ME7100 Assignment - 1: Perturbation Methods

Harish N Dixit

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Note: In the case of solutions to differential equations, plotting your solution is compulsory. Preferably use MATLAB to make your plots.

I. Solve the following algebraic equations using perturbation techniques you learnt in class.

1. $x^2 + x + 6\epsilon = 0$.
2. $x^3 - \epsilon x - 1 = 0$.
3. $\epsilon x^3 + x^2 - 2x + 1 = 0$.

II. **Regular perturbations:** Solve the following IVP's using regular perturbation technique:

1. $y'' = (\sin x)y; \qquad y(0) = 1; \quad y'(0) = 1.$

Plot your solution and comment on the nature of the solution as $x \to \infty$. Is your perturbation expansion uniformly valid in this large x limit?

2. $y'' + (1 - \epsilon x)y; \qquad y(0) = 1; \quad y'(0) = 0.$

Obtain the exact solution using Maple or Mathematica. Plot your perturbation solution against the exact solution and comment on the nature of the solution as $x \to \infty$.

Are the above two problems regular or singular pertubation problems in the limit $x \to \infty$?

III. **Boundary layers:** Solving the following boundary layer problems using singular perturbation techniques. Clearly explain where the boundary layer is present and the show the relevant boundary layer scalings. Obtain the uniform (composite) solution and plot all three solutions (outer, inner and composite) for a few values of ϵ.

1. $\epsilon y'' + y' = e^{-x}; \qquad y(0) = 1, \quad y(1) = 1$
2. $\epsilon y'' - x^2 y' - y = 0; \qquad y(0) = 1, \quad y(1) = 1$

IV. **Multiple scales:** Use the method of multiple scales to solve the following problems:

1. $y'' + y + \epsilon(y')^3 = 0; \qquad y(0) = 1, \quad y'(0) = 0.$

Comment on the nature of your solution for $\epsilon > 0$ and $\epsilon < 0$. Solve the problem numerically using codes given to you from class.

2. Show that the following oscillator has a limit cycle: $y'' + y = \epsilon\left(y' - \frac{1}{3}(y')^3\right); \qquad y(0) = 0, \quad y'(0) = \alpha$. Assume α is a positive number in your analysis. Now solve the same problem numerically and make a plot of $y(t)$ vs t using $\epsilon = 0.2$ and $\alpha = 0.02$ also showing your perturbation solution. Also plot y vs dy/dt to see the limit cycle behaviour by using various values of α. In the second plot, choose two values of α such that your solution approaches the limit cycle from inside and outside. Does your limit cycle agree with your perturbation solution for small ϵ?