

H314

Hydrostatics and Properties of Fluids

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NOMENCLATURE

| | |
|----------------|---|
| A | Area |
| d, D | Diameter |
| F | Force |
| g | Acceleration due to gravity |
| h | Height of depth of fluid |
| Δh | Height difference (e.g. differential head of manometer) |
| M | Moment |
| p | Pressure |
| R | Radius |
| u | Velocity |
| W and ω | Weight |
| m | Mass |
| ρ | Density of fluid |
| σ | Density of solid, surface tension |
| θ | Angle |
| τ | Shear stress |
| μ | Coefficient of viscosity |
| ν | Kinematic viscosity |

SECTION 1.0 INTRODUCTION

The TecQuipment H314 Hydrostatics and Properties of Fluids apparatus provides a comprehensive range of experiments and demonstrations which are designed to give the student a thorough understanding of the basic principles of fluid mechanics and properties of fluids. Properties such as density, viscosity and surface tension can be determined and basic principles such as Pascal's Law and Archimedes' Law demonstrated. From these, the student can progress to a wide range of practical applications of hydrostatic principles, including buoyancy, centre of pressure, flotation and stability of floating bodies, measurement of barometric pressure, operation and calibration of a Bourdon pressure gauge, and the very important subject of manometry. In addition to these topics, a demonstration of static head due to friction along a pipe can be given.

SECTION 2.0 DESCRIPTION OF THE APPARATUS

2.1 General Description



Figure 1 H314 Hydrostatics and Properties of Fluids Apparatus

The TecQuipment H314 Hydrostatics and Properties of Fluids apparatus is shown in Figure 1. The unit consists of an integral plastic moulding mounted on lockable castors and is complete with all the necessary equipment for a wide range of experiments. Much of the equipment is rigidly mounted on the bench, with the remainder being free-standing items suitable for use with the bench top. The water required for the experiments is supplied from a reservoir tank via a lift pump. An additional tank is mounted on the unit and this can be filled from the reservoir for experiments which require a free water surface. A large drain tray is fitted in the top for collecting and returning water to the reservoir.

The right-hand side of the unit has moulded features to locate a number of experiments, such as centre of pressure, Archimedes and so on, together with storage for loose items. The back panel supports a set of U-tubed manometers (pressurised by an air pump), a pressure gauge, and Pascal's tubes. This is covered in an easy-to-clean plastic sheet. The left-hand side of the unit provides covered storage for viscosity jars and small items.

