Post Correspondence Problem (PCP)

INPUT: $\{\left[\frac{t_1}{b_1}\right], \left[\frac{t_2}{b_2}\right], \ldots, \left[\frac{t_k}{b_k}\right]\}$

PROBLEM: Is there $i_1, i_2, \ldots, i_m$ such that $t_{i_1} t_{i_2} \ldots t_{i_m} = b_{i_1} b_{i_2} \ldots b_{i_m}$?

$t_1, t_2, \ldots, t_k, b_1, b_2, \ldots, b_k$ are strings over some alphabet Σ.

A solution, if it exists, is called a *match*.

PCP - example

For the collection of dominos below:

$$\left[\frac{ab}{aba}\right], \left[\frac{ba}{abb}\right], \left[\frac{b}{ab}\right], \left[\frac{abb}{b}\right], \left[\frac{a}{bab}\right]$$

here is a match:

$$\left[\frac{ab}{aba}\right] \left[\frac{a}{bab}\right] \left[\frac{ba}{abb}\right] \left[\frac{b}{ab}\right] \left[\frac{abb}{b}\right] \left[\frac{abb}{b}\right] \left[\frac{b}{ab}\right] \left[\frac{abb}{b}\right]$$

For the collection of dominos below:

$$\left[\frac{ab}{aba}\right], \left[\frac{ba}{abb}\right], \left[\frac{b}{ab}\right]$$

there is no match.

Modified Post Correspondence Problem (MPCP)

Require that the match start with the first domino.

INPUT: $\{\left[\frac{t_1}{b_1}\right], \left[\frac{t_2}{b_2}\right], \ldots, \left[\frac{t_k}{b_k}\right]\}$

PROBLEM: Is there $i_2, \ldots, i_m$ such that $t_1 t_{i_2} \ldots t_{i_m} \; = \; b_1 b_{i_2} \ldots b_{i_m}$?

$t_1, t_2, \ldots, t_k, b_1, b_2, \ldots, b_k$ are strings over some alphabet Σ.

Reduction from MPCP to PCP

Let $u = u_1 u_2 \ldots u_n$ be a string. Define

$$*u = *u_1 * u_2 * \ldots * u_n.$$

$$u* = u_1 * u_2 * \ldots * u_n *.$$

$$*u* = *u_1 * u_2 * \ldots * u_n *.$$

Given the collection of dominos:

$$\{\left[\frac{t_1}{b_1}\right], \left[\frac{t_2}{b_2}\right], \ldots, \left[\frac{t_k}{b_k}\right]\}$$

output the collection of dominos:

$$\{\left[\frac{*t_1}{*b_1*}\right], \left[\frac{*t_1}{b_1*}\right], \left[\frac{*t_2}{b_2*}\right], \ldots, \left[\frac{*t_k}{b_k*}\right], \left[\frac{*\diamond}{\diamond}\right]\}.$$

Reduction from A_{TM} to MPCP

Given $M = (Q, \Sigma, \Gamma, \delta, q_s, q_{acc}, q_{rej})$ and $w = w_1 w_2 \dots w_n$ a reduction machine constructs dominos as described below:

1. $\left[\frac{\#}{\# q_s w_1 w_2 \dots w_n \#}\right]$.

2. For all $a, b \in \Gamma$, for all $q, r \in Q$ so that $q \neq q_{rej}$:

   If $\delta(q, a) = (r, b, R)$ add dominos $\left[\frac{qa}{br}\right]$.

3. For all $a, b, c \in \Gamma$, for all $q, r \in Q$ so that $q \neq q_{rej}$:

   If $\delta(q, a) = (r, b, L)$ add dominos $\left[\frac{cqa}{rcb}\right]$.

4. For all $a \in \Gamma$, add dominos $\left[\frac{a}{a}\right]$.

5. Add dominos $\left[\frac{\#}{\#}\right]$ and $\left[\frac{\#}{\sqcup \#}\right]$.

6. For all $a \in \Gamma$, add dominos $\left[\frac{a q_{acc}}{q_{acc}}\right]$ and $\left[\frac{q_{acc} a}{q_{acc}}\right]$.

7. Add domino $\left[\frac{q_{acc}\#\#}{\#}\right]$

Correctness

- Any solution must begin with $\left[\frac{\#}{\#q_s w_1 w_2 \ldots w_n \#}\right]$.
- q_{acc} is not in the dominos: bottom string longer than top string.
- Growing the top part makes the bottom part represent the next configuration:

$$\frac{\alpha\#}{\alpha\#x\#} \longrightarrow \frac{\alpha\#x\#}{\alpha\#x\#y\#}$$

  (y is the configuration next to x.)

- If M does not accept w, q_{acc} never appears in the bottom.

  The lengths are always different and hence no match.