Relativistic dark matter

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Keywords:

– dark matter + quantum statistics

– real scalar dark matter

– thermalization

– relativistic freeze-in

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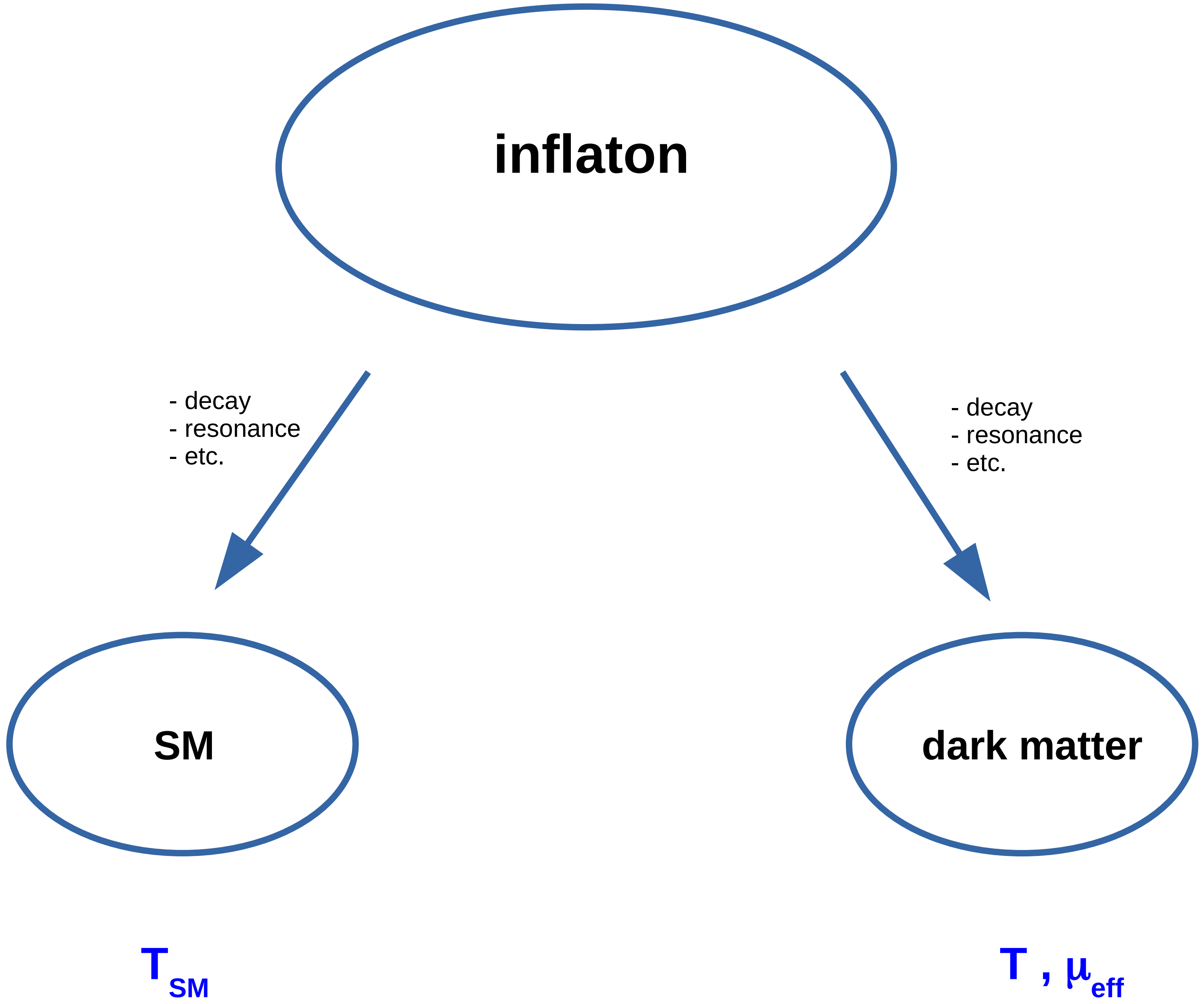

inflaton
- decay
- resonance
- etc.
- decay
- resonance
- etc.
SM
dark matter
T_{SM}
T , μ_{eff}

Typical dynamics:

initial state → thermalization → freeze-out

Common approach:

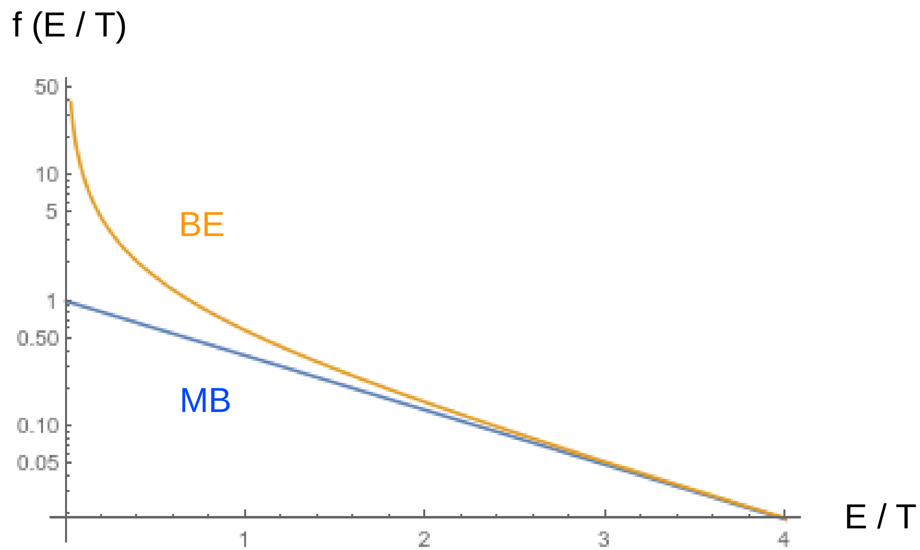
$$\begin{cases} f(p) = \dfrac{1}{e^{\frac{E-\mu}{T}} - 1} \quad \rightarrow \quad f(p) = e^{-(E-\mu)/T} \\ \mu \rightarrow 0 \end{cases}$$

Dœs not work in the relativistic regime T >>m !

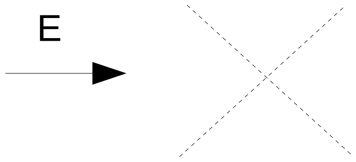
(and μ dœs not factorize)

Maxwell-Boltzmann vs Bose-Einstein

$$f(p) = e^{-(E-\mu)/T} \qquad \textbf{vs} \qquad f(p) = \frac{1}{e^{\frac{E-\mu}{T}} - 1}$$



“Massless” scalar scattering:



$\sigma \sim 1 / E^2$

rate ~ $\int d^3\mathbf{p_1} d^3\mathbf{p_2}\ f(p_1) f(p_2)\ \sigma(p_1, p_2) v_{\text{Møl}}$

IR sensitive!

Carlson,Machacek,Hall '92
Bernal, Chu '16

Simplest dark matter model

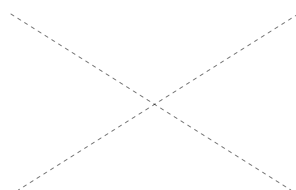
$$V = \frac{m^2}{2} S^2 + \frac{\lambda}{4!} S^4$$

Arcadi,OL,Pokorski,Toma ' 19

Thermodynamics:

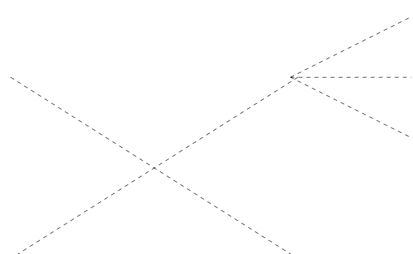
Bose-Einstein statistics $$f(p) = \frac{1}{e^{\frac{E-\mu}{T}} - 1}$$

2 ⟷ 2



$\Gamma_{22} > H$ ⇒ T , μ_{eff}

2 ⟷ 4



$\Gamma_{24} > H$ ⇒ T

Arcadi,OL,Pokorski,Toma ' 19

Relativistic reaction rates

$$\Gamma_{a\to b} = \int \left(\prod_{i\in a} \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i} f(p_i)\right) \left(\prod_{j\in b} \frac{d^3\mathbf{p}_j}{(2\pi)^3 2E_j}(1+f(p_j))\right) |\mathcal{M}_{a\to b}|^2 \,(2\pi)^4 \delta^4(p_a - p_b)$$

2 → 4 :

$$\sigma(p_1,p_2) = \frac{1}{4F(p_1,p_2)} \int |\mathcal{M}_{2\to 4}|^2 (2\pi)^4 \delta^4(p_1+p_2-\sum_i k_i) \prod_i \frac{d^3\mathbf{k}_i}{(2\pi)^3 2E_{k_i}} (1+f(k_i))$$

can't neglect!

$$\Gamma_{2\to 4} = (2\pi)^{-6} \int d^3\mathbf{p_1} d^3\mathbf{p_2} \; f(p_1) f(p_2) \; \sigma(p_1,p_2) v_{\text{Møl}}$$

Issue: can only compute σ(2 → 4) in the center-of-mass frame (CalcHEP, etc.)