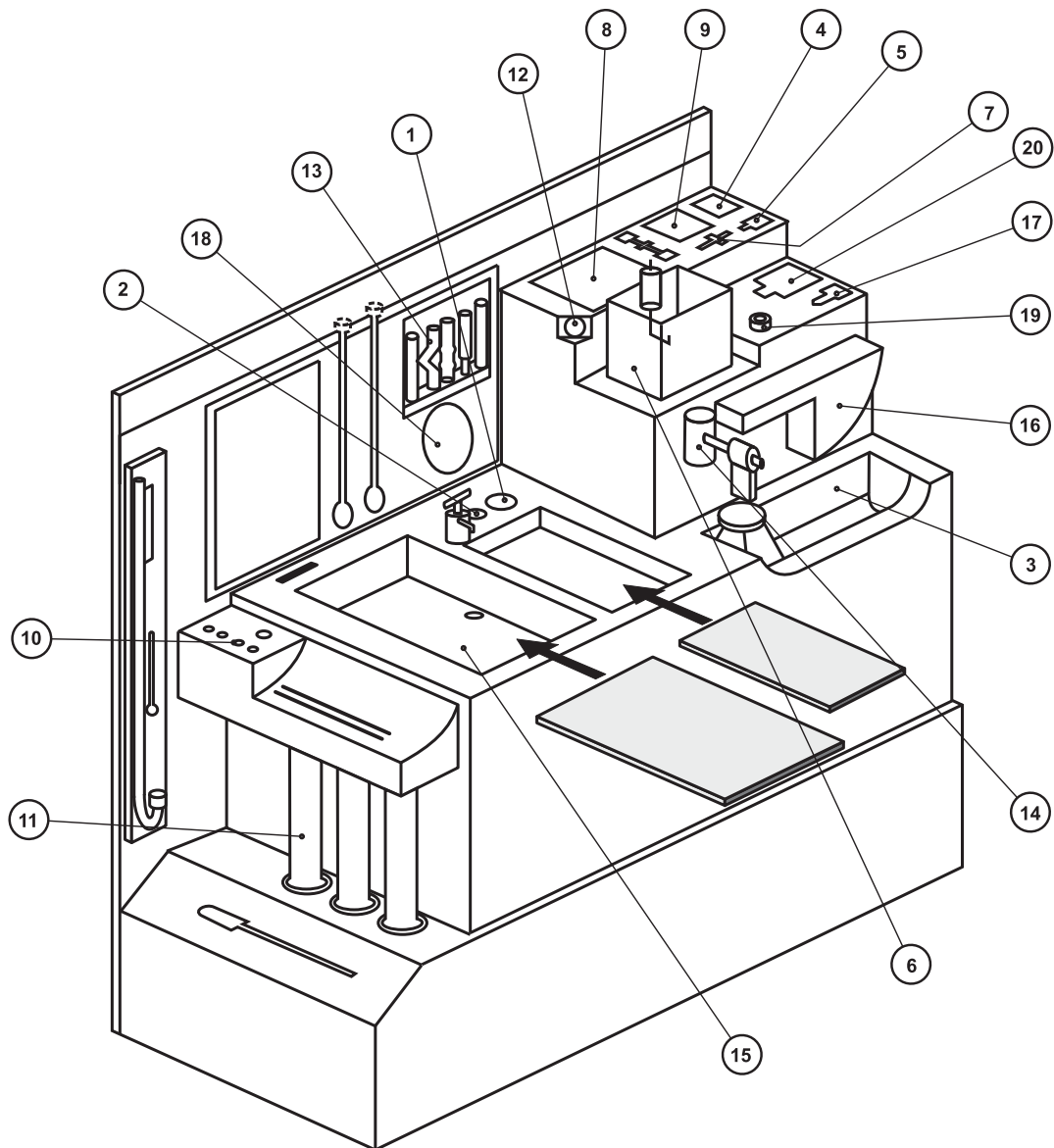
A portion of the working surface can be removed to access a reservoir which stores a rectangular stability pontoon with adjustable weights, and can be filled to float the pontoon for experiments.

Apparatus supplied for determining a variety of fluid properties includes a Eureka can, a specific gravity bottle, a hydrometer capillarity apparatus, a falling sphere viscometer and a point gauge for fluid level measurement.

The range of experiments that can be undertaken are:

