A Pragmatic Non-Blocking Concurrent Directed Acyclic Graph

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Outline of the Presentation

1. Introduction
2. The System Model
3. The ADT Operations
4. Modified ADT Operations: For Maintaining Acyclicity
5. The Data Structure
6. Acyclic Add Edge Operation
7. Reachability Methods to Test a Cycle
8. Correctness and Progress Guarantees
9. Simulation Results

Introduction

- Common real world objects can be modeled as graphs, which build the pairwise relations between objects.
- Graph algorithms applied in many applications, including social networks, communication networks, VLSI design, graphics, etc.
- Often these graphs are dynamic in nature and the updates are real-time.

Introduction

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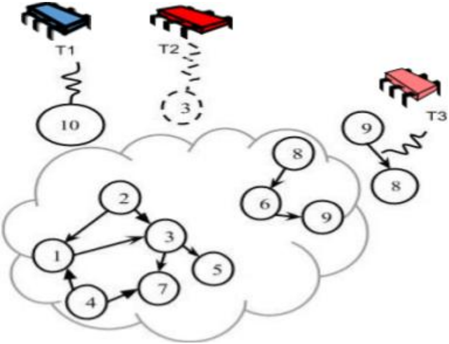

Acyclic Graph

Several applications which maintain dynamic graphs require it to be acyclic such as:

- Databases: *Serialization Graph Testing (SGT)* in the field of Databases and Transactional Memory Systems (TM).
- Blockchains: Several blockchains maintain acyclic graphs such as tree structure (Bitcoin, Ethereum) or general DAGs (Tangle).
- Deadlock Detection: Several deadlock detection algorithms have been proposed in literature that require maintenance of acyclic graphs.
- Data processing, Data compression etc.

The System Model

- Asynchronous shared-memory model with a finite set of p processors accessed by a finite set of n threads.
- The non-faulty threads communicate with each other by invoking methods on the shared objects.
- Execution on a shared-memory multi-processor system which supports atomic `read`, `write`, `fetch-and-add (FAA)` `and` `compare-and-swap (CAS)` instructions.

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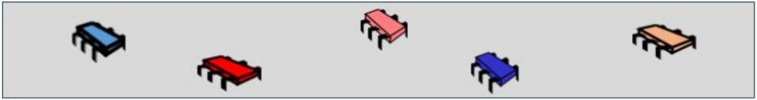
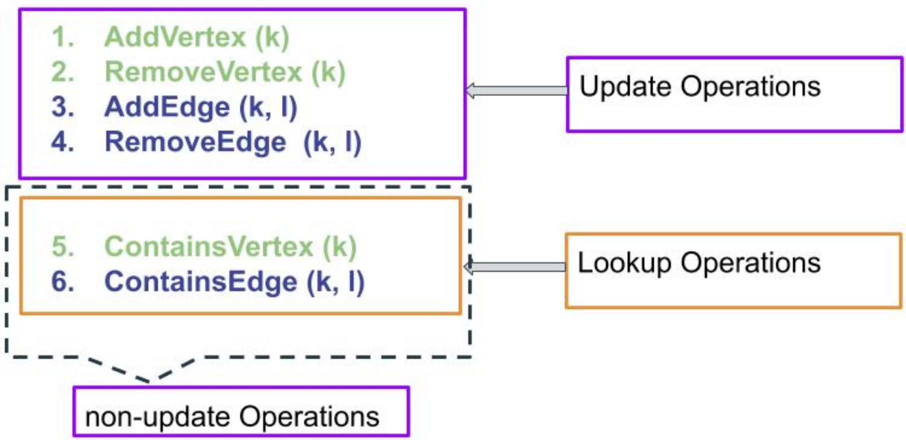


Figure: Concurrent Threads.

The ADT Operations [a]



1. AddVertex (k)
2. RemoveVertex (k)
3. AddEdge (k, l)
4. RemoveEdge (k, l)

Update Operations

5. ContainsVertex (k)
6. ContainsEdge (k, l)

Lookup Operations

non-update Operations

---

[a] Bapi Chatterjee, Sathya Peri, Muktikanta Sa, and Nandini Singhal. *A Simple and Practical Concurrent Non-blocking Unbounded Graph with Linearizable Reachability Queries*,ICDCN 2019.

