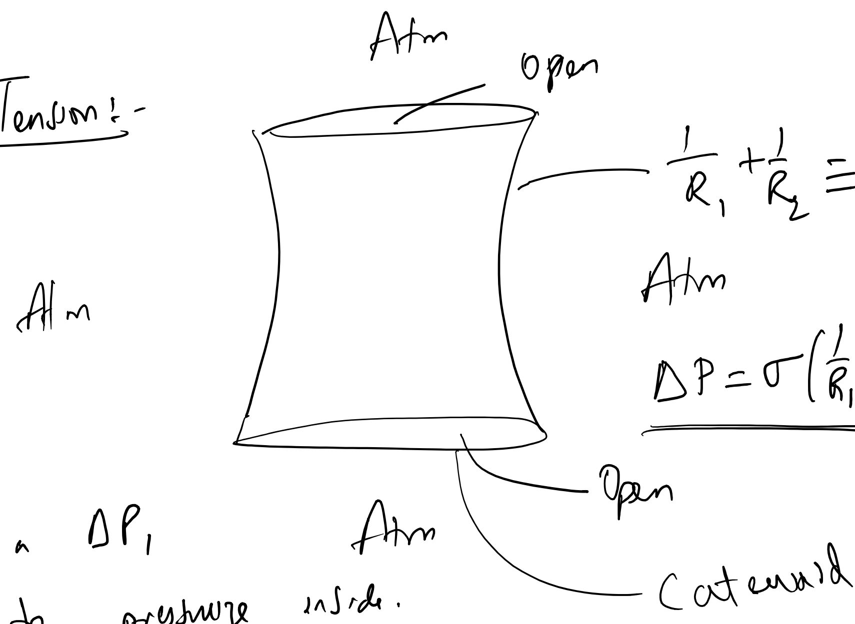
Surface Tension:-



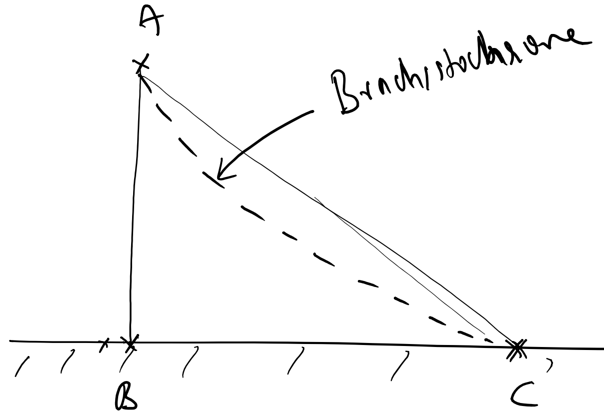
$$\frac{1}{R_1} + \frac{1}{R_2} = 0$$

Atm

$$\Delta P = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

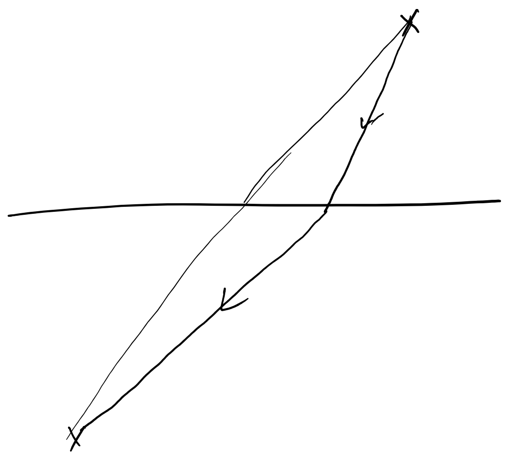
$\Delta P = 0$

To create a ΔP, we need to pressurize inside.

