Exam - 1

CS5580

04-Feb-2019 (6:30pm-7:30pm)

Note: Please answer the questions using rigorous and succinct mathematical justifications. Simplify expressions as much as possible.

Problem 1. Let $\mathcal{M}_n$ be the vector space of all symmetric matrices of size n. Let $\mathcal{S}_n$ denote the subset of all psd matrices in $\mathcal{M}_n$. Prove or disprove the statement: "Set $\mathcal{S}_n$ is closed under the vector addition operation of $\mathcal{M}_n$".

Problem 2. Let $\mathcal{M}_n$ be the vector space of all symmetric matrices of size n. Let $\mathcal{S}_n$ denote the subset of all psd matrices in $\mathcal{M}_n$. Prove or disprove the statement: "Set $\mathcal{S}_n$ forms a subspace in $\mathcal{M}_n$".

Problem 3. Consider an inner-product space $\mathcal{I} = (V, +, \cdot, \langle \cdot, \cdot \rangle)$. Assuming only the axioms that define the vector addition $(+)$, the scalar multiplication $(\cdot)$, and the inner-product $(\langle \cdot, \cdot \rangle)$, prove the statement:

$$(\langle x, y \rangle)^2 \leq \langle x, x \rangle \langle y, y \rangle \ \forall \ x, y \in V.$$

Problem 5. Is the set of all bisymmetric[1] matrices of size n an affine set?

Problem 6. What is the dimensionality of the set of all bisymmetric matrices of size n?

Problem 7. Is the set of all square matrices, of size n, with non-negative entries, and each row sum being equal to unity[2] an affine set?

Problem 8. What is the least number of vectors required to represent/describe the set of all square matrices, of size n, with each row sum being equal to unity?

Problem 9. Let L_1, L_2 be two linear sets. Show that $5L_1 - 12L_2 = LIN(-3L_1 \cup 7L_2)$.

[1] Matrices that are symmetric wrt both the diagonals.

[2] Such matrices are called as Stochastic or Markov matrices.