Conversion to the CM frame:

$$p_1\,,\,p_2 \quad\to\quad p=\frac{p_1+p_2}{2}\;,\;k=\frac{p_1-p_2}{2}$$

$$p=\Lambda(p)\begin{pmatrix}E\\0\\0\\0\end{pmatrix}$$

$$\Rightarrow\quad \int\frac{d^3\mathbf{p_1}}{2E_1}\frac{d^3\mathbf{p_2}}{2E_2}\;\ldots\;=\;2\int_m^\infty dE\,\sqrt{E^2-m^2}\,E^2\int_0^\infty d\eta\,\sinh^2\eta\int d\Omega_p\,d\Omega_k\;\ldots$$

Relativistic analog of the Gelmini-Gondolo formula:

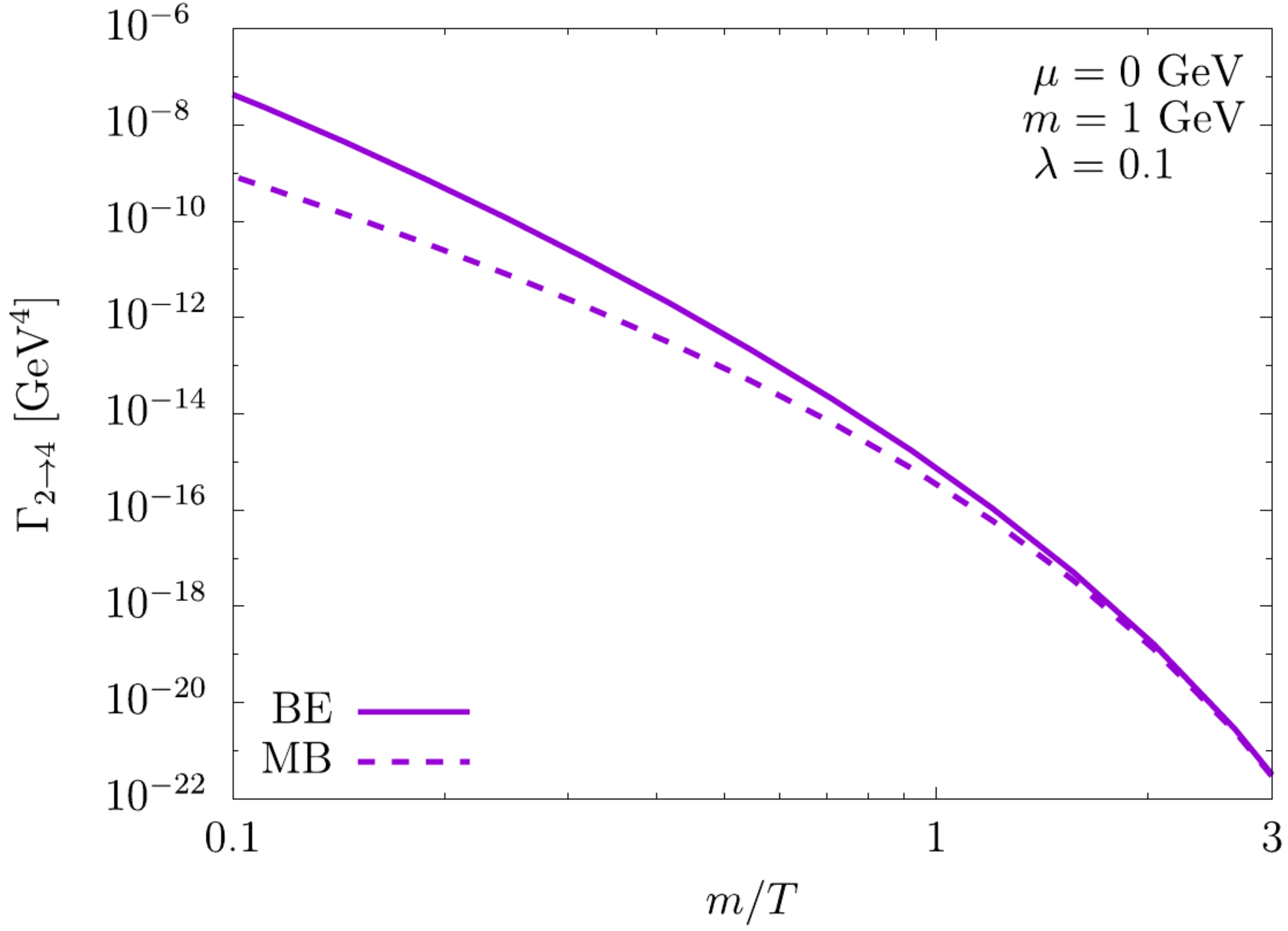
Arcadi,OL,Pokorski,Toma ' 19

$$\Gamma_{2\to4}\;=\;\frac{4T}{\pi^4}\int_m^\infty dE\;E^3\sqrt{E^2-m^2}\int_0^\infty d\eta\frac{\sinh\eta}{e^{2(E\cosh\eta-\mu)/T}-1}\;\ln\frac{\sinh\frac{E\cosh\eta+\sqrt{E^2-m^2}\sinh\eta-\mu}{2T}}{\sinh\frac{E\cosh\eta-\sqrt{E^2-m^2}\sinh\eta-\mu}{2T}}\;\sigma_{\rm CM}(E,\eta)$$

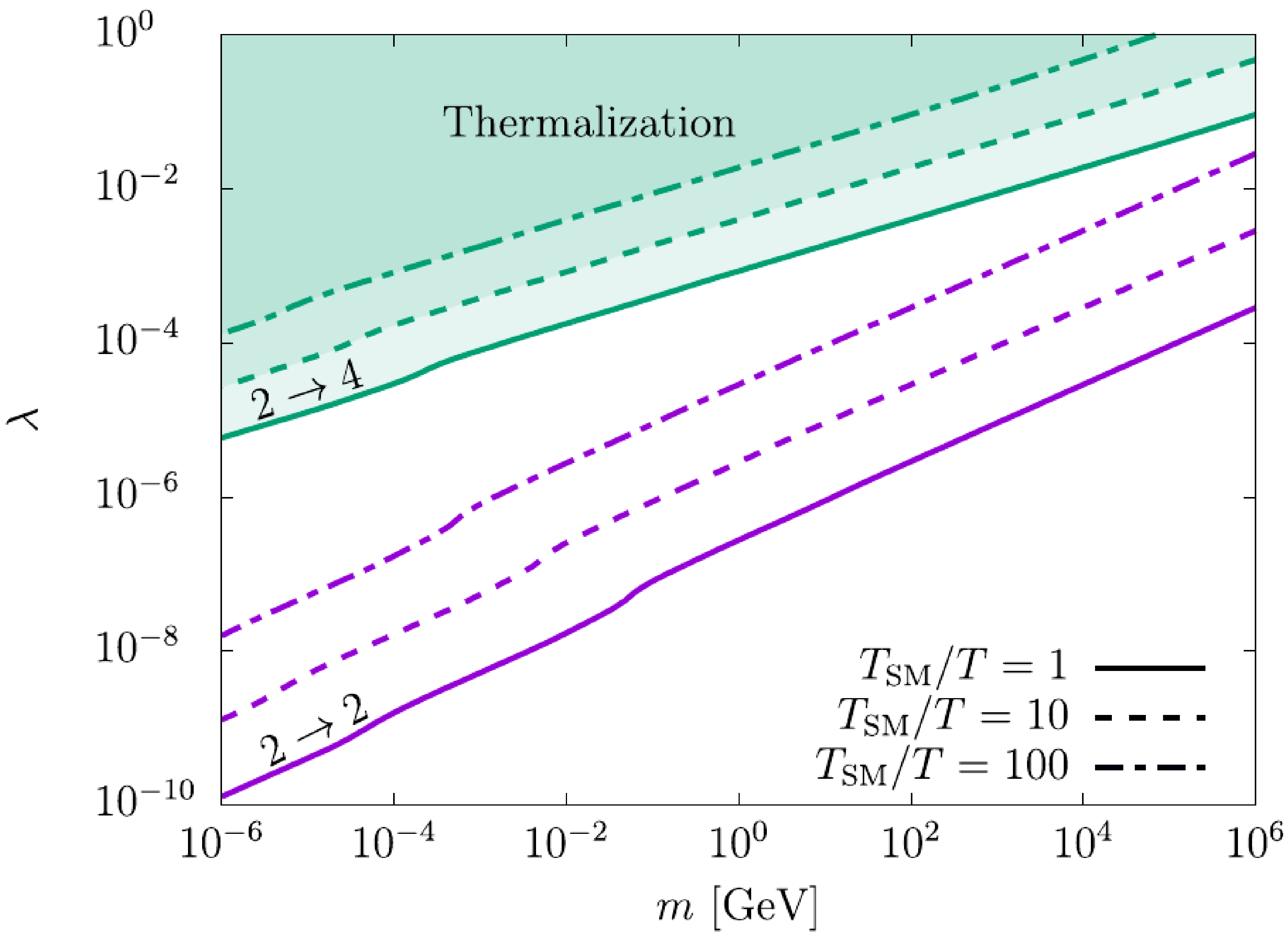
includes BE final state factors

(same for 2 → n)

