CS5560: Exam2

6:45pm-8:15pm, 24-10-2018 (Wed).

Note: Please write **relevant**, **precise**, and **concise** answers. Please write **legibly**. Do not forget to write your ROLL NO. on the answer sheet.

1. Consider the Cauchy-model defined in the previous exam. Recall that its likelihood function is given by:

   $$p_\theta(x) \equiv \frac{\theta}{\pi\left(x^2+\theta^2\right)} \;\forall\; x \in \mathbb{R},$$

   where $\theta > 0$ is the parameter. Does this model belong to the exponential family? If yes, then write down the likelihood function in its canonical form and identify the canonical parameters. If not, then prove that it does not belong to the exponential family.

   [**4**Marks]

2. Write down the likelihood function of inverse-gamma model in canonical form and identify the canonical parameters.

   [**3**Marks]

3. Consider the (joint input-output) generative model (belonging to the exponential family) defined by the sufficient statistic: $\phi(x,y) \equiv \psi(x) \otimes \tau(y)$, where $\psi : \mathcal{X} \to \mathbb{R}^{n_1}$ and $\tau : \mathcal{Y} \to \mathbb{R}^{n_2}$ are given functions[1]. Let the training set be $\mathcal{D} = \{(x_1, y_1), \ldots, (x_m, y_m)\}$.

   (a) From first principles, derive a simplified expression for the (moment matching) condition that the MLE for the parameters must satisfy.

   [**3**Marks]

   (b) Write down a simplified expression for the sufficient statistics, and the canonical parameters of the predictive distribution corresponding to this joint model. Your expression for the canonical parameters must be in terms of the original joint model's parameters, denoted by $w \in \mathbb{R}^{n_1 n_2}$.

   [**3**Marks]

   (c) Motivated by this predictive (generative) model, write down a corresponding discriminative model, while identifying the parameters.

[1] Assume that the scaling function is unity.

[**1**Mark]

(d) Write down a simplified expression for the (partial moment matching) condition(s) that the MLE for the parameter(s) of the discriminative model must satisfy. You need not present its derivation.

[**2**Marks]

(e) Write down the canonical expression for the likelihood function of the conjugate prior corresponding to the (joint) generative model. Clearly identify this conjugate prior model's sufficient statistics, canonical (hyper) parameters and scaling function. In your expression, denote the partition function of this conjugate prior model by $Z(\cdot)$ i.e., the normalization constant is the partition function evaluated at the hyper-parameters.

[**2**Marks]

(f) Using this conjugate prior, write down a simplified expression for the posterior distribution of the parameters and the (moment matching) condition that the MAP estimate for the parameters must satisfy.

[**2**Marks]

(g) Using this conjugate prior, write down a simplified expression for the Bayesian predictive (Bayesian Average Model) likelihood in terms of Z, hyper-parameters and samples in $\mathcal{D}$.

[**3**Marks]

4. Consider the following[2] model that captures the relations between the observed inputs, X, the hidden variables, Z, and the output variable, Y:

$$
\begin{aligned}
p_{w,w_0}(y/z,x) = p_{w,w_0}(y/z) &\sim \mathcal{N}_{(w^\top z + w_0, 1)}(y),\\
p_{V,v_0}(x/z) &\sim \mathcal{N}_{(Vz + v_0, I)}(x),\\
p_{\mu}(z) &\sim \mathcal{N}_{(\mu, I)}(z),
\end{aligned}
$$

Suppose the training set is $\mathcal{D} = \{(x_1, y_1), (x_2, y_2), \ldots, (x_i, y_i), \ldots, (x_m, y_m)\}$.

(a) Write down a simplified expression for the discrmincative model that satifies the given modeling assumptions.

[**5**Marks]

(b) Provide rough details an EM algorithm for MLE with this discriminative model.

[**7**Marks]

[2] This model is known as the Latent Factor Regression model (or sometimes as Bayesian Factor Regression or sometimes as Supervised PCA), and such models are described in Chapter 12 of Kevin Murphy's book.