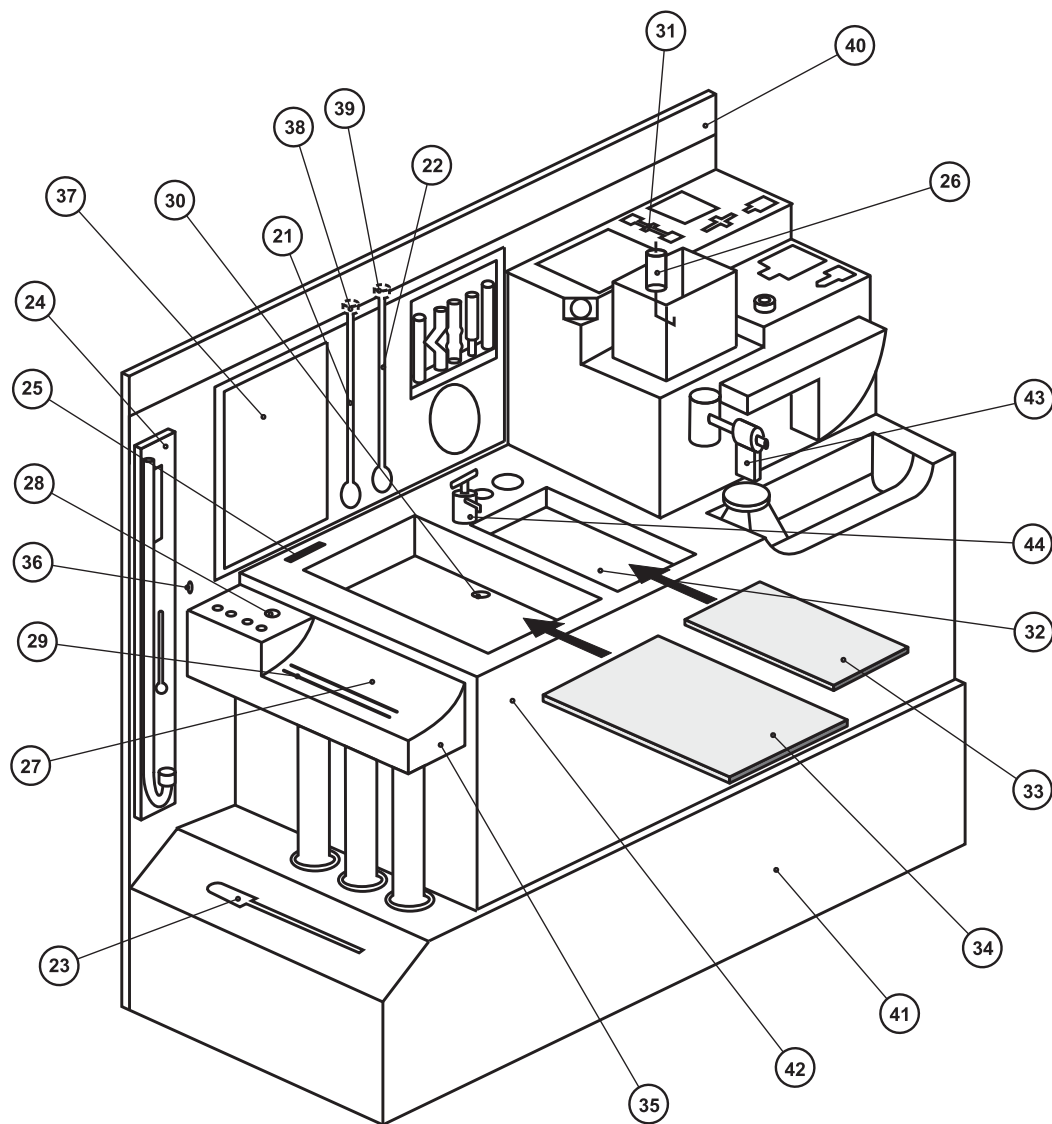
- Determination of fluid density and specific gravity
- Principles and use of a hydrometer
- Capillarity (capillary action) in tubes and between plates
- Measurement of viscosity by falling sphere method
- Demonstration of Pascal's Law
- Measurement of fluid levels by hook gauge
- Fluid flow head relationship
- Verification of Archimedes' Law and demonstration of the principles of flotation
- Stability of a floating body and determination of metacentric height
- Periodicity of a floating body
- Measurement of force and centre of pressure on a plane surface
- Operation and calibration of a Bourdon pressure gauge
- Principle of a single limb mercury barometer
- U-tube manometers: fluid/air, and mercury under water

2.2 Item List



- | | | | |
|----|--------------------------|----|----------------------|
| 1 | Measuring Beaker – 800ml | 11 | Graduated Jars |
| 2 | Measuring Beaker – 100ml | 12 | Stopwatch |
| 3 | Beam Balance | 13 | Pascal's Tubes |
| 4 | Eureka Can | 14 | Archimedes |
| 5 | Density Bottle | 15 | Pontoon |
| 6 | Header Tank | 16 | Toroidal Segment |
| 7 | Capillary Tubes | 17 | Weights (Brass) |
| 8 | Capillary Plate | 18 | Bourdon Gauge |
| 9 | Shims | 19 | Calibration Cylinder |
| 10 | Spheres | 20 | Weights (Iron) |

