CS5580 Quiz-8

StartWriting at S=5:30pm, StopWriting at D=6:30pm, Submit by D+15=6:45pm

NOTE: Please write your ROLL NO. clearly on ALL answer sheets.

1. Consider the function $f : V \mapsto \mathbb{R}$ defined by $f(x) \equiv \frac{1}{2}\|x\|^2$, where V is set of all vectors in an arbitrary space and $\|\cdot\|$ is an arbitrary norm.
   (a) Write down the definition of sub-gradient of f at some $x_0 \in V$.
   (b) From this, obtain a simplified, single scalar-valued, inequality condition that any sub-gradient of f at x_0 should satisfy. In particular, the LHS/RHS expressions in this inequality must NOT involve any min or max symbols etc., and must be scalar/number valued.
   (c) Using standard inequalities you learnt in this course and the above inequality, obtain a simplified, scalar-valued, single equality condition that any sub-gradient of f at x_0 should satisfy.
   (d) In the special case of n-dimensional Eucldiean space and the norm-squared function, f, defined by $f(x) \equiv \frac{1}{2}x^\top Mx$, where $M \succ 0$, write down this equality condition and simplify it. Show that this condition is satisfied by a unique sub-gradient vector and hence f is differentiable. Note the simplified expression for this unique sub-gradient vector.
2. Replacement for Quiz7: Show that f defined by $f(M)$ is the sum of singular values of M is a conic function. Is f a norm? If not, find the dual of f, else find it's dual norm.