

# Proving Correctness of Concurrent Objects by Validating Linearization Points

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- 1 Introduction
  - Sequential Object and Sequential Specification
  - Sequential vs Concurrent History
- 2 Linearizability
- 3 Proposed Technique: Validating LPs
- 4 Conclusion
- 5 Future Work

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# Introduction

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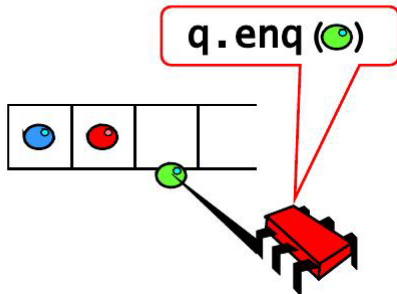
- Set of correct histories which can be generated by single threaded execution.
- Pre-condition : state before you call the method.
- Post-condition : other state after the method returns.



# Introduction

## Example of Sequential Specification

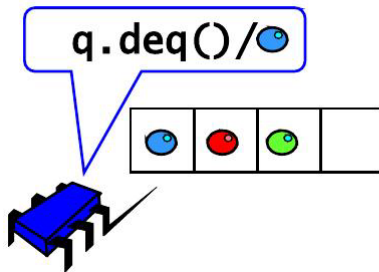
### FIFO Queue: Enqueue Method



# Introduction

## Example of Sequential Specification

### FIFO Queue: Dequeue Method



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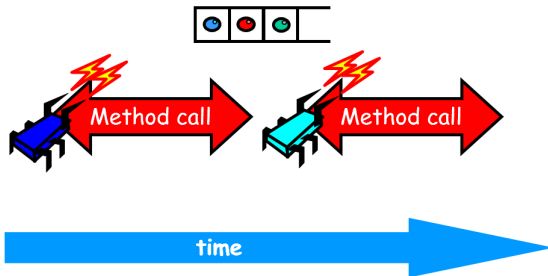


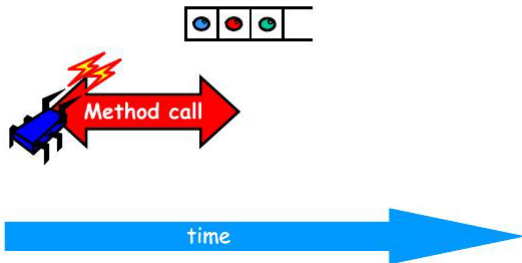
Figure: Sequential History

# Sequential vs Concurrent History

## Concurrent History

- Object methods can be invoked by concurrent processes.
- It is necessary to give a meaning to possible interleavings of operations invocation.

## Concurrent Methods Take Overlapping Time

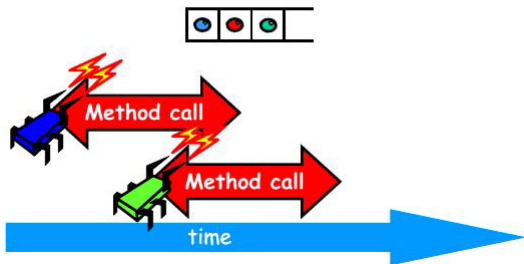


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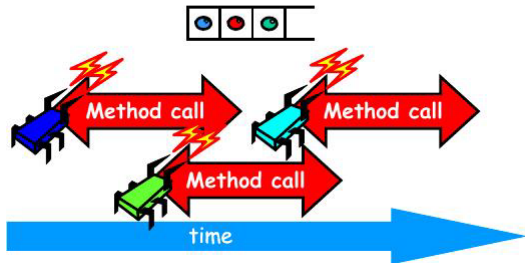


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What does it mean for a concurrent object to be correct?

Correctness Criteria: **Linearizability**.

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- 2 **Linearizability**
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## Linearizability

- Each method should
  - Take effect
  - Instantaneously
  - Between invocation and response events.

## Linearizability

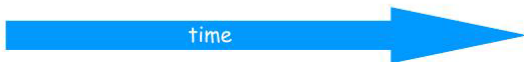
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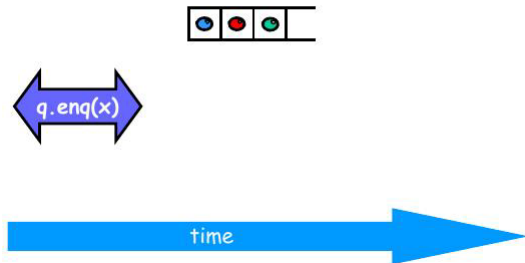
## Linearizability

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- Object is correct if it adheres to its sequential specification.
- Any such concurrent object is *Linearizable*.