Figure 2



- | | | | |
|----|---------------------------|----|------------------------|
| 21 | Mercury Manometer | 33 | Right Hand Sink Cover |
| 22 | Water Manometer | 34 | Left Hand Sink Cover |
| 23 | Cycle Pump | 35 | Side Moulding |
| 24 | Barometer | 36 | Schrader Valve (V4) |
| 25 | Valve (V1) | 37 | User Guide |
| 26 | Depth Gauge | 38 | Mercury Manometer Trap |
| 27 | Hydrometer | 39 | Water Manometer Trap |
| 28 | Ball Guide | 40 | Top Moulding |
| 29 | Pipette Tube | 41 | Main Unit |
| 30 | Rubber Bung | 42 | Sump Tank (Internal) |
| 31 | 50g Weight Hanger (Brass) | 43 | Vertical Scale |
| 32 | Drain Cover | 44 | Bilge Pump |

Figure 3



NOTE

Items 1 to 28 in Figures 2 and 3 are not consistent with the numbers on the Packing Contents List. Items on the Packing Contents List should be checked against the descriptions in these lists.

2.3 Circuit Layout

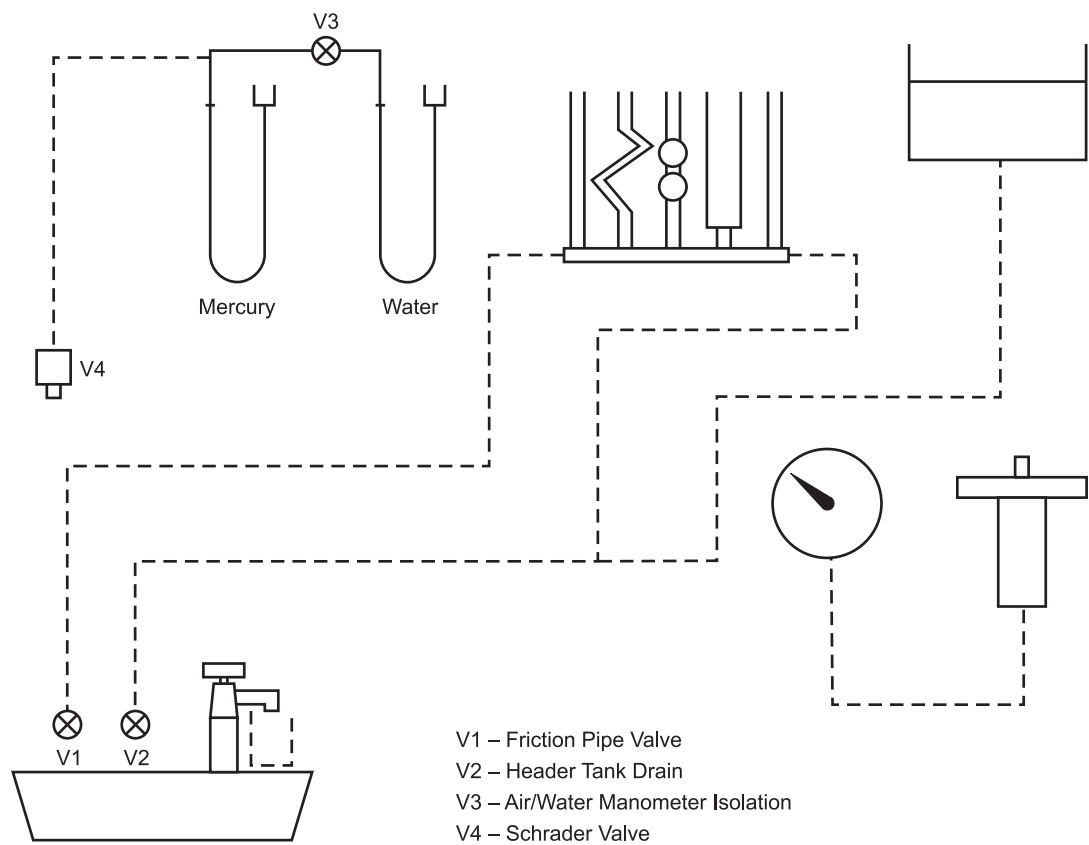


Figure 4 H314 Circuit Diagram

The circuit diagram of the apparatus, shown in Figure 4, is given for reference.

SECTION 3.0 SETTING UP AND GENERAL INSTRUCTIONS

3.1 Unpacking

The TecQuipment H314 Hydrostatics and Properties of Fluids apparatus is dispatched in a partially-assembled condition. Check against the packing list that all items have been received. Missing or damaged items must be reported immediately to TecQuipment Ltd, or your TecQuipment agent.

Before commencing assembly, examine and thoroughly clean all components to ensure freedom from traces of packing materials. Pay particular attention to the inside of the sump tank, as foreign bodies can cause blockages in the pump and the valve.

3.2 Assembly

Refer to Figure 2 and Figure 3 for a list of the various parts and their numbers which are used below.

- a) Place all loose items in their respective moulded storage area on the unit.
- b) Fill the sump tank (42) to within approximately 25 mm of the drain hole of the left-hand reservoir. Add a little water colouring.
- c) Remove the cover (40) and fill the manometers via the manometer traps. The right-hand side manometer is air/water, the left-hand side manometer is air/mercury. Note that mercury is not supplied with this apparatus; you need approximately 40 g of general-purpose laboratory mercury, which you should buy locally.



WARNING

Mercury and its vapours are poisonous and should be treated with great care. When the barometer is full, a small amount of water should be poured into the reservoir to cover the mercury and so prevent vapour escaping into the air.

- d) The mercury barometer is supplied empty for safety reasons and will therefore need to be removed and filled. **If you are wearing any items containing gold or silver, remove them prior to filling the barometer.** The object is to fill the dead-ended limb with a continuous column of mercury and then invert the column so that a vacuum is formed in the closed end of the tube.

Hold the manometer upside down and support it firmly. Thread the supplied catheter tube into the short piece of tubing and then into the manometer tube itself. Ensure the catheter tube end is touching the sealed end of the glass column – the short tubing helps to prevent buckling in the bowl. Fill the supplied syringe with 10 ml of mercury and connect to the catheter tube. Slowly fill the glass column using the syringe, and, as the mercury fills the column, withdraw the tube ensuring there are no air bubbles left. Fill up to the bend and return the barometer to its normal position. Calibration can be achieved by sliding the column up or down, or by adding a bit more mercury, until the reading is correct. When the barometer is full, pour a small quantity of water into the reservoir to cover the mercury.

- e) Carefully unpack the pressure gauge calibration cylinder (19) and place into the moulded recess provided. Connect the length of clear plastic tube from the rear of the pressure gauge to the base of the cylinder. The cylinder may now be filled with water and bled at the gauge end to remove all traces of air.
- f) Use the bilge pump (44) and the 800 ml beaker (1) to fill the header tank (6).
- g) Screw the depth gauge (26) to the block on the header tank using the stainless steel screws provided.
- h) Untie the knot in the black cord attached to the stopwatch and remove the brass weight from the cord. Thread both ends of the cord through the two holes in the stepped corner of the moulding, re-thread the weight on the cord and re-knot. The watch should now sit upright on the step.

- i) Assemble the Stability of a Floating Body apparatus by fitting the sail into its housing on the pontoon and tightening the clamp screws. Check that the plumb bob hangs vertically downwards on its cord and is free to swing across the lower scale.

The unit now ready for use.