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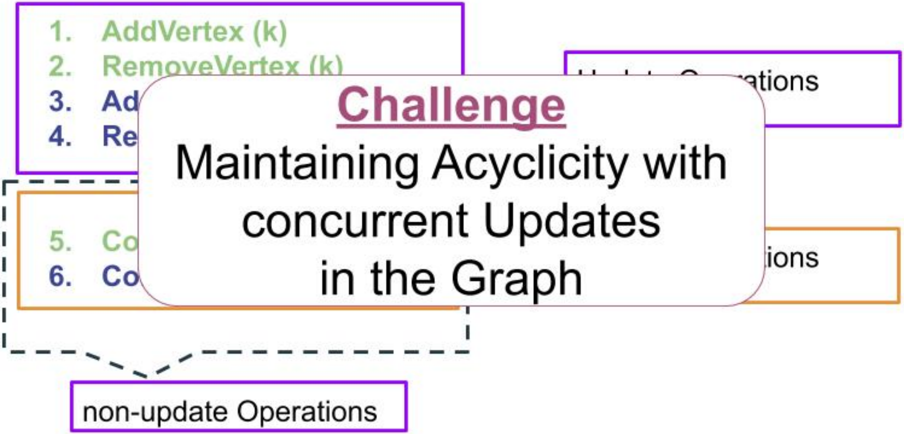

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3. Ad
4. Re
5. Co
6. Co

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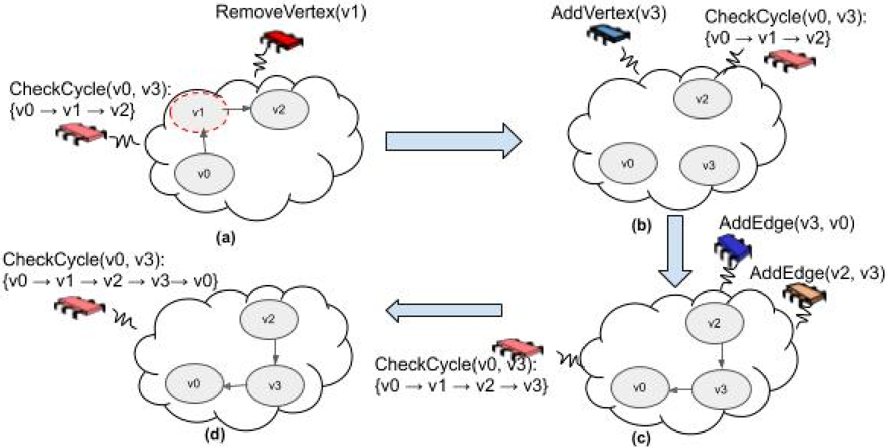
ions

non-update Operations

Challenge

Maintaining Acyclicity with concurrent Updates in the Graph

Difficulty with Maintaining Acyclicity



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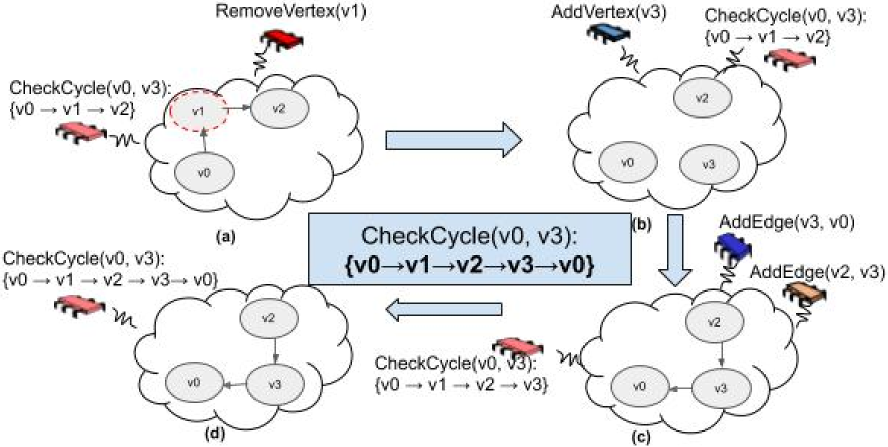