Maxwell-Boltzmann vs Bose-Einstein:



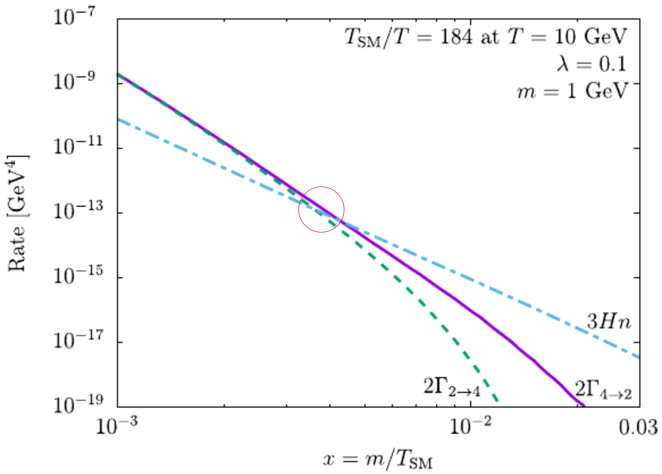
Thermal or kinetic equilibrium: Γ_{24} or Γ_{22} > n H

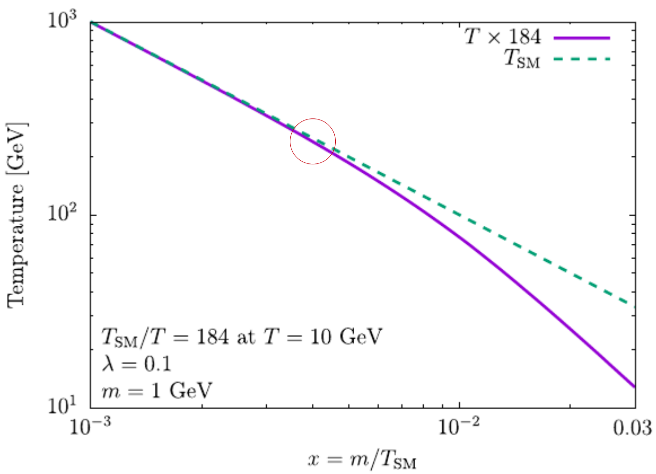


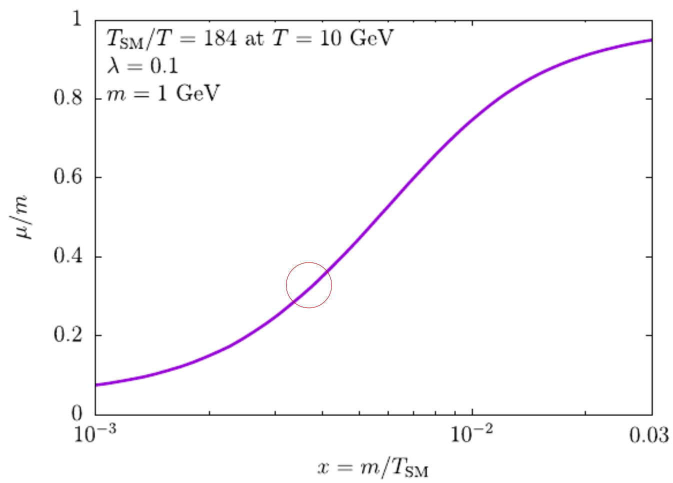
Boltzmann equation + entropy conservation:

$$\begin{cases} \dfrac{dn}{dt} + 3Hn = 2\,(\Gamma_{2\to4} - \Gamma_{4\to2}) \\ \dfrac{s}{s_{\rm SM}} = \text{const} \end{cases} \quad \Rightarrow \quad T(t)\,,\ \mu(t)$$

Freeze-out $2\,\Gamma_{24} < 3\,n\,H$:







Thermal mass effect at high T:

$$m^2 \to m^2 + \frac{\lambda}{24} T^2$$

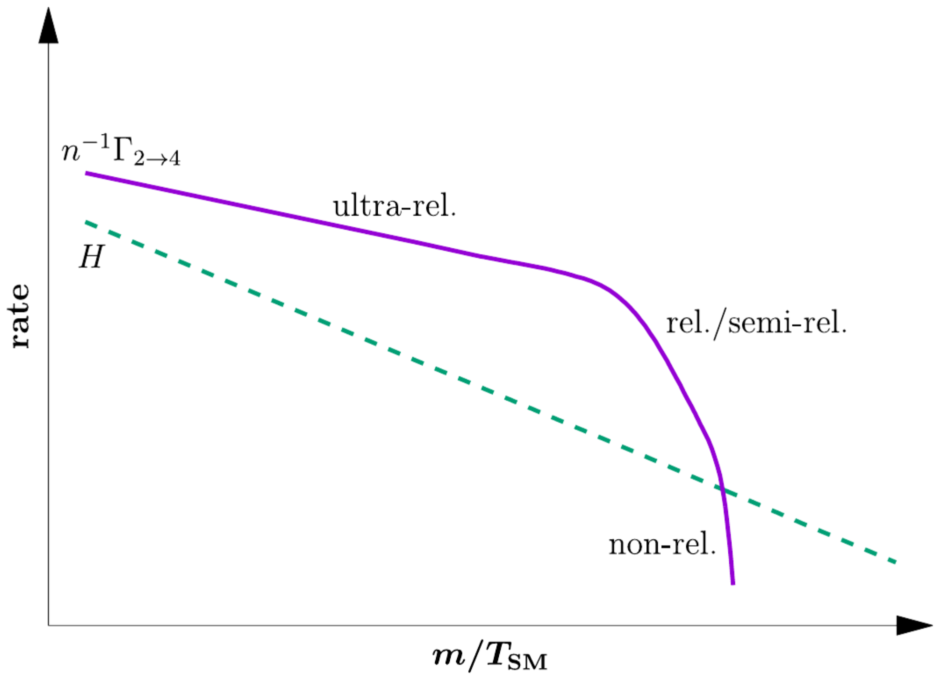
$$\Gamma_{2\to 2} \propto T^4 \ln \frac{T}{m_{\rm eff}} \;\; \to c\, T^4$$

IR divergent without
thermal mass

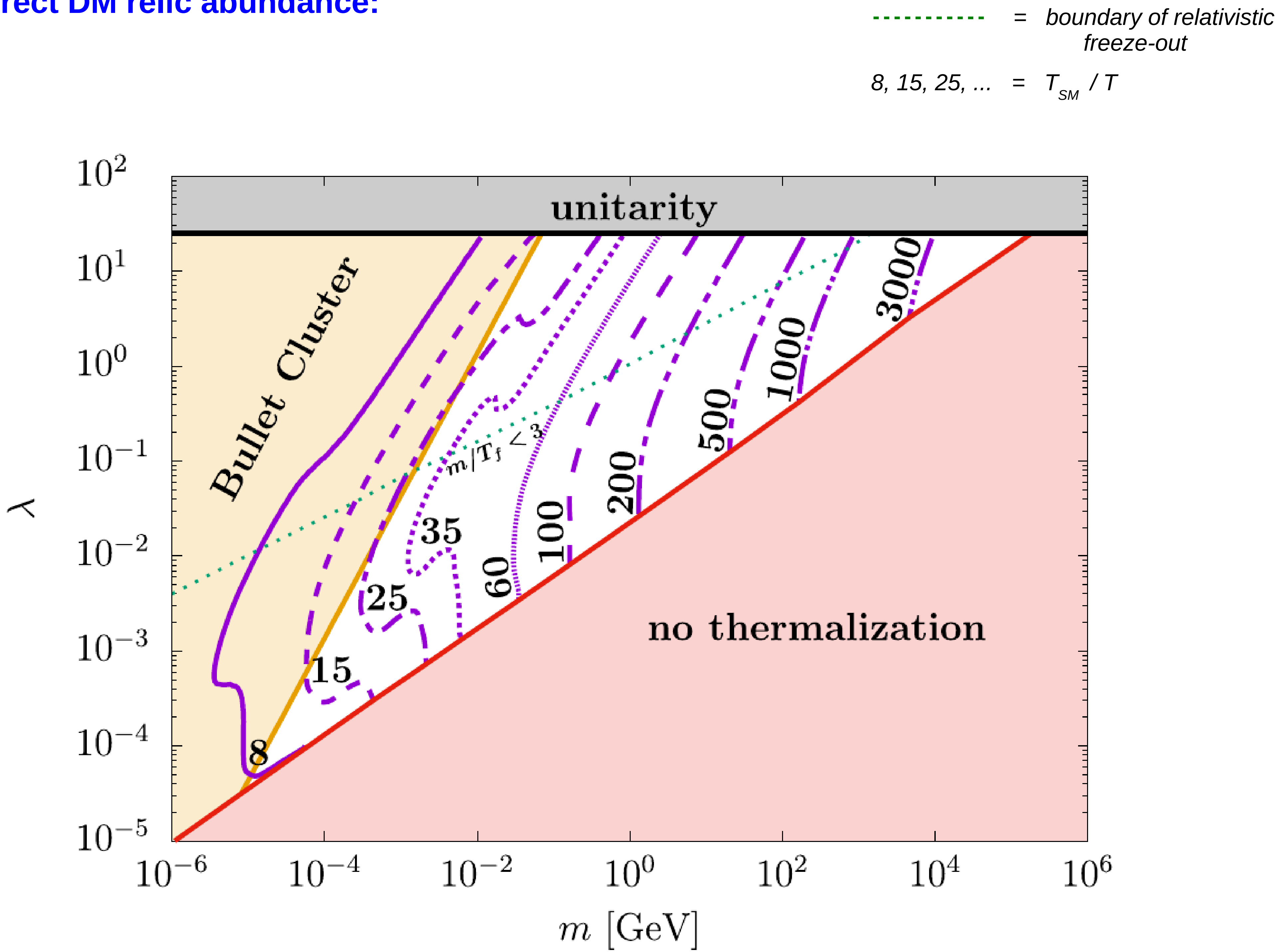
Freeze-out can't be ultra-relativistic:

$\Gamma_{24} \sim T^4$

$n H \sim T^5$



Correct DM relic abundance:



Arcadi,OL,Pokorski,Toma ‘ 19

Relativistic freeze-in

$$V_{hs} = \frac{1}{2}\lambda_{hs} H^\dagger H s^2$$

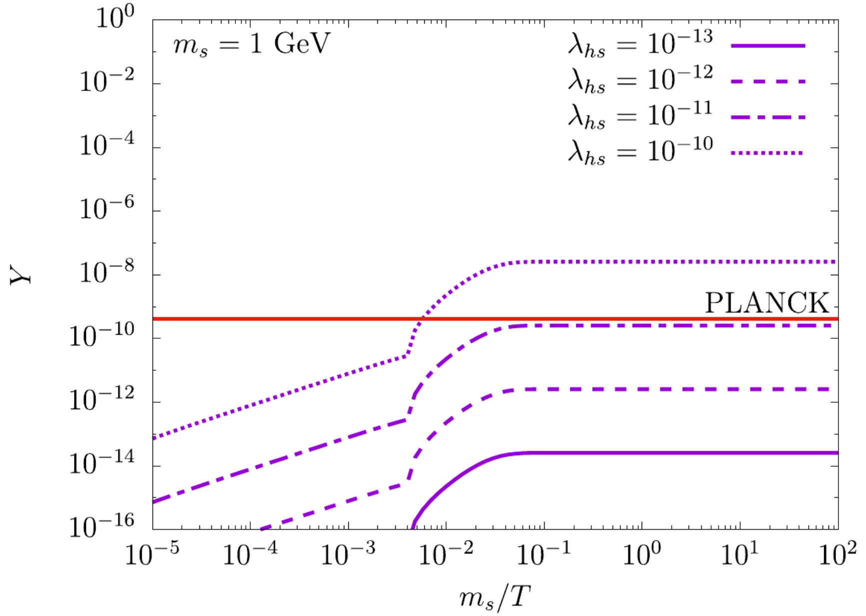
$T > T_{EW}$ regime: $h_i h_i \rightarrow ss$

$$\Gamma_{2\to2} = \frac{1}{2!2!}\frac{\lambda_{hs}^2 T}{16\pi^5}\int_{m_h}^{\infty} dE\, E\sqrt{E^2-m_s^2}\int_0^\infty d\eta \frac{\sinh\eta}{e^{\frac{2E}{T}\cosh\eta}-1}\ \ln\frac{\sinh\frac{E\cosh\eta+\sqrt{E^2-m_h^2}\sinh\eta}{2T}}{\sinh\frac{E\cosh\eta-\sqrt{E^2-m_h^2}\sinh\eta}{2T}}$$

$$m_h^2 \simeq m_{h0}^2 + \left(\frac{3}{16}g_2^2 + \frac{1}{16}g_1^2 + \frac{1}{4}y_t^2 + \frac{1}{2}\lambda_h\right)T^2$$

$$m_s > m_h \quad \rightarrow \quad \lambda_{hs} \simeq 2.2\times10^{-11}$$

$$m_s \ll m_h \quad \rightarrow \quad \lambda_{hs} \simeq 1.2\times10^{-11}\sqrt{\frac{\text{GeV}}{m_s}}$$