SECTION 4.0 EXPERIMENTS

4.1 Properties of Fluids

The term fluid relates to both gases and liquids (for example, air and water) and, although there are differences between them, they both have the same essential property that when acted upon by any unbalanced external force, an infinite change of shape will occur if the force acts for a long enough time. Alternatively, one may say that if acted on by a force, a fluid will move continuously, while a solid will distort only a fixed amount. If a shear force is applied to one surface of a volume of fluid, the layers of fluid will move over one another, so producing a velocity gradient in the fluid. For a given shear stress, a property called the viscosity determines the velocity gradient and hence the velocity of the fluid in the plane of the applied stress. The viscosity is a measure of the fluid's resistance to motion. Viscosity is a very important property in fluid mechanics since it determines the behaviour of fluids whenever they move relative to solid surfaces.

Liquids and gases both share the property of fluidity described previously, but they differ in other respects. A quantity of liquid has a definite volume and, if in contact with a gas, it has a definite boundary or free surface. Gases, on the other hand, expand to fill the space available and cannot be considered as having a definite volume unless constrained on all sides by fixed boundaries, such as a totally enclosed vessel. The volume of a liquid changes slightly with pressure and temperature, but for a gas these changes can be very large. For most engineering purposes, liquids can be regarded as incompressible, by which we mean that volume and density do not change significantly with pressure, whereas gases usually have to be treated as compressible. Similarly, the effects of varying temperature can often be ignored for liquids, except in certain special cases, but must be taken into account with gases.

The engineer is often concerned with determining the forces produced by static or moving fluid and, when doing this, the above differences between liquids and gases can be very important. Generally, it is much easier to deal with liquids because, for most purposes, it can be assumed that their volume and density do not change with pressure and temperature. In the study of hydrostatics, we are primarily concerned with the forces due to static liquids. The forces result from the pressure acting in the liquid and, at a given point, this depends on the depth below the free surface. Density, or mass per unit volume, is a basic property which must be known before any calculation of forces can be made.

When considering the interfaces between liquids, solids and gases there is a further property which can produce forces and this is called the surface tension. When a liquid/gas interface is in contact with a solid boundary, the edge of the liquid will be distorted upwards, or downwards, depending on whether the solid attracts or repels the liquid. If the liquid is attracted to, or 'wets' the solid, it will move upwards at the edge and the surface tension will cause a small upwards force in the body of the liquid. If the liquid is in a tube, the force will act all round the periphery and the liquid may be drawn up the tube by a small amount. This is sometimes called the capillarity effect or capillary action. The forces involved are small and the effect need only be considered in a limited number of cases.

4.2 Determination of Density

To determine the density of a liquid it is necessary to measure the mass of a known volume of liquid. The volume is the more difficult quantity to determine, and an outline of three methods follows. Any liquid may be used but, for demonstration purposes, water is the most convenient.

Measuring Beaker

- Weigh the empty measuring beaker (1) using the triple-beam balance (3) and record the mass.
- Fill the beaker with water and read the volume as accurately as possible.
- Weigh the beaker plus water and record the mass. The mass of water can then be determined by subtraction and the density ρ obtained as:

$$\rho = \frac{\text{Mass in grammes}}{\text{Volume in ml}} \times \frac{10^6}{10^3} \text{ (kg/m}^3\text{)}$$

The density of pure water at 20°C is 998.2 kg/m³ and this is often rounded up to 1000 kg/m³ for engineering purposes. The experimental result should be within 1% of this value. The measurement of volume is not very precise and depends on the accuracy of the graduations on the beaker and this cannot be checked.

Eureka Can

The Eureka can (4) is a container with a fixed spout. If filled until liquid overflows from the spout, the final level when the liquid has stopped flowing will always be the same, provided the can is level and the liquid is not contaminated. If the can is initially full and a solid object is placed in it, a volume of liquid will be displaced equal to the volume of the object. This gives us a basic method of obtaining a known volume of liquid.

- a) Take a solid object which will fit in the can (for example, a cylinder or cube) and accurately measure its dimensions and calculate its volume.
- b) Place the Eureka can at the edge of the working surface and fill it with liquid until it overflows.
- c) Weigh an empty beaker (2) and place this under the spout.
- d) Gently lower the object into the can until fully immersed and collect the liquid in the beaker. Now re-weigh the beaker plus liquid.