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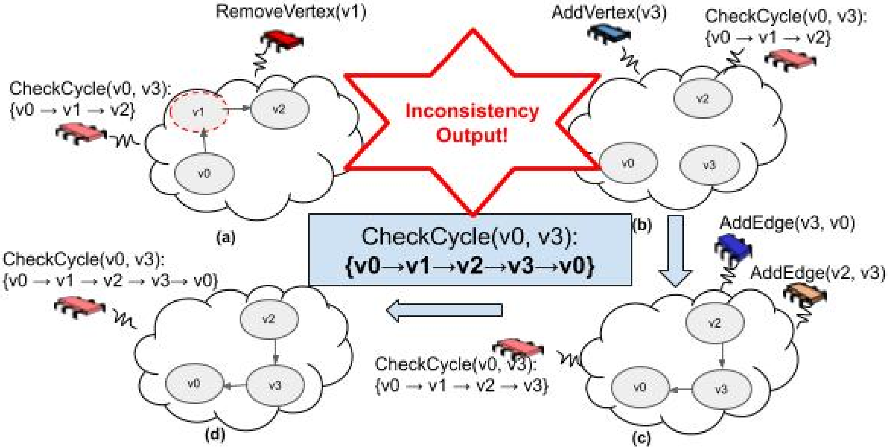

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- The correctness-criterion that we consider is *linearizability*.
- A concurrent data-strcture d is linearizable if for any history (execution) H output by d:
    - Assign an atomic step as a linearization point (LP) inside the execution interval of each of the operations.
    - The history H is equivalent to a valid sequential execution obtained by ordering the operations by their LPs.

Linearizability Example: Set Data-Structure

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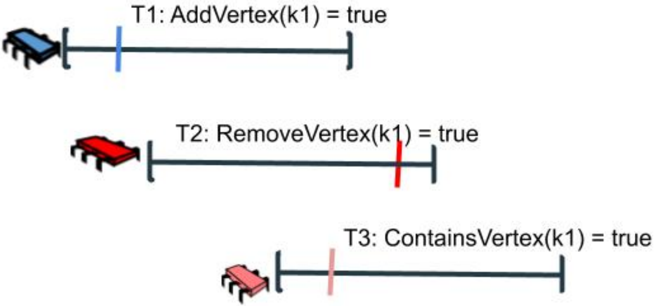

Progress Guarantees

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Lock-free

A method is lock-free if it guarantees that infinitely often some method call finishes in a finite number of steps.

Obstruction-free

A method is obstruction-free if, from any point after which it executes in isolation, it finishes in a finite number of steps (method call executes in isolation if no other threads take steps).

Modification of ADT Operations: For Maintaining Acyclicity

1. AddVertex
2. RemoveVertex
3. ContainsVertex
4. AddEdge
5. RemoveEdge
6. ContainsEdge

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1. AddVertex
2. RemoveVertex
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4. **AddEdge**
5. RemoveEdge
6. ContainsEdge

Modification of AddEdge Operation: For Maintaining Acyclicity

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1. Only edge addition can cause cycle
2. So, we modify AddEdge to ensure that no cycles are formed.

The Data Structure

A directed graph $G = (V, E)$

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- based on [Chatterjee et. al., ICDCN 2019]

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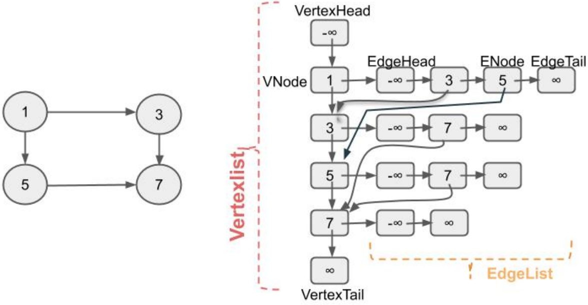
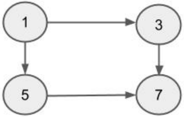
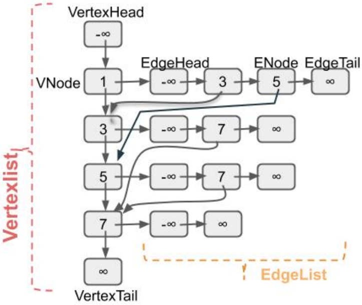


