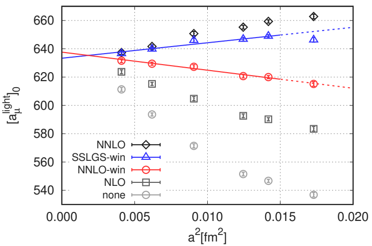


[A. Keshavarzi et al, arXiv:1802.02995]

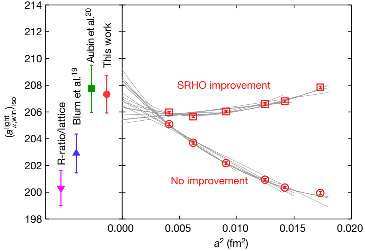




SRHO (>0.4 fm)
SRHO (>1.3 fm)
SRHO (0.4–1.3 fm) + NNLO (>1.3 fm)
Number of fits (×10^4)
a^2 (fm^2)



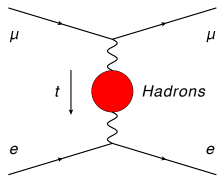
NNLO
SSLGS-win
NNLO-win
NLO
none
a^2[fm^2]

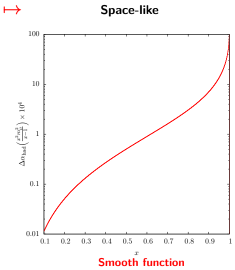


R-ratio/lattice
Blum et al.19
Aubin et al.20
This work
SRHO improvement
No improvement
a^2 (fm^2)

μ-e elastic scattering to measure a_μ^{HVP}

LOI June 2019 [P. Banerjeei et al, arXiv:2004.13663, Eur.Phys.J.C 80 (2020)]





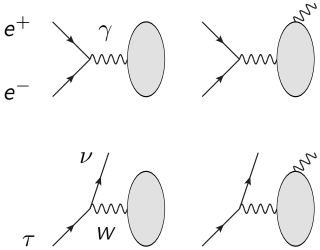
CC, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

- use CERN M2 muon beam (150 GeV)
- Physics beyond colliders program @ CERN
- LOI June 2019
- pilot run in 2021
- full apparatus in 2023-2024

μONe

Contribution	PdRV(09) [471]	N/JN(09) [472, 573]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	− 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

Dispersive method - Overview



$e^+e^- \to \mathrm{hadrons}(\gamma)$

$$J_\mu = V_\mu^{I=1,I_3=0} + V_\mu^{I=0,I_3=0}$$

$\tau \to \nu\mathrm{hadrons}(\gamma)$

$$J_\mu = V_\mu^{I=1,I_3=\pm1} - A_\mu^{I=1,I_3=\pm1}$$

Knowledge of isospin-breaking corrections and separation of vector and axial-vector components needed to use τ decay data.

Can have both energy-scan and ISR setup.

Experiment	$a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau]$ (10^{-10}) $2m_{\pi^\pm} - 0.36$ GeV	$0.36 - 1.8$ GeV
ALEPH	$9.80 \pm 0.40 \pm 0.05 \pm 0.07$	$501.2 \pm 4.5 \pm 2.7 \pm 1.9$
CLEO	$9.65 \pm 0.42 \pm 0.17 \pm 0.07$	$504.5 \pm 5.4 \pm 8.8 \pm 1.9$
OPAL	$11.31 \pm 0.76 \pm 0.15 \pm 0.07$	$515.6 \pm 9.9 \pm 6.9 \pm 1.9$
Belle	$9.74 \pm 0.28 \pm 0.15 \pm 0.07$	$503.9 \pm 1.9 \pm 7.8 \pm 1.9$
Combined	$9.82 \pm 0.13 \pm 0.04 \pm 0.07$	$506.4 \pm 1.9 \pm 2.2 \pm 1.9$

Davier et al. 2013: $a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau] = 516.2(3.5) \times 10^{-10}$ ($2m_\pi^\pm$ – 1.8 GeV)

Compare to e^+e^-:

- $a_\mu^{\mathrm{had,LO}}[\pi\pi, e^+e^-] = 507.1(2.6) \times 10^{-10}$ (DHMZ17, $2m_\pi^\pm$ – 1.8 GeV)
- $a_\mu^{\mathrm{had,LO}}[\pi\pi, e^+e^-] = 503.7(2.0) \times 10^{-10}$ (KNT18, $2m_\pi^\pm$ – 1.937 GeV)

Here treatment of isospin-breaking to relate matrix elements of $V_\mu^{I=1,I_3=1}$ to $V_\mu^{I=1,I_3=0}$ crucial. Progress towards a first-principles calculation from LQCD+QED (arXiv:1811.00508).

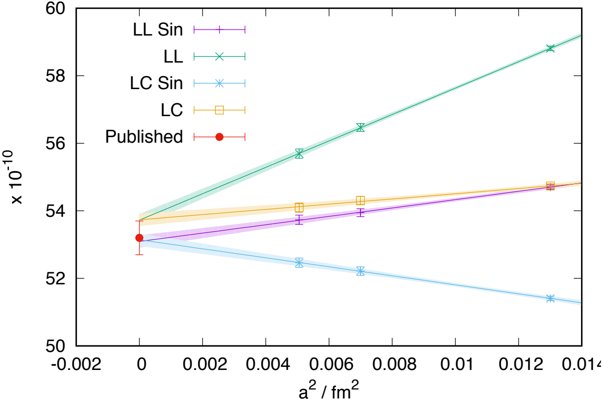
Analysis of the Hadronic Light-by-Light Contributions to the Muon $g-2$

Johan Bijnens, Elisabetta Pallante, Joaquim Prades

We calculate the hadronic light-by-light contributions to the muon $g-2$. We use both $1/N_c$ and chiral counting to organize the calculation. Then we calculate the leading and next-to-leading order in the $1/N_c$ expansion low energy contributions using the Extended Nambu--Jona-Lasinio model as hadronic model. We do that to all orders in the external momenta and quark masses expansion. Although the hadronic light-by-light contributions to muon $g-2$ are not saturated by these low energy contributions we estimate them conservatively. A detailed analysis of the different hadronic light-by-light contributions to muon $g-2$ is done. The dominant contribution is the twice anomalous pseudoscalar exchange diagram. The final result we get is $a_\mu^{\text{light-by-light}} = (-9.2 \pm 3.2) \cdot 10^{-10}$. This is between two and three times the expected experimental uncertainty at the forthcoming BNL muon $g-2$ experiment.

Add $a^{-1} = 2.77$ GeV lattice spacing

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):



- For light quark need new ensemble at physical pion mass. Data still being generated on Summit in USA and Booster in Germany ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV)