INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

Problem Sheet 3

Date : 26.02.17 MA 1140 : Linear Algebra

1.
   - Let $T : \mathbb{R}^2 \to \mathbb{R}^1$ be defined by $T((x_1, x_2)) = x_1^2 + x_2^2$. Is T a linear transformation?
   - Let $T : \mathbb{C}^2 \to \mathbb{C}^2$ defined by $T((z_1, z_2)) = (z_1 + z_2, z_1 - 2z_2)$. Is T a linear transformation?

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by
$$T(x_1, x_2) = (x_1 - x_2, 3x_2, 4x_1 + 5x_2)$$
Find the matrix A such that $T(\underline{x}) = A\underline{x}$ for all $\underline{x} \in \mathbb{R}^2$.

3. Consider the basis $A = \{(1, 0, 1), (1, 1, 0), (2, 2, -3)\}$ and $B = \{(3, 1, 1), (0, 0, 1), (-1, 2, 0)\}$ of $\mathbb{R}^3$. Find the matrix for a change of basis from B to A.

4. Are the matrices A and B similar, where
$$A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 2 & 3 & -1 \\ 4 & 1 & -2 & 1 \\ 1 & 2 & -1 & 0 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 4 & 2 \\ 3 & 5 & 1 & 0 \\ -2 & 0 & -2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}$$

5. Find the determinant of the following matrix
$$\begin{bmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{bmatrix}.$$

6. Find the eigen values and eigen vectors of the following matrix
$$\begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix}.$$

7. If A is a matrix with distinct eigen values, then the matrix formed with columns as eigen vectors of A is called the modal matrix P. If $A = \begin{bmatrix} -1 & 4 \\ 0 & 3 \end{bmatrix}$, then calculate the modal matrix of A. What can you say about the matrix obtained from $P^{-1}AP$?

8. Find the eigenvalues of the following matrix
$$\begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}.$$

9. Find the characteristic and minimal polynomial of

$$A = \begin{bmatrix} 8 & -6 & 12 \\ -18 & 11 & 18 \\ -6 & -3 & 26 \end{bmatrix}.$$

10. Verify the Cayley-Hamilton theorem for the matrix A and also find A^{-1}, where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$