Tutorial 2: Modeling

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Problem 1: A farmer has 10 acres to plant in wheat and rice. He has to plant at least 7 acres. However, he has only 1200 to spend and each acre of wheat costs 200 to plant and each acre of rice costs 100 to plant. Moreover, the farmer has to get the planting done in 12 hours and it takes an hour to plant an acre of wheat and 2 hours to plant an acre of ricee. If the profit is 500 per acre of wheat and 300 per acre of ricee how many acres of each should be planted to maximize profits? Model this a *linear programming problem*

Problem 2: A gold processor has two sources of gold ore, source A and source B. In order to keep his plant running, at least three tons of ore must be processed each day. Ore from source A costs 20 per ton to process, and ore from source B costs 10 per ton to process. Costs must be kept to less than 80 per day. Moreover, Federal Regulations require that the amount of ore from source B cannot exceed twice the amount of ore from source A. If ore from source A yields 2 kg of gold per ton, and ore from source B yields 3 kg of gold per ton, how many tons of ore from both sources must be processed each day to maximize the amount of gold extracted subject to the above constraints? Model this a *linear programming problem*

Problem 3: There are I persons available for J jobs. The value of person i working 1 day at job j is a_{ij}, for $i = 1, 2, \ldots, I$, and $= 1, \ldots, J$. The problem is to choose an assignment of persons to jobs for a day to maximize the total value. How do you model as *linear programming problem*

Problem 4: We are given a directed graph D with a distinguished source node s and a distinguished sink node t. In addition, each edge e has a capacity c_e, which is the largest amount of flow that can go over this edge. Our goal is to route as much flow from the source s to the sink t as we can, along the edges of the graph. Formulate this problem as a *linear programming problem.*

Let G be a bipartite graph $G = (U \cup V; E)$, where $E \subseteq U \times V$. A matching in G is a set of edges $M \subseteq E$ such that each vertex appears in at most one edge of M. Maximum-cardinality matching in G is a matching of largest size.

Problem 5: Model the maximum-cardinality bipartite matching problem as a *Integer linear programming problem.* The input is a bipartite graph $G = (U \cup V; E)$, where $E \subseteq U \times V$; the output is the largest matching in G.

Let G be a graph. A proper vertex coloring of G means assigning colors to vertices of G such that if uv is an edge in G then vertices u, v should get different colors. The chromatic number of G is the minimum number of colors needed in a proper vertex coloring of G

Problem 6: How do you model the problem of computing the chromatic number of G as a *Integer linear programming problem*

Note: 4^{th} *and* 6^{th} *are slightly involved*