Via epigraph defn

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$$f(x) \equiv \min_y \; g(y) \quad \text{is convex}$$

$$\text{s.t.} \quad h(y) \leq x$$

($\because$ Theorem 17.0.3)

$$\Updownarrow$$

$$epi(f) \equiv \left\{ (x,t) \;\middle|\; \min_{y \in \mathbb{R}^n} g(y) \leq t \;\; \text{s.t. } h(y) \leq x \right\} \text{ is convex}$$

$$\Updownarrow$$

$$epi(f) = \{ (x,t) \mid \exists\, y \in \mathbb{R}^n \ni g(y) \leq t,\ h(y) \leq x \}$$

Proof: Let $(x_1, t_1) \in epi(f) \Rightarrow \exists\, y_1 \in \mathbb{R}^n \ni g($

$(x_2, t_2) \in epi(f) \Rightarrow \exists\, y_2 \in \mathbb{R}^n \ni g($

via Hessian Definition

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This is clearly mentioned in Boyd's book on page 74, first example: log-sum-exp.

The theorem used is Theorem21.0.5 in the summary notes.

via 1-d double derivatives

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The theorem used is theorem 21.0.4 in summary notes.

$$f(x) = \log\left(\sum_{i=1}^{n} e^{x_i}\right)$$

$$g_{xu}(t) = \log\left(e^{x_1+tu_1} + \cdots + e^{x_n+tu_n}\right)$$

$$\frac{d\,g_{xu}(t)}{dt} = \frac{\sum_{i=1}^{n} u_i e^{x_i+tu_i}}{\sum_{i=1}^{n} e^{x_i+tu_i}}$$

$$\frac{d^2 g_{xu}(t)}{dt^2} = \frac{\left(\sum_{i=1}^{n} u_i^2 e^{x_i+tu_i}\right)\left(\sum_{i=1}^{n} e^{x_i+tu_i}\right) - \left(\sum_{i=1}^{n} u_i e^{x_i+tu_i}\right)^2}{\left(\sum_{i=1}^{n} e^{x_i+tu_i}\right)^2}$$

$\left(\sum_{i=1}^{n} e^{x_i+tu_i}\right)^2 \longrightarrow$ always ≥ 0

Via epigraph defn

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$f(x) = \log(e^{x_1} + \dots + e^{x_n})$ is convex

$\Updownarrow$

$epi(f) = \{(x,t) \mid \log(e^{x_1} + \dots + e^{x_n}) \leq t\}$ is convex

$\Updownarrow$

$\{(x,t) \mid e^{x_1} + \dots + e^{x_n} \leq e^t\}$ is convex

($\because$ log is

$\Updownarrow$

$\{(x,t) \mid e^{x_1 - t} + \dots + e^{x_n}$