Q1

08 February 2019 23:53

Claim: S_n is closed under vector addition.

Proof: Let $M, N \in S_n$

$\Rightarrow$ (i) M, N are symmetric

(ii) $x^T M x \geq 0$, $x^T N x \geq 0$ $\forall x \in \mathbb{R}^n$.

TST $M+N \in S_n$

i.e. TST (i) $M+N$ is symmetric

(ii) $x^T(M+N)x \geq 0$ $\forall x \in \mathbb{R}^n$

ij^{th} entry of $M+N$ = ij^{th} entry of M + ij^{th} entry of N

= ji^{th} " " + ji^{th} " " ($\because M, N$ are symmetric)

= ji^{th} entry of $M+N$

$x^T(M+N)x = x^T M x + x^T N x$

≥ 0 ($\because x^T M x \geq 0, x^T N x \geq 0$)

Q2

09 February 2019 00:01

Claim S_n does not induce a subspace

Proof: **Claim** S_n is not closed under scalar multiplicati

Because if $M \in S_n \Rightarrow x^T M x \geq 0 \ \forall x$

$$\Rightarrow x^T(-M)x \leq 0 \ \forall x$$

$$\Rightarrow -M \notin S_n \quad (\text{unless } M=0)$$

$\Bigg(\because \ x^T M x = 0 \ \forall x \iff M = 0$

easy to prove by taking $x = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ldots \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ldots \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 1 \end{bmatrix} \Bigg)$

Q3

09 February 2019 00:07

Let $x, y \in V$ be arbitrary vectors.

Then by closure under lin. comb., we have

$$\alpha x + y \in V \quad \forall \alpha \in \mathbb{R}.$$

$$\Rightarrow \langle \alpha x + y, \alpha x + y \rangle \geq 0 \quad \forall \alpha \in \mathbb{R}$$

($\because$ non-negativity of $\langle \rangle$)

$$\Rightarrow \langle \alpha x, \alpha x + y \rangle + \langle y, \alpha x + y \rangle \geq 0 \;\forall \alpha \in \mathbb{R}$$

($\because$ linearity of $\langle \rangle$)

$$\Rightarrow \alpha \langle x, \alpha x + y \rangle + \langle y, \alpha x + y \rangle \geq 0 \;\forall \alpha \in \mathbb{R}$$

($\because$ linearity of $\langle \rangle$)

$$\Rightarrow \alpha\langle \alpha x+y, x\rangle + \langle \alpha x+y, y\rangle \geq 0 \ \forall \alpha \in \mathbb{R}$$

$(\because$ Symmetry of $\langle\,\rangle)$

$$\Rightarrow \alpha^2\langle x, x\rangle + 2\alpha\langle x, y\rangle + \langle y, y\rangle \geq 0 \ \forall \alpha \in \mathbb{R}$$

$(\because$ repeated linearity of $\langle\,\rangle)$

$$\Rightarrow (2\langle x, y\rangle)^2 - 4(\langle x, x\rangle)(\langle y, y\rangle) \leq 0$$

$(\because b^2 - 4ac \leq 0)$

Proof done by re-arranging terms.

Q 4

09 February 2019 00:15

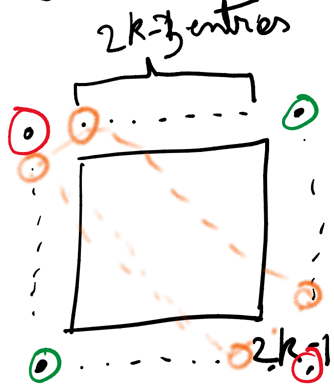
Claim: Set of bi-symmetric matrices is linear.

Proof: Since vect. add & scalar multiplication are entry-wise, by arguments similar to Q1, this set is linear, hence affine too!

Q6

09 February 2019 00:17

Here is the way to "generate" 2k-1 size matrix from 2k-3 size matrix:



$$\Rightarrow B_{2k-1} = B_{2k-3} + 2k-1$$

where B_n is basis size for bisymmetric matrices for size n

$$\Rightarrow B_{2k-1} = 1 + 3 + 5 \dots + 2k-1$$
$$= k^2$$

IIIly, $B_{2k} = B_{2k-2} + 2k$

$$\Rightarrow B_{2k} = 2 + 4 + 6 + \cdots + 2k$$
$$= k(k+1)$$

Q7

09 February 2019 00:34

<u>claim</u>: It is <u>Not</u> an affine set.

In fact not closed under scalar multiplication.

<u>eg</u>: If $M \neq 0$ is in this set, then $-M \notin$ to this set

($\because$ $-M$ has negative entries)

Q8

09 February 2019 00:36

Dual representation requires 'n' vectors and hence is the most efficient:

$$\left\langle \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ 0 & \cdots & & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & & \cdots & 0 \end{bmatrix}, M \right\rangle = 1$$

$$\vdots$$

$$\left\langle \begin{bmatrix} 0 & \cdots & & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & & \cdots & 0 \\ 1 & 1 & \cdots & 1 & 1 \end{bmatrix}, M \right\rangle = 1$$

Q9

09 February 2019 00:40

For any $\alpha \in \mathbb{R}$ and linear set L, $\alpha L = L$.

$\therefore$ <u>TST</u> $L_1 + L_2 = LIN(L_1 \cup L_2)$.

Proof $L_1 + L_2 \subseteq LIN(L_1 \cup L_2)$

Let $x \in L_1 + L_2 \Rightarrow x = l_1 + l_2,\ l_1 \in L_1,\ l_2 \in L_2.$

$\Rightarrow x = l_1 + l_2,\ l_1 \in L_1 \cup L_2,$
$l_2 \in L_1 \cup L_2.$

$\Rightarrow x \in LIN(L_1 \cup L_2).$

Proof $LIN(L_1 \cup L_2) \subseteq L_1 + L_2$

Let $x \in LIN(L_1 \cup L_2) \Rightarrow x = \sum_i \lambda_i v_i,\ v_i \in L_1 \cup L_2$

$(\because$ defn of union$)$ $\Rightarrow x = \sum_i \lambda_i l_i^1 + \sum_j \lambda_j l_j^2,\ l_i^1 \in L_1$
$l_i^2 \in L_2$

(∵ defn of union) $\Rightarrow x = \sum_i \lambda_i l_i + \sum_j \lambda_j l_j, \; l_i \ldots$

$l_j^2 \in L_2$

$\Rightarrow x = l_1 + l_2$ (∵ L_1, L_2 are linear)

$\Rightarrow x \in L_1 + L_2.$