

Exam - 3

CS5580

26-Apr-2019 (8:30pm-9:30pm)

Note: Please answer the questions using rigorous and succinct mathematical justifications. Simplify expressions as much as possible. PLEASE WRITE YOUR ROLL NO. ON YOUR ANSWER SCRIPT.

Problem 1,2,3. Using three different definitions/theorems regarding convex functions that are covered in lectures, prove that $f(x_1, \dots, x_n) \equiv \log(e^{x_1} + \dots + e^{x_n})$ is convex¹. Clearly write down the definition/theorem-statement you use in each case.

Problem 4. Let g, h be convex functions defined over \mathbb{R}^n . Suppose $\min_{x \in \mathbb{R}^n} h(x) > -\infty$ and assume all non-empty level-sets of h are bounded. Consider f defined by:

$$f(x) \equiv \min_{y \in \mathbb{R}^n} g(y), \\ \text{s.t. } h(y) \leq x.$$

Is f a convex function?

¹Your proof using different definitions will be treated as answers to different questions *viz.*, 1,2,3. If you prove correctly using three different definitions, your marks for the third question will be normalized against others who also give three correct proofs. Marks for the second question will be normalized against others who give atleast two correct proofs. And marks for the first question will be normalized against others who give atleast one correct proof. And so on....