The Data Structure - Node States





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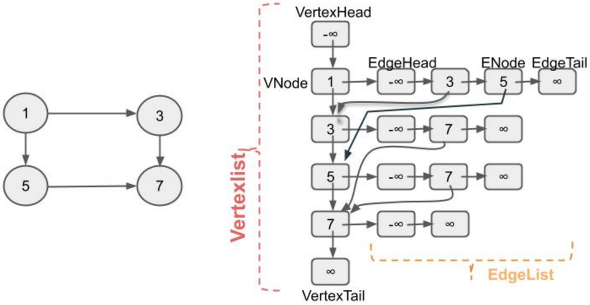
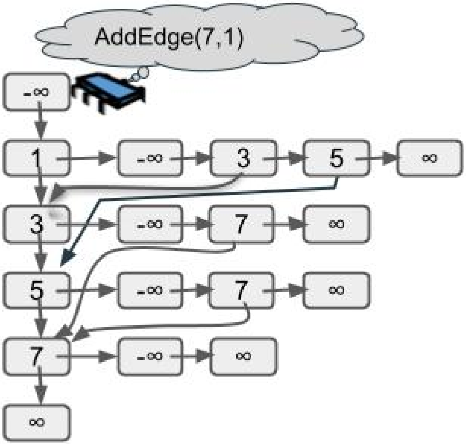
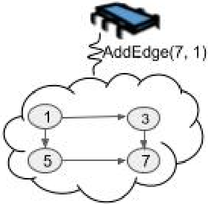


The states of the nodes

- VNode: `MARKED` or `UNMARKED`. Similar to a concurrent list-based set.
- ENode: `MARKED` or `ADDED` or `TRANSIT`.

Acyclic Add Edge Operation





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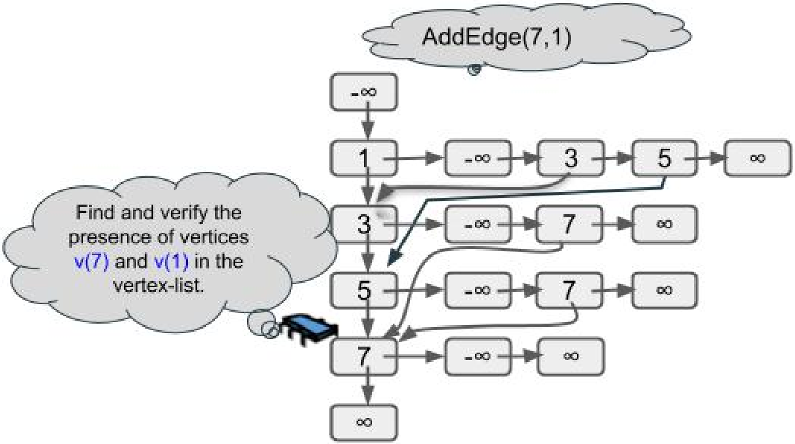
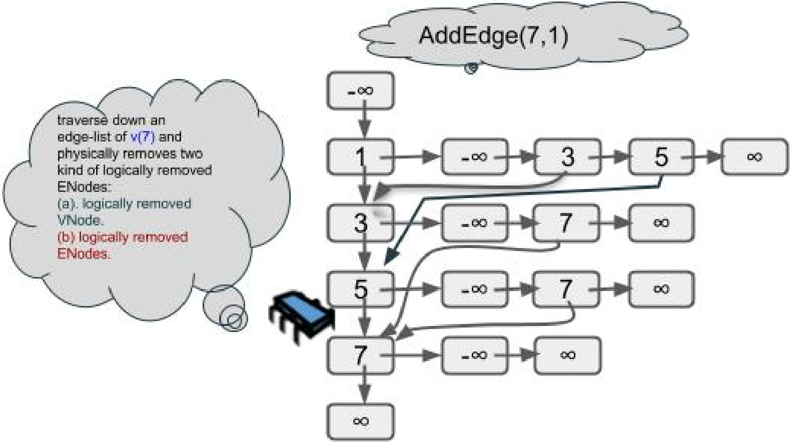