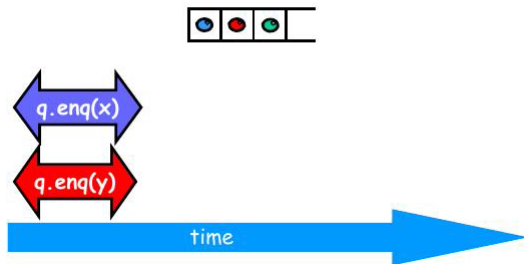
## Example



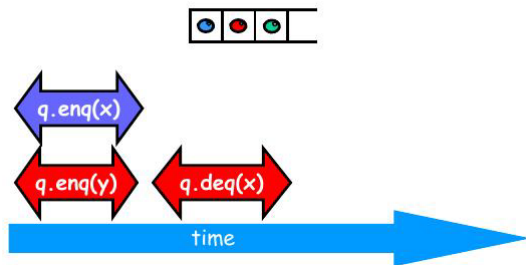
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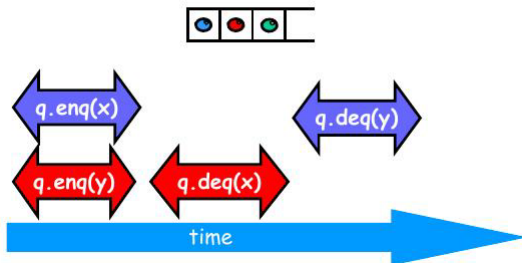


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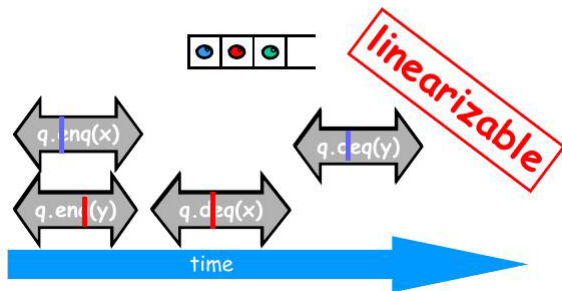




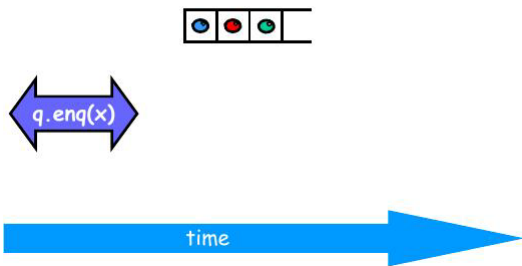
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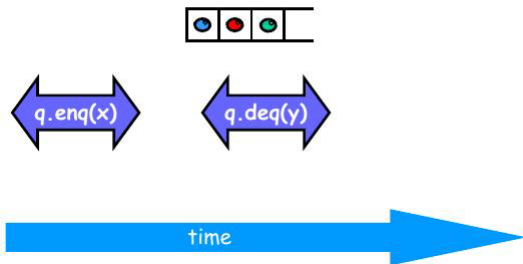


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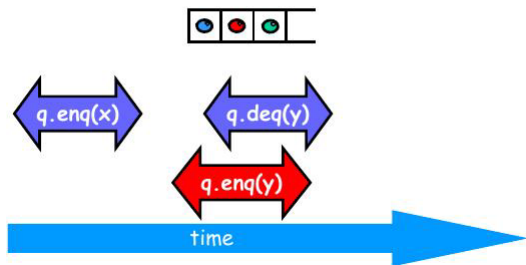


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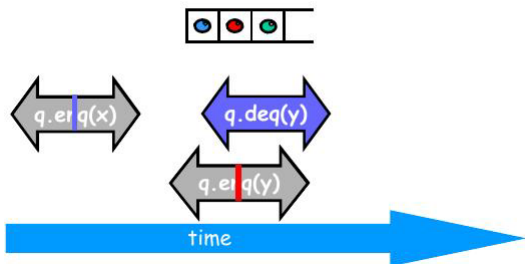


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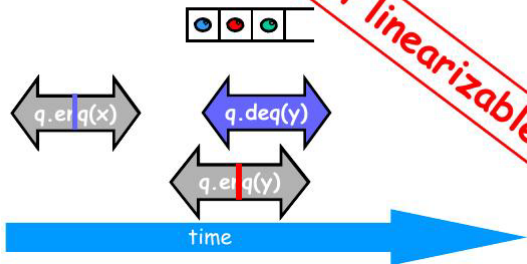


## Example





Example



# Proving Linearizability Manually

- Challenging even for simple data structures.

