ME7100 Assignment - 1: Perturbation Methods

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Note: In most cases, plotting your solution is compulsory. Preferably use MATLAB to make your plots. Plots in excel are unacceptable. All problems to be solved by hand without making use of Mathematica or Maple.

I. Solve the following algebraic equations using perturbation techniques you learnt in class.

1. $x^2 + x + 6\varepsilon = 0$.
2. $x^3 - \varepsilon x - 1 = 0$.
3. $(1 - \varepsilon)x^2 - 2x + 1 = 0$. (Hint: You will have a non-integral power for ε. You will get a repeated root for leading term which will lead to difficulties).
4. $\varepsilon x^3 + x^2 - 2x + 1 = 0$.
5. $\varepsilon x^3 - x + 1 = 0$.

For problems (1), (2), (3), make plots of x vs ε at $O(1)$, $O(\varepsilon)$ and $O(\varepsilon^2)$.

II. Consider potential flow around a slightly distorted sphere whose surface is given by

$$r = R(\theta, \varepsilon) \equiv 1 + \varepsilon P_2(\cos\theta)$$

where P_2 is a Legendre function and $\varepsilon \ll 1$. The flow outside the sphere is given by the potential flow equations:

$$\begin{array}{rccc} & \nabla^2\phi = 0 & \text{in} & r \geq R(\theta;\varepsilon) \\ \text{subject to} & \phi = 1 & \text{on} & r = R(\theta;\varepsilon) \\ \text{and} & \phi \to 0 & \text{as} & r \to \infty. \end{array}$$

This problem can be solved with regular perturbation expansion

$$\phi(r,\theta;\varepsilon) = \phi_0(r,\theta) + \varepsilon\phi_1(r,\theta) + \varepsilon^2\phi_2(r,\theta) + ...$$

Simply derive the system of equations at various orders of ε. There is no need to solve the problem.

III. Solve the following IVP using regular perturbation technique and obtain the solution up to second order:

$$y'' = -e^{-x}y; \qquad y(0) = 1; \quad y'(0) = 1.$$

Plot your solution. The exact solution has a zero crossing for some positive value of x. Is your perturbation expansion uniformly valid in this large x limit?

IV. The equation of a projectile vertically thrown up, written in non-dimensional form, is given by

$$\frac{d^2y}{dt^2} = -\varepsilon\frac{dy}{dt} - 1; \quad y(0) = 0, \quad \frac{dy}{dt}(t = 0) = 1.$$

The first term on RHS with ε represents the friction from air. In the limit of $\varepsilon \ll 1$, obtain a perturbation solution of the governing equation up to second order. Make a comparative plot showing the exact solution with $\varepsilon = 0.1$, exact solution with $\varepsilon = 0$ and perturbative solution with $\varepsilon = 0.1$. Also make a comment on how the solution changes with increase in ε.

V. The equation of a pendulum of length l is written in non-dimensionless form as

$$\frac{d^2\theta}{dt^2} = -\sin\theta; \quad \theta(0) = \phi, \quad \frac{d\theta}{dt}(t=0) = 0,$$

where time is non-dimensionalized by $\sqrt{l/g}$. Using regular perturbation techniques, obtain the solution for the case when $0 < \phi \ll 1$. Compare this to the case of a simple pendulum when $\sin\theta$ is replaced by θ. Is your approximate solution uniformly valid in time?