CS5560: Exam1

7:30pm-8:30pm, 30-08-2018.

Note: Please write **relevant**, **precise**, and **concise** answers. Please write **legibly**. Do not forget to write your ROLL NO. on the answer sheet.

1. From first principles[1], derive the (generic[2]) optimization problem associated with Maximum-Conditional Likelihood Estimation (MCLE).

   [**2**Marks]

2. Write down the expression for the likelihood function associated with the (multi-class) logistic regression model. Identify the parameters. For this model, write down the MCLE problem in a simplified form.

   [**2**Marks]

3. Consider the ML application where output (y) is the number of views of an ML course video and input (x) is the (vector of) factors affecting y like time of the day, day of week, season, content of video, speaker in video, comments on the video etc. The goal is to estimate the (unknown) distribution of y given (any) x using a training set of sample (x_i, y_i) pairs.

   (a) Argue that the logistic regression model, Bayes classifier model, and the linear regression model are not well-suited for this application.

   [**1**Mark]

   (b) Which named discrete distribution in chapter 2 of your textbook[3] would you use to model the outputs? Justify your answer.

   [**1**Mark]

   (c) Write down an appropriate discriminative model for this problem using your modeling choice (above in step (b)). Identify the parameters.

   [**1**Mark]

[1] i.e., starting only from the intuition that the "best" parameter is that which minimizes the "distance" between the corresponding distribution in the model and the unknown distribution being modeled.

[2] i.e., agnostic to details of the model.

[3] In case you forgot the simple distributions in textbook, then you are free to choose from any distribution that you know. You may also "design" a distribution now!

4. Consider the model defined by the following likelihood function:

$$p_\theta(x) \equiv \frac{\theta}{\pi\left(x^2+\theta^2\right)} \;\forall\; x \in \mathbb{R},$$

where $\theta > 0$ is the parameter.

(a) Show that for any training data, the MLE exists, and is unique[4].

[**5**Marks]

(b) Explain what is the difficulty (if any) in applying the method of moments for parameter estimation with this model.

[**3**Marks]

[4] In other words, for any training data, show that there exists a value of the parameter at which the likelihood of the training data is maximum. Moreover, show that at no other value of the parameters the likelihood is maximum. In other words, show that the optimization problem associated with MLE in this model always has an optimal solution, moreover it is unique.