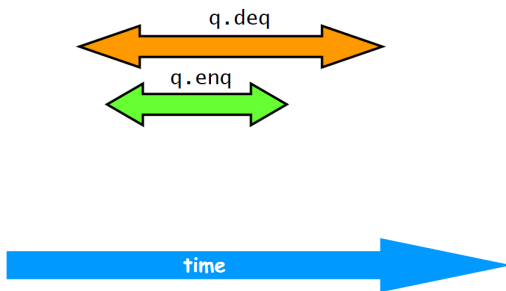
# Proving Linearizability Manually

- Challenging even for simple data structures.
- Several techniques have been proposed
  - Linearization Points [HerlihyWing90]
  - Rely Guarantee [Vafeiadis, et al. 06]
  - Hindsight Lemma [O'Hearn10]
  - Base Point Analysis [KfirKeidar15]

# Linearization Points [HerlihyWing90]

- Every operation "appears to happen" at some individual instruction between invocation and response.
- In coarse locks, LP could be anywhere in the critical section.

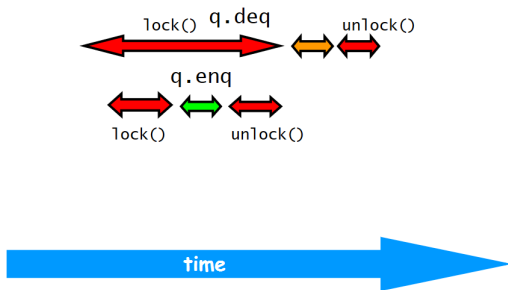
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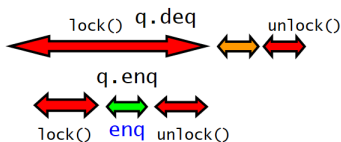




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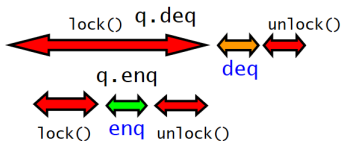
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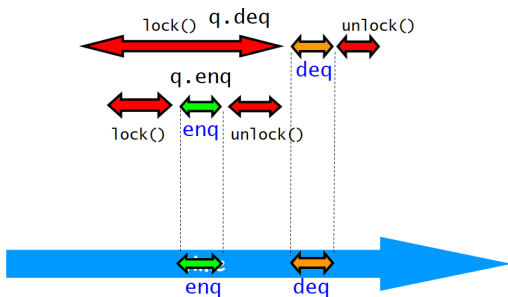
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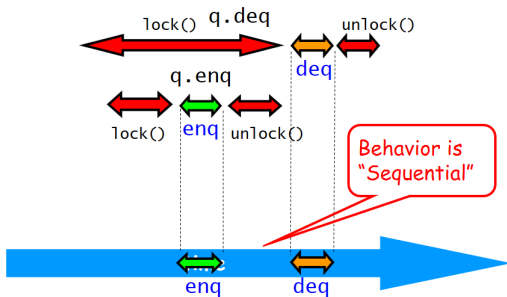
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## Problem!

How do you know if you have identified the correct LPs indeed?

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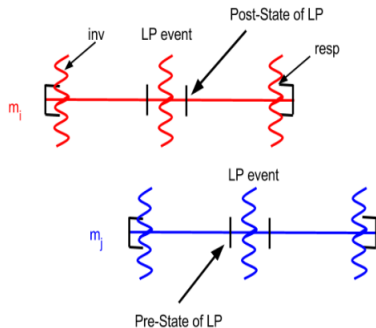
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# Proposed Technique: Validating LPs

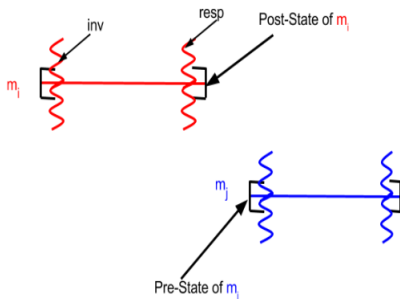
Hand-crafted generic technique for validating LPs

- Terminology:
  - Events, Methods
  - State
  - History, Execution, Complete History
  - Abstract data structure (AbDS)

Concurrent Execution  $E^H$



Sequential Execution  $E^S$





# Proposed Technique: Validating LPs

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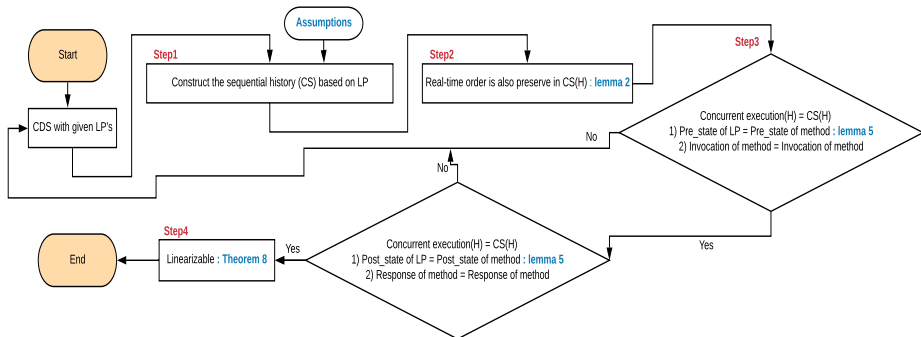


Figure: Iterative steps to prove linearizability of a CDS with given LP's.