The mass of liquid displaced can be obtained by subtraction and the density calculated as before. The result may be less accurate than with the measuring beaker, but it demonstrates a more fundamental way of determining the volume of liquid. The errors might be reduced if the can and the solid object were much larger. A good question for discussion might be whether the best accuracy would be obtained with a narrow deep can, or a wide shallow can.

Density Bottle

The problem of accurately measuring a volume of liquid can be overcome by using a special vessel with a known volume, such as a density bottle (5). This is accurately made and has a glass stopper with a hole in it through which excess liquid is expelled. When the liquid is level with the top of the stopper, the volume of liquid is 50 cm³ (ml).

- a) Dry and weigh the bottle and stopper.
- b) Fill the bottle with liquid and replace the stopper.
- c) Carefully dry the outside of the bottle with a cloth or tissue paper and remove any excess liquid from the stopper, such that the liquid in the hole is level with the top of the stopper.
- d) Re-weigh the bottle plus liquid and determine the mass of liquid and hence the density.

This method should give an accurate result and is limited more by the accuracy of the balance than by the volume of liquid.

Density of Solids

Having determined the density of a liquid, such as water, it is interesting to note that the methods used can be adapted to measure the density of irregular solids, for example sand. If a measured weight of sand is put into the Eureka can instead of a solid object (as described in the Eureka can experiment above), we can determine the volume of sand from the mass of water displaced (since we now know the density of water). The density of the sand is its mass divided by the volume of water displaced. The density bottle could also be used to determine the density of sand and the student could be asked to work out how to do this as a further exercise.

4.3 Specific Gravity

Specific gravity, or relative density as it is sometimes called, is the ratio of the density of a fluid to the density of water. Typical values are 0.8 for paraffin, 1.6 for carbon tetrachloride and 13.6 for mercury. Specific gravity should not be confused with density, even though in some units (for example, the c.g.s. system) it has the same numerical values.

Similarly, specific weight should not be confused with density or specific gravity. Specific weight is used in some text books in place of density and is the weight force per unit volume of a fluid. It only has a fixed value when the gravitational acceleration is constant. In determining density, we have used a beam balance to 'weigh' quantities of liquid and this is calibrated in grammes (i.e. units of mass). The quantity to be weighed is balanced by sliding weights along the lever arms. A useful question for discussion could be:

"Would the density of water be the same on the Moon, where gravity is one-sixth that on the Earth, and would you obtain the same result for density if you used the methods in Section 4.2 on the Moon?"

Specific gravity can be determined directly from the density of a liquid as measured, for example, by using a density bottle. The value is simply divided by the density of water to obtain the specific gravity. A convenient alternative method is to use a specially-calibrated instrument called a hydrometer (27). This takes the form of a hollow glass float which is weighted to float upright in liquids of various densities. The depth to which the stem sinks in the liquid is a measure of the density of the liquid and a scale is provided which is calibrated to read specific gravity. The sensitivity of the hydrometer depends on the diameter of the stem. A very sensitive hydrometer would have a large bulb and a thin stem (see Figure 5).

To determine the specific gravity of any liquid, place one of the tall glass cylinders (11) on the measuring surface and fill with a liquid and allow the air to rise to the top. Carefully insert the hydrometer and allow it to settle in the centre of the cylinder. Take care not to let it touch the sides, otherwise surface tension effects may cause errors. When the hydrometer has settled, read the scale at the level of the free water surface at the bottom of the meniscus, as in Figure 5.