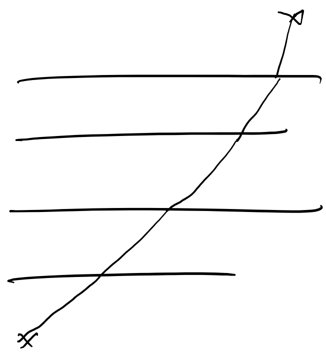


$\downarrow g$

AC is not the quickest route



Vsaule



Governing equations:-

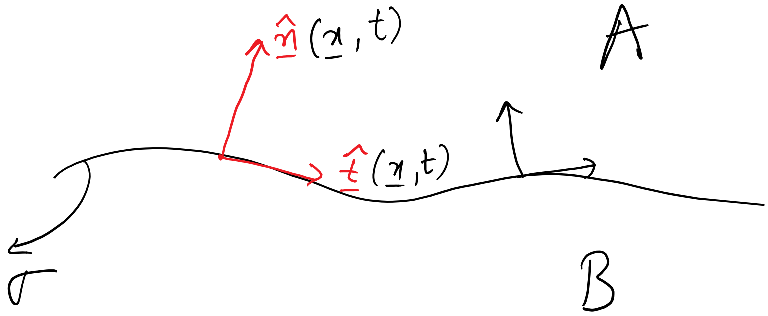
Constant density, ρ

" viscosity, μ or ν

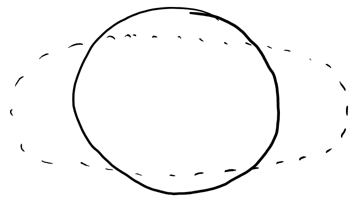
Body force, $\underline{F} = \rho \underline{g}$

$$\nabla \cdot \underline{u} = 0$$

$$\rho \left(\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u} \right) = -\nabla p + \underline{F} + \mu \nabla^2 \underline{u}$$



Boundary conditions at an interface:-



Normal Stress Balance:-

$$\hat{n} \cdot \underline{\underline{T}} \cdot \hat{n} = \sigma (\nabla \cdot \hat{n})$$

$$\Delta P = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

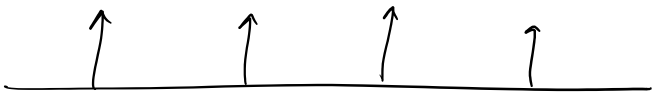
(Mean curvature)

$$= \sigma (\nabla \cdot \hat{n})$$

$\underline{\underline{T}}$: Stress tensor

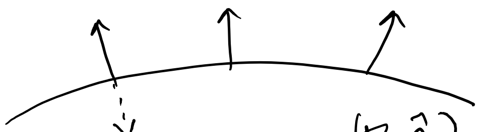
$$= -p \underline{\underline{I}} + 2\mu \underline{\underline{S}}$$

$$= -p \underline{\underline{I}} + \mu (\nabla u + \nabla u^T)$$



Flat interface, Curvature = 0

Tangential Stress Balance:-



Curvature $= -(\nabla \cdot \hat{n})$

$$\underline{n} \cdot \underline{\underline{T}} \cdot \hat{t} = \nabla \sigma \cdot \hat{t}$$

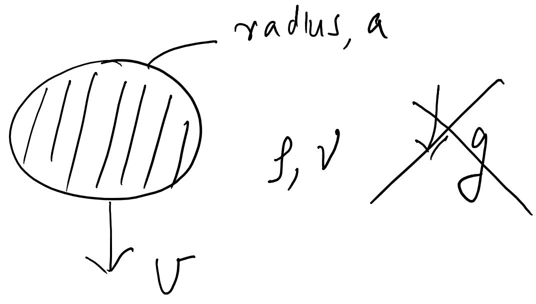
Scaling of Surface tension:-

$$[\sigma] : \frac{N}{m} \text{ or } \frac{\text{dynes}}{\text{cm}}$$

$[\sigma]$ for air-water interface $= 70 \ \frac{\text{dynes}}{\text{cm}}$

$$= 70 \times 10^{-3} \frac{N}{m}$$

Sedimenting sphere:-



Drag = ?

Physical variables:-

$$\boxed{D, V, a, \rho, \nu}, \cancel{g}$$

$M = 5$ (physical variables)

$N = 3$ (fundamental units : Mass, length, time)

No. of dimensionless groups $= M - N = 5 - 3 = 2$

$$C_d = \frac{D}{\rho V^2} \quad ; \quad Re = \frac{Va}{\nu}$$

$$C_d = f(Re)$$

For interfacial flows:-

physical variables : $\{\rho, \nu, \sigma, a, V, g\} \Rightarrow M = 6$

Fundamental units : $3 = N$

No. of non-dimensional groups $= M - N = 3$

---

forces involved: Inertia, Viscous, Surface tension, gravity

$$Re = \frac{\text{Inertia}}{\text{Viscous}} = \frac{\rho V a}{\nu} \quad : \text{Reynolds number}$$

$$Fr = \frac{\text{Inertia}}{\quad} = \frac{V^2}{ga} \quad : \text{Froude number}$$

$$Fr = \frac{\text{Inertia}}{\text{Gravity}} = \frac{v^2}{ga} \quad : \text{Froude number}$$

$$Bo = \frac{\text{Gravity}}{\text{Surface tension}} = \frac{\rho g a^2}{\sigma} = \text{Bond number}$$

If $Bo \approx 1$, $\frac{\rho g a^2}{\sigma} = 1$ or $a^2 = \frac{\sigma}{\rho g}$

Calling this scale l_c, we have

$$l_c = \sqrt{\frac{\sigma}{\rho g}} \quad : \text{Capillary length.}$$

$\rho = \rho_A - \rho_B$

Air - Water interface:-

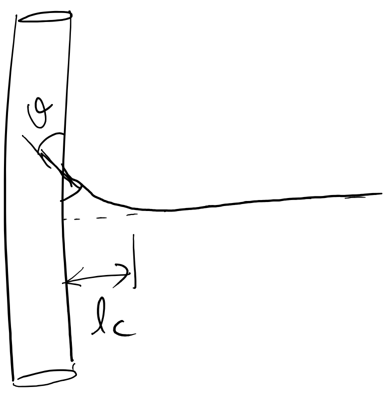
$$\sigma = 70 \frac{mN}{m}$$

$$g \simeq 9.81 \ m/sec^2,$$

$$\Delta\rho = 999 \ kg/m^3,$$

$$l_c \approx 2.7 \ mm$$

Typically determines size of pendant drops, water-walking insects, rain drops, etc.



Other choices:

$$We = \frac{\text{Inertia}}{\text{Surface tension}} = \frac{\rho U^2 a}{\sigma} = \text{Weber number}$$

$$Ca = \frac{\text{Viscous}}{\text{Surface tension}} = \frac{\rho \nu U}{\sigma} \; : \text{Capillary number}$$

If there are surface tension gradients:-

$$Ma = \left(\frac{\Delta\sigma}{L}\right)\frac{a}{\sigma} = \text{Marangoni number} = \frac{\text{Marangoni stress}}{\text{Curvature of surface tension}}$$

Non-dimensionalization:-

Dynamic pressure, $p_d = p - \rho \underline{g} \cdot \underline{x}$

$= p - \rho g z$

Assume high Re flow:-

$\underline{u} = U \underline{u}'$, $\underline{x} = a \underline{x}'$, $p_d = \rho U^2 p_d'$

$t = \frac{a}{U} t'$

$$\frac{\partial \underline{u}'}{\partial t} + \underline{u}' \cdot \nabla' \underline{u}' = -\nabla p_d' + \frac{1}{Re} \nabla'^2 u'$$

$$\nabla' \cdot \underline{u}' = 0$$

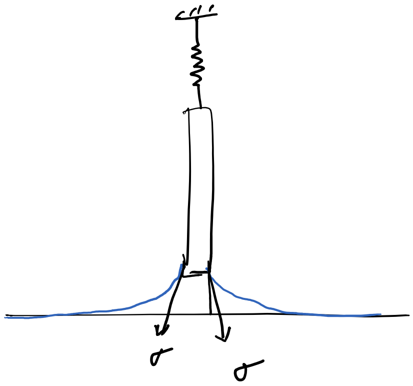
$$-p_d' + \frac{1}{Fr} z' + \frac{2}{Re} \hat{n} \cdot \underline{\underline{S}}' \cdot \hat{n} = \frac{1}{We} \nabla' \cdot \hat{n}$$

Measuring surface tension:-

Force-based technique:-

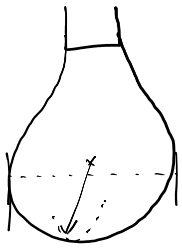
(Wilhelmy plate method)

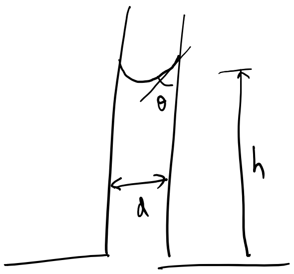
(Wilhelmy plate method)



Pendant drop technique:-

$$\Delta P = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$





h to σ

$$h = f(\sigma, g, \theta, d)$$

Thu – 4 to 5:30

Sat – 10:00 to 1:00

Mon – 2:30 to 4:00