Gauged models: Bandyopadhyay, Mitra, Roy ‘20

General freeze-in scalar production: h h → ss , hh → s , h → ss

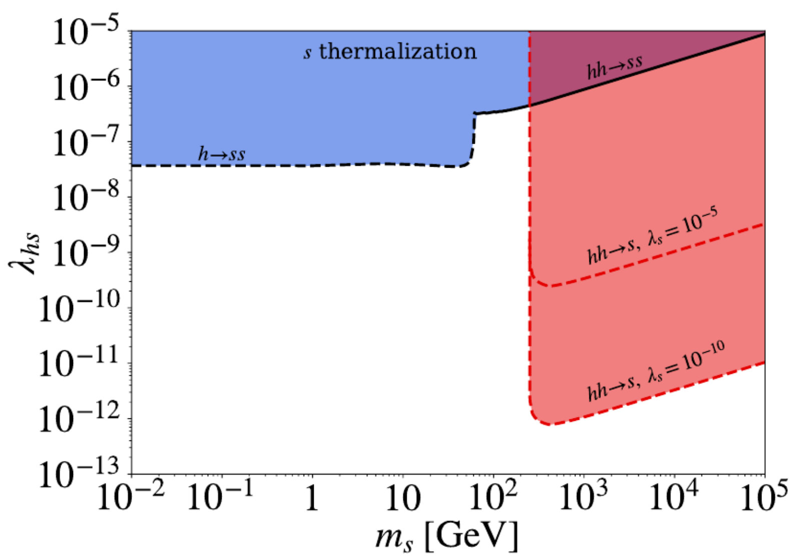
$$\Gamma_{21} = \frac{\lambda_{hs}^2 u^2 m_s T}{32\pi^3}\theta(m_s - 2m_h)\int_0^\infty d\eta \frac{\sinh\eta}{e^{\frac{m_s\cosh\eta}{T}}-1}\ \ln\frac{\sinh\frac{m_s\cosh\eta+\sqrt{m_s^2-4m_h^2}\sinh\eta}{4T}}{\sinh\frac{m_s\cosh\eta-\sqrt{m_s^2-4m_h^2}\sinh\eta}{4T}}$$

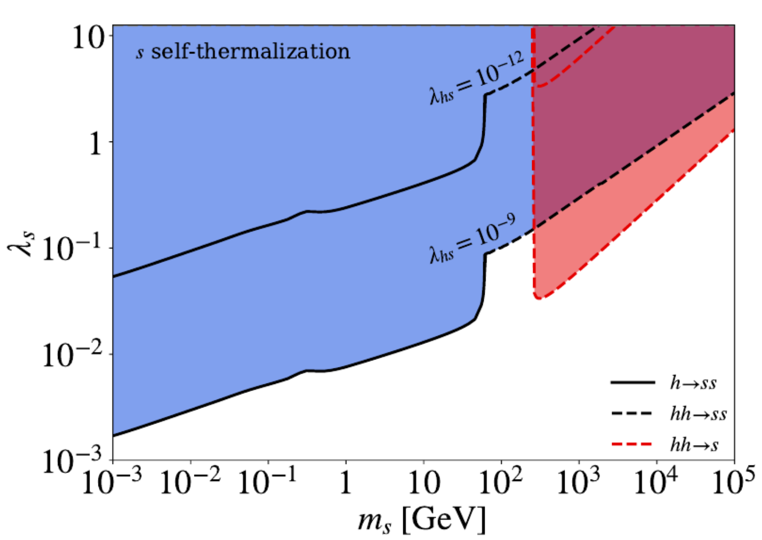
$$\Gamma_{12} = \frac{\lambda_{hs}^2 v^2 m_h^2}{64\pi^3}\sqrt{1-\frac{4m_s^2}{m_h^2}}\int_1^\infty dx\,\frac{\sqrt{x^2-1}}{e^{\frac{m_h}{T}x}-1}\,,$$

$$\Gamma_{22} = \frac{1}{2!2!}\,\frac{\lambda_{hs}^2 T}{16\pi^5}$$

$$\times\ \int_{m_h}^\infty dE\ E\sqrt{E^2-m_s^2}\int_0^\infty d\eta\frac{\sinh\eta}{e^{\frac{2E}{T}\cosh\eta}-1}\ \ln\frac{\sinh\frac{E\cosh\eta+\sqrt{E^2-m_h^2}\sinh\eta}{2T}}{\sinh\frac{E\cosh\eta-\sqrt{E^2-m_h^2}\sinh\eta}{2T}}$$

Domain of validity of freeze-in (*non-thermalization*):





CONCLUSION

– *breakdown of Maxwell-Boltzmann approximation*

– *quantum statistics can be very important*

– *possible rate enhancement by orders of magnitude*

– *relativistic freeze-out / freeze-in*