CS 4510 : Automata and Complexity
Simulating NTMs using DTMs

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Simulation of NTMs using DTMs

Theorem 1. *Every nondeterministic Turing machine has an equivalent deterministic Turing machine.*

We need to simulate a nondeterministic Turing machine (NTM) by a deterministic Turing machine (DTM). Given an NTM N, we need to simulate it using a DTM D on an input w. N is a 1-tape NTM. We shall use a 3-tape DTM M to simulate it. Since we have seen that multitape DTMs are equivalent to single tape DTMs, this is good enough.

We look at the configuration graph of N to understand the simulation. By the definition of the NTM, there are a finite set of choices that N can move from each given configuration. Call this number b. Order the possible successor configurations of a given configuration with $1, 2, \ldots, b$. This gives us the configuration tree of N, each node having at most b children. The DTM D simulates N by doing a breadth first search on the configuration graph. The tapes of D contain the following information.

1. Input w
2. The simulation tape. This contains the contents of N's tape as it is working.
3. The counter for the node in the tree.

Please see the next page for the algorithm described formally.

Guess and verify NTMs

If we have an NTM which is a decider, i.e., one which is known to halt within t steps, we could make all the guesses (nondeterministic choices) right at the beginning of the computation. We could guess one of the strings p from $\{1, 2, \ldots, b\}^t$ and proceed the computation guided by the string p. At each step, we follow the respective symbol of p, when faced with choices. Notice that once we make the guess of p, the remaining computation is completely **deterministic**. The machine has to just look up the corresponding symbol in p and decide on the choice to move. The machine accepts if the computation leads to an accept, and rejects if it leads to a reject. The configuration graph of this "guess and verify" NTM, it would look like a tree with a paths attached to each of the leaves.

Algorithm 1 Simulating NTM using DTM. Given NTM N and input string w.

1: Tape 3 $\leftarrow \varepsilon$
2: $l = 0$ // l corresponds to the length of the counter in tape 3
3: $l \leftarrow l + 1$; Tape 3 $\leftarrow 1^l$
4: Set Rejectedsofar to TRUE
5: Tape 2 $\leftarrow$ Tape 1
6: Simulate N on contents of tape 2, guided by contents of tape 3.
    Use tape 3 contents to choose from the options available and go to the required node
    - Current configuration is not defined or rejecting: GOTO step 7
    - Current configuration is accepting: HALT and ACCEPT
    - Current configuration is defined but neither accepting nor rejecting:
      Rejectedsofar = FALSE; GOTO step 7
7:
8: **if** Tape 3 = b^l **then**
9: **if** Rejectedsofar = TRUE **then**
10: HALT and REJECT
11: **else**
12: GOTO step 3
13: **else**
14: Increment string in Tape 3 and GOTO step 5