CS5580 EndSem 2

StartWriting at S=4pm, StopWriting at D=4:55pm, Submit by D+15=5:10pm

NOTE: Please write your ROLL NO. clearly on ALL answer sheets.

1. Consider the following mathematical program:
$$\min_{x\in\mathbb{R},y\in\mathbb{R}} 4x + 5y \text{ s.t. } 2x^2 + 3y^2 \leq 10$$
   (a) Is this a convex program? Justify your answer by explicitly stating the definitions and/or theorem/result statements that you may use.
   (b) Is this a regular convex program? Justify.
   (c) If so, clearly write down the KKT conditions for this program.
   (d) From KKT conditions argue that the constraint is active at optimality.
   (e) Find out all optimal solutions by simplifying these conditions.
2. Find a simplified expression for the conjugate of f defined by $f(M) \equiv trace(M^{-1}), M \succ 0$.
3. Prove that the negative harmonic mean function, $f : \mathbb{R}^n_{++} \mapsto \mathbb{R}$ given by $f(x) \equiv \frac{-n}{\frac{1}{x_1}+\ldots+\frac{1}{x_n}}, x_i > 0 \; \forall \; i = 1, \ldots, n$ is convex.
4. Indicate whether the following statements are true or false. Marks will be awarded only if appropriate justifications are provided. You may quote specific results/theorems discussed in lectures:
   (a) In the vector space of all square matrices of size n, the semi-definite cone is self-dual.
   (b) Can it happen that a regular convex program is solvable even though there is no KKT satisfying point ?
   (c) All convex functions on $\mathbb{R}^n$ are continuous.
   (d) All convex functions on $\mathbb{R}^n$ are sub-differentiable.
   (e) Affine sets can always be expressed as intersection of a finite number of hyperplanes and convex sets can always be expressed as intersection of possibly infinite number of halfspaces.