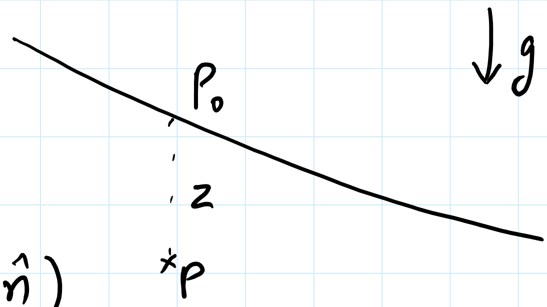
Fluid Statics : Floating objects

17 February 2017 16:11

Floating objects experience both buoyancy and surface tension.

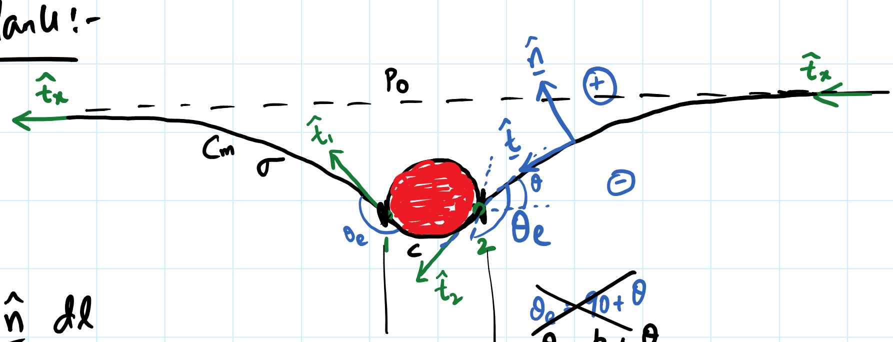
Pressure beneath an interface,



$$p = P_0 + \rho g z + \sigma(\nabla\cdot\underline{\hat{n}})$$

Contour $= C + C_m$

Vertical force balance:-



$\cancel{\theta_e = 90 + \theta}$

$\theta_e = \beta + \theta$

$$Mg = \hat{e}_z \cdot \int -p\underline{\hat{n}}\, dl$$

$$\underbrace{\text{Weight}} = \underbrace{F_b}_{\text{Buoyancy force}} + \underbrace{F_c}_{\text{Curvature force}}$$

C : contour from 1 to 2.

C_m : Contour from $(-\infty$ to $1) + (2$ to $\infty)$

Buoyancy force:-

$$F_b = \hat{e}_z \cdot \int_C \rho g z\, \hat{n}\, dl$$

$$= \rho g V_b$$

Curvature force:-

$$F_c = \hat{e}_z \cdot \int \sigma \underbrace{(\nabla\cdot\underline{\hat{n}})\,\underline{\hat{n}}}\, dl$$

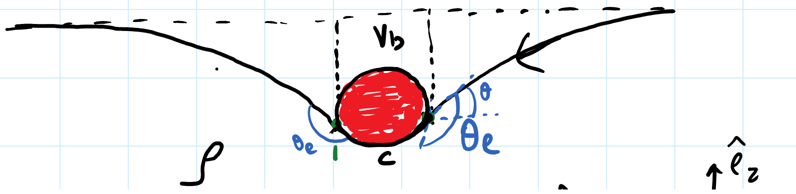
$$= \hat{e}_z \cdot \int_C \sigma \frac{d\hat{t}}{dl}\, dl$$

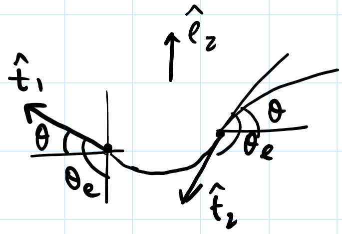
$$\int_C d\bar{t}$$

$$= \sigma\, \hat{e}_z \cdot (\hat{t}_1 - \hat{t}_2)$$

$$= \sigma(\sin\theta - (-\sin\theta)) = 2\sigma\sin\theta$$

$\star$





Frenet - Serret Equations:-

These are well Known results from differential geometry for 2D interface.

$$(\nabla\cdot\hat{\underline{n}})\,\hat{\underline{n}} = \frac{d\hat{\underline{t}}}{dl}$$

$$-(\nabla\cdot\hat{\underline{n}})\,\hat{\underline{t}} = \frac{d\hat{\underline{n}}}{dl}$$

If we add curvature 4 buoyancy forces, we get

$$Mg = \rho g V_b + 2\sigma\sin\theta$$

Now we do a force balance for an interface, ie; only along contour C_m separating air-water interface :-