AddEdge(7,1)

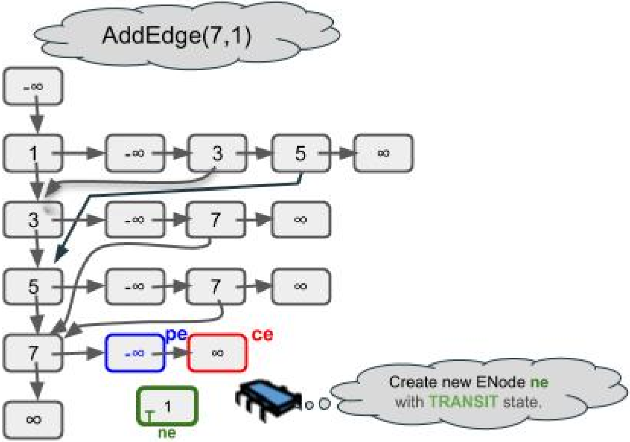
Find and verify the presence of vertices v(7) and v(1) in the vertex-list.

Acyclic Add Edge Operation Cont...



traverse down an edge-list of v(7) and physically removes two kind of logically removed ENodes:
(a). logically removed VNode.
(b) logically removed ENodes.

Acyclic Add Edge Operation Cont...



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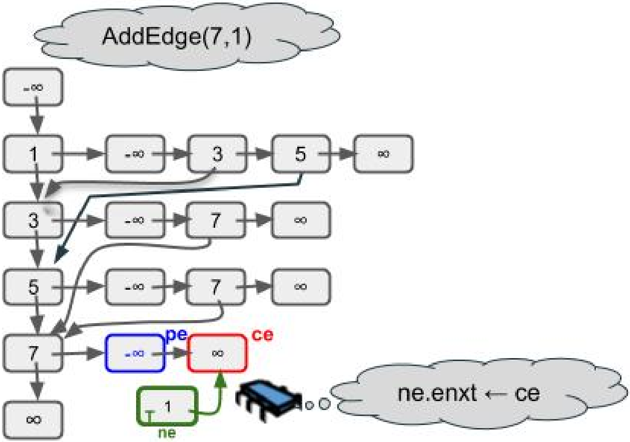

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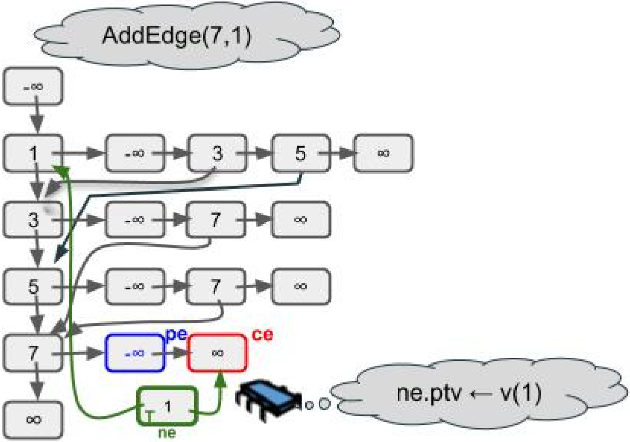

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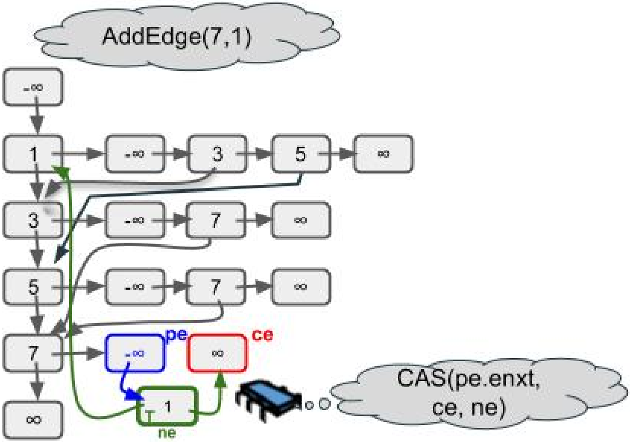

