INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

Problem Sheet 4

Date : 06.03.17 MA 1140 : Linear Algebra

1. Find the algebraic and geometric multiplicities of each distinct eigenvalues of the following matrix
$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}.$$

2. Using diagonalisation, find A^5, where
$$A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$$

3. Solve the initial value problem of the following system of differential equations using diagonalisation of matrices.
$$\frac{dx}{dt} = 4x - 2y,$$
$$\frac{dy}{dt} = x + y,$$
and initial conditions are $x(0) = 3$; $y(0) = 5$.

4. Suppose $u, v \in V$, where V is a real inner product space. Then prove that
   - $\langle u + v, u - v \rangle = ||u||^2 - ||v||^2$
   - $||u + v||^2 + ||u - v||^2 = 2(||u||^2 + ||v||^2)$.

5. Compute the orthonormal basis for the subspace generated by the span $\{(1, 1, 1, 1), (-1, 4, 4, -1), (4, -2, 2, 0)\}$ in $\mathbb{R}^4$.

6. Suppose $u, v \in V$, where V is a real inner product space. Then prove that u and v are orthogonal vectors iff $||u + v|| = ||u - v||$.

7. The matrices A and B have same eigenvalues, where
$$A = \begin{bmatrix} a & 1 \\ -2 & d \end{bmatrix}; B = \begin{bmatrix} 114/25 & 48/25 \\ 48/25 & 86/25 \end{bmatrix}$$
Find the values of a and d.

8. Determine the orthogonal projection of the vector $(2, 0, -1, 3)$ onto the plane generated by the span of $\{(0, 1, 1, 1), (-1, 1, 0, 1)\}$ in $\mathbb{R}^4$.