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- Assumptions:
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- Assumptions:

- Every sequential history  $S$  generated by the concurrent data structure(CDS) is *legal*.
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- Only the LP events of a method can change AbDS of CDS.

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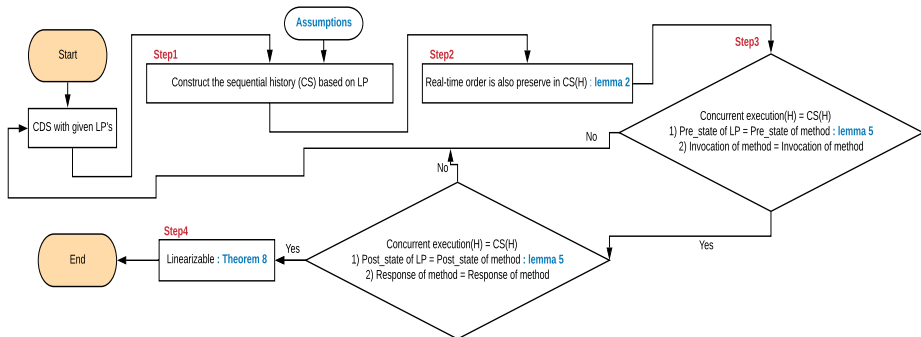
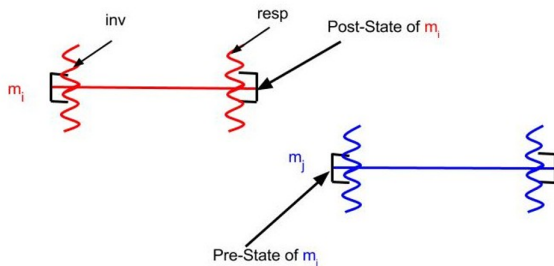


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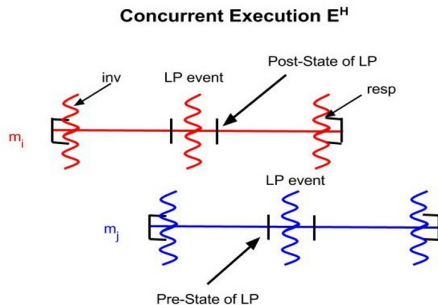
## Sequential Execution $E^S$



$E^S$  : Post-state of  $m_i$  = Pre-state of  $m_j$

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$E^H$  : Post-state of  $m_i.LP$  = Pre-state of  $m_j.LP$

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Hand-crafted generic technique for validating LPs

$$\forall m : \langle \text{Pre-state of } E^H.m_i.LP = \text{Pre-state of } E^S.m_i \rangle \wedge \langle E^H.m_i.inv = E^S.m_i.inv \rangle \rightarrow \langle \text{Post-state of } E^H.m_i.LP = \text{Post-state of } E^S.m_i \rangle \wedge \langle E^H.m_i.resp = E^S.m_i.resp \rangle$$

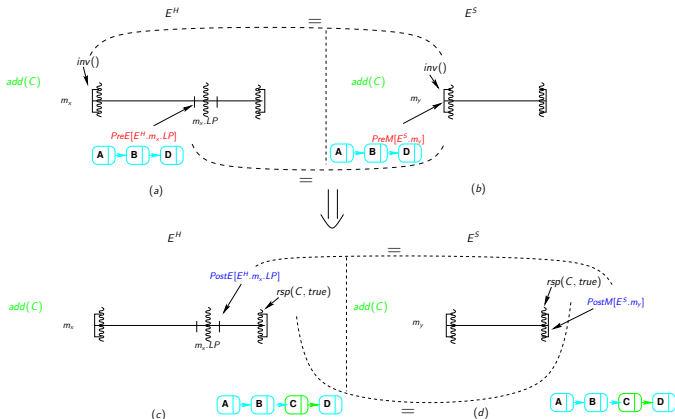


Figure: CDS Specific lemma



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- We have shown the correctness of **lazy-list** and **hand-over-hand** locking list in technical report.

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- We will extend it to the concept of **Linearization Blocks**.

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- We will try to develop the automatic tool for validating LPs.



- 1 John Derrick, Gerhard Schellhorn, and Heike Wehrheim. Verifying linearisability with potential linearisation points. In Proceedings of the 17th International Conference on Formal Methods, FM'11, pages 323–337, Berlin, Heidelberg, 2011. Springer-Verlag.
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**Thank you for your attention!**

Any Questions or  
Comments?



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