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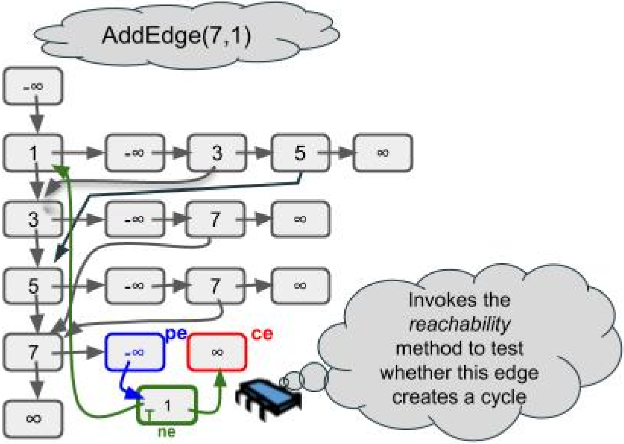

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    1. If so, we delete the edge by setting its state from `TRANSIT` to `MARKED` and return `false` along with an indicative string `CYCLE DETECTED`.
    2. Otherwise, we set the state from `TRANSIT` to `ADDED` and return `true` along with an indicative string `EDGE ADDED`.

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3. An edge in `TRANSIT` state is not visible to containsEdge operations.

Reachability Methods to Test a Cycle

We present two approaches for maintaining acyclicity:

1. First one is based on a Wait-free Reachability query (SCR: Single Collect Reachable)

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Reachability Methods to Test a Cycle

We present two approaches for maintaining acyclicity:

1. First one is based on a Wait-free Reachability query (SCR: Single Collect Reachable)
2. Second one is based on a Obstruction-free Reachability query (DCR: Double Collect Reachable), similar to the `GetPath`[b] algorithm.

---

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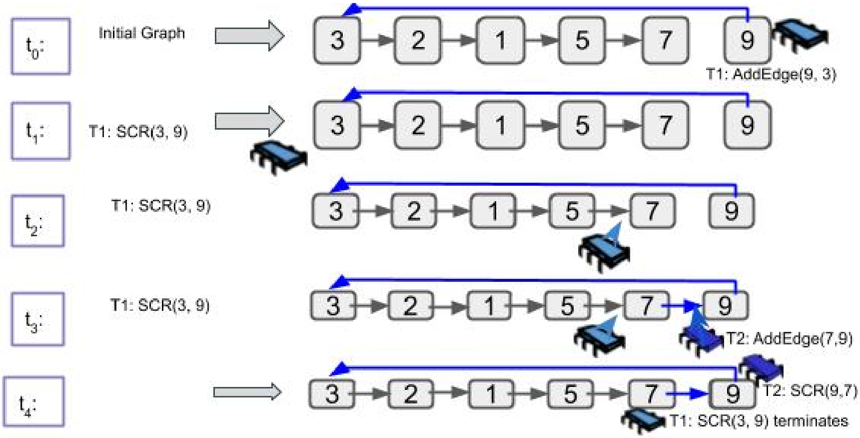
SCR(k, l) : Wait-free Reachability



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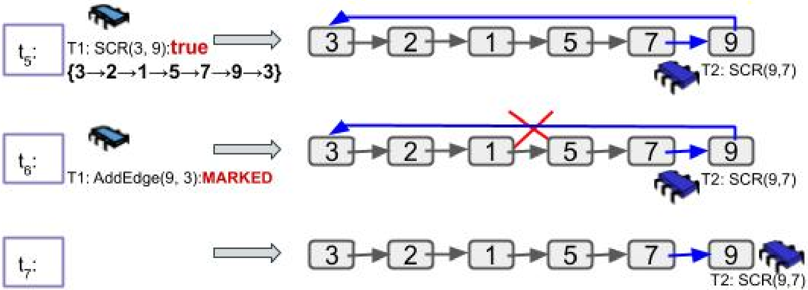

OR

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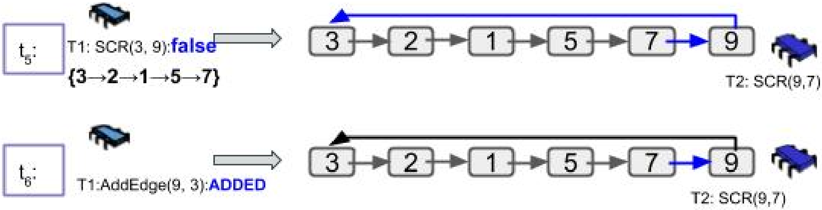