Since interface is of zero thickness,

$$0 = \rho g z + \sigma(\nabla\cdot\hat{\underline{n}})$$

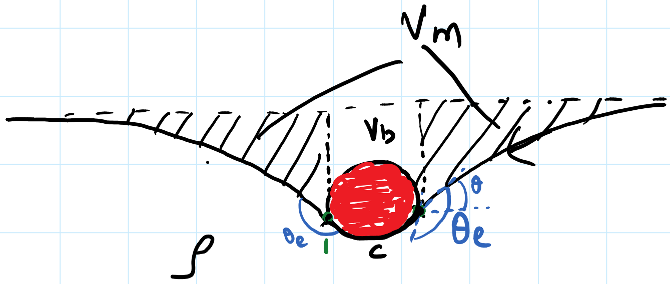
$$\Rightarrow 0 = F_b^m + F_c^m$$

where $F_b^m = \hat{e}_z \cdot \int_{C_m} \rho g z\,\hat{n}\,dl$

$$= \rho g \cdot \underbrace{\hat{e}_z \cdot \int z\,\hat{n}\,dl}_{V}$$

$$V_m$$

$$= \rho g V_m$$



$$F_c^m = \hat{e}_z \cdot \int_{C_m} \sigma (\nabla \cdot \hat{\underline{n}}) \hat{\underline{n}} \, dl = \sigma \hat{e}_z \cdot \int_{C_m} \frac{d\hat{t}}{dl} \, dl$$

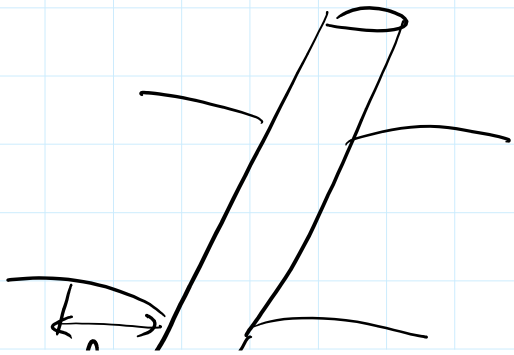
$$= \sigma (\hat{t}_1 - \hat{t}_2) = 2\sigma \sin\theta \qquad (\star\star)$$

Equation ($\star$) and ($\star\star$) show that the curvature force acting on a floating body is expressible in terms of fluid volume displaced "outside" the the line of tangency.

ie; $F_c = \rho g V_m$

The relative magnitude of buoyancy & curvature force is given by

$$\frac{F_b}{F_c} = \frac{\rho g V_b}{\rho g V_m} = \frac{V_b}{V_m}$$



For 2D bodies, distortion is over the capillary length.

For 2D bodies, [illegible]
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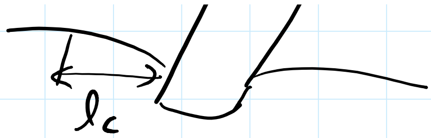
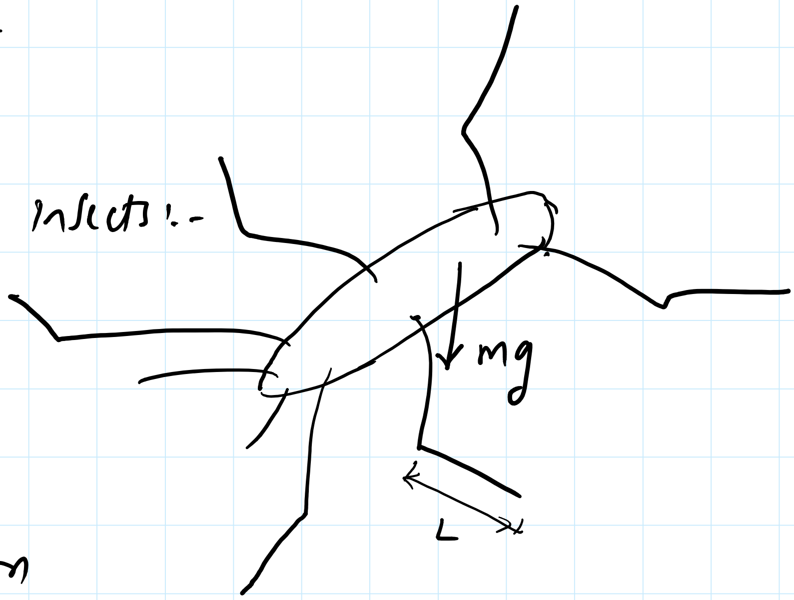


$$\Rightarrow \quad \frac{F_b}{F_c} \sim \frac{a}{l_c}$$

For water-walking insects:-



$F_b \propto a$

$F_c \propto L \times 2\sigma \sin\theta$

L ↓ length of legs on water

Weight of the insect $\approx Mg$

Buoyancy is negligible for $a << L$

$\therefore$ Insect will float if $Mg < L \cdot 2\sigma \sin\theta$

or $\dfrac{Mg}{2\sigma L \sin\theta} < 1$

3D extension

3D