CS5580 Quiz-4

StartWriting at S=7pm, StopWriting at D=7:25pm, Submit by D+15=7:40pm

NOTE: Please write your ROLL NO. clearly on ALL answer sheets.

Consider two spaces $\mathcal{V}_1 = (V_1, +_1, \cdot_1, \langle\cdot,\cdot\rangle_1)$ and $\mathcal{V}_2 = (V_2, +_2, \cdot_2, \langle\cdot,\cdot\rangle_2)$. Here V_1, V_2 are the sets of vectors, $+_1, +_2$ are vector sum operations, $\cdot_1, \cdot_2$ are scalar multiplication operations, $\langle\cdot,\cdot\rangle_1, \langle\cdot,\cdot\rangle_2$ are the inner product operations in these two spaces. Then, the direct sum space, $\mathcal{V}_1 \oplus \mathcal{V}_2$, is defined as that with set of vectors as $V_1 \times V_2$ (Cartesian product), vector addition given by $(v_1, v_2) + (w_1, w_2) \equiv (v_1 +_1 w_1, v_2 +_2 w_2)$, scalar multiplication given by $\alpha \cdot (v_1, v_2) \equiv (\alpha \cdot_1 v_1, \alpha \cdot_2 v_2)$ and inner-product defined by $\langle (v_1, v_2), (w_1, w_2) \rangle \equiv \langle v_1, w_1 \rangle_1 + \langle v_2, w_2 \rangle_2$. Go home and show that the direct sum is a valid (inner-product) space if the given two are. This is infact how $\mathbb{R}^2$ space constructed from $\mathbb{R}$ i.e., $\mathbb{R}^2$ space is direct sum of $\mathbb{R}$ and $\mathbb{R}$, $\mathbb{R}^3$ is direct sum of $\mathbb{R}^2$ and $\mathbb{R}$, so on... For now assume that direct sum is a valid space.

Consider the set $K = \{(X, y) \mid \|X\|_* \leq y, X \in \mathcal{S}^n, y \in \mathbb{R}\}$, where $\mathcal{S}^n$ is set of all symmetric matrices of size n, $\|X\|_*$ is the nuclear norm of X, which is sum of absolute values of eigen values of X. Assume that the nuclear norm is a valid norm (Go home and try proving this).

1. Prove or disprove that K is cone.
2. Write down the definition of dual cone of K in terms of the Frobenius inner-product. Assume the space in which K lies is the direct sum of the (standard) space of $\mathcal{S}^n$ and the space of reals.
3. Go home and simplify the expression for the dual cone as much as possible. In particular, express the dual cone in terms of the so-called spectral norm.
4. Write down a highly simplified expression for the dual cone of $\{(X, y) \mid \|X\|_F \leq y, X \in \mathcal{S}^n, y \in \mathbb{R}\}$.