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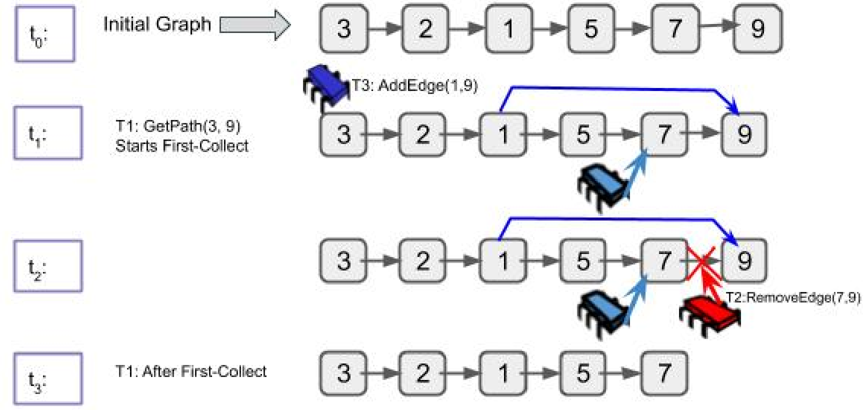
4. If it unable to reach l then it terminates by returning `false` to the AddEdge operation.

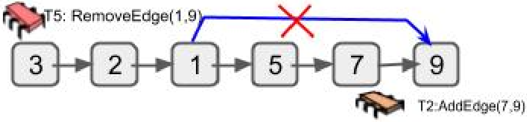
DCR(k,l) : Obstruction-free Reachability

Double Collect Problem: First Collect

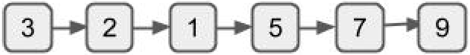


After First Collect Graph Restored

t_4:



Graph has been restored



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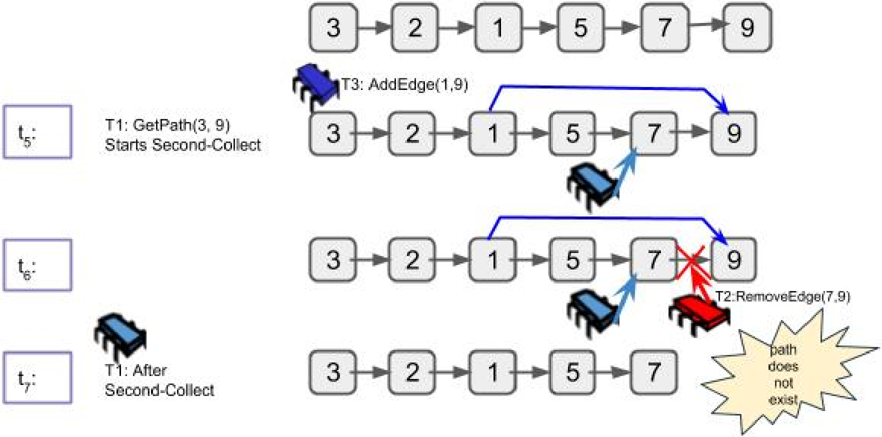

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4. To verify the double collect we compare with BFS-tree alone with counter.
5. If the both the double collects are same
    1. We have valid snapshot
    2. We analyse the the valid snapshot for the presence or absence of the path.

Correctness and Progress Guarantees

Theorem 1:

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Theorem 2:

The ADT operations are non-blocking:

1. The operations *ContainsVertex*, *ContainsEdge* and *SCR* are wait-free.
2. The operation *DCR* is obstruction-free.
3. The operations *AddVertex*, *RemoveVertex*, *ContainsVertex*, *AddEdge*, *RemoveEdge*, and *ContainsEdge* are lock-free.

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Proofs of the Theorem 1 and 2 are shown in the paper.

Experimental Setup

Experimental Setup

- Intel(R) Xeon(R) E5-2690 v4 CPU containing 14 cores running at 2.60GHz on two sockets. Each core supports 2 logical threads.
  - Thus, a total of 56 logical cores.
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- Each data point is obtained by averaging over 7 iterations.
- We compare the SCR and DCR with its sequential and coarse-grained counterparts.

Workload Distributions

Graph Operations: AddVertex, RemoveVertex, ContainsVertex, AddEdge, RemoveEdge and ContainsEdge

- **Lookup Intensive:** (2.5%, 2.5%, 45%, 2.5%, 2.5%, 45%)
- **Equal Lookup and Updates:** (12.5%, 12.5%, 25%,12.5%, 12.5%, 25%)
- **Update Intensive:** (22.5%, 22.5%, 5%, 22.5%, 22.5%, 5%)

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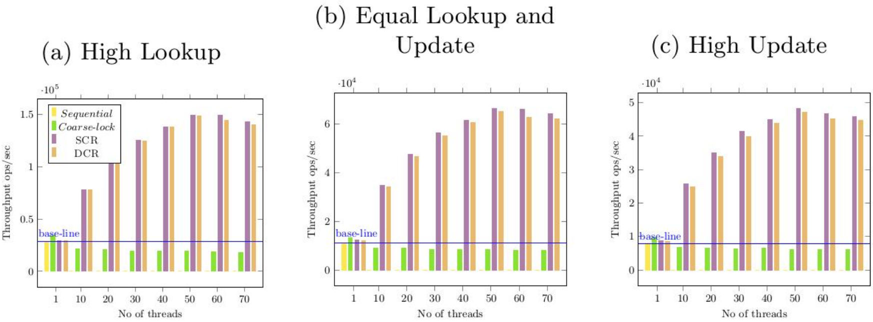
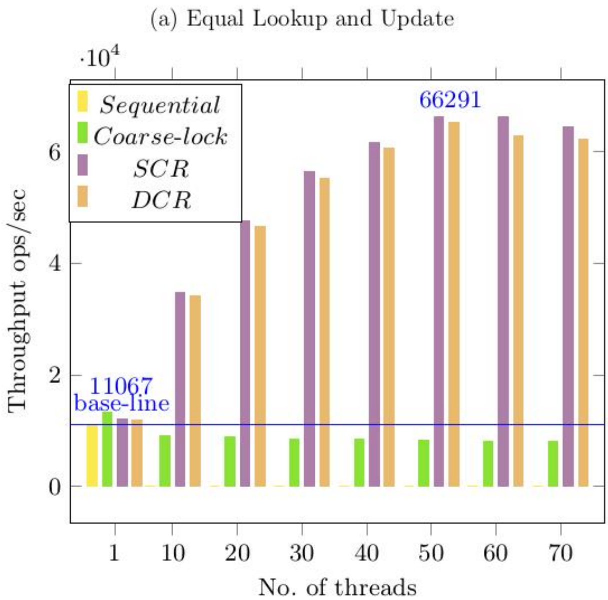
We have compared the following cases.

S. No	Label	Explanation
1	Sequential	Sequential execution of all the operations
2	Coarse-lock	Coarse lock execution of all the operations
3	SCR	AddEdge based on Single Collect Reachable Algorithm
4	DCR	AddEdge based on Double Collect Reachable Algorithm

Results


(a) High Lookup
(b) Equal Lookup and Update
(c) High Update
·10^5
·10^4
·10^4
Sequential
Coarse-lock
SCR
DCR
base-line
Throughput ops/sec
No of threads
1
10
20
30
40
50
60
70
0
0.5
1
1.5
0
2
4
6
0
1
2
3
4
5

Results


(a) Equal Lookup and Update
$\cdot 10^4$
Sequential
Coarse-lock
SCR
DCR
66291
11067
base-line
Throughput ops/sec
0
2
4
6
1
10
20
30
40
50
60
70
No. of threads

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3. Identify ways to reduce them.

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4. We extensively evaluate a sample C/C++ implementation of the algorithm through a number of micro-benchmarks.
5. Our experiments show that the proposed algorithm scales 7X with the number of threads in commonly available multi-core systems.

For More Information

1. The Technical Report is available at: *https://arxiv.org/abs/1611.03947*
2. And the complete source code is available at: *https://github.com/PDCRL/ConcurrentGraphDS*

Thank You!

For Further Reading..

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