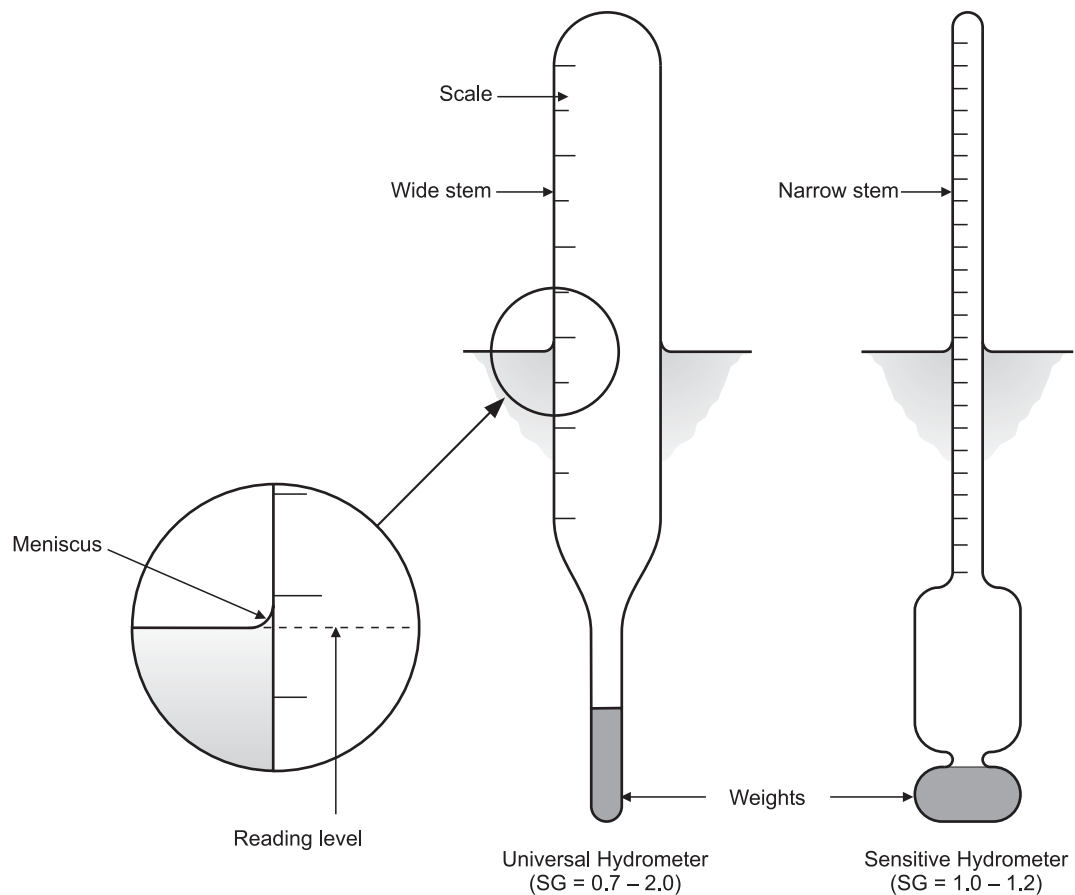


Figure 5 Types of hydrometer

4.4 Capillarity (Capillary action)

The capillarity apparatus is provided with three glass capillary tubes having bores of 0.4 mm, 0.8 mm and 1.6 mm (7). Glass plates (8) are provided, together with plastic shim material (from which strips can be cut to hold the plates a fixed-distance apart). The thicknesses are as follows:

| | |
|-----------|----------|
| Dark blue | 0.050 mm |
| Green | 0.075 mm |
| Amber | 0.100 mm |
| Slate | 0.125 mm |
| Natural | 0.190 mm |
| Black | 0.250 mm |
| Red | 0.400 mm |
| Yellow | 0.500 mm |

For demonstration purposes, it is recommended that a single thickness of the yellow shim should be used to give a plate separation of approximately 0.5 mm. One strip 5 to 10 mm wide should be placed down two edges of a plate and the second plate clamped to it using the clips. The plate assembly is then placed in the header tank (6) in the slot provided, and supported by the extended clips (Figure 6). If necessary, adjust the water height in the header tank by means of the drain valve. The water will creep up the tubes and between the glass plates. The levels can be measured and compared. As a simple demonstration, this shows firstly that capillarity (capillary action) does in fact take place, and secondly that the height to which the water rises depends on the size of the tube. The effect is clearly only significant if the gap is small and is generally ignored for tubes with a bore larger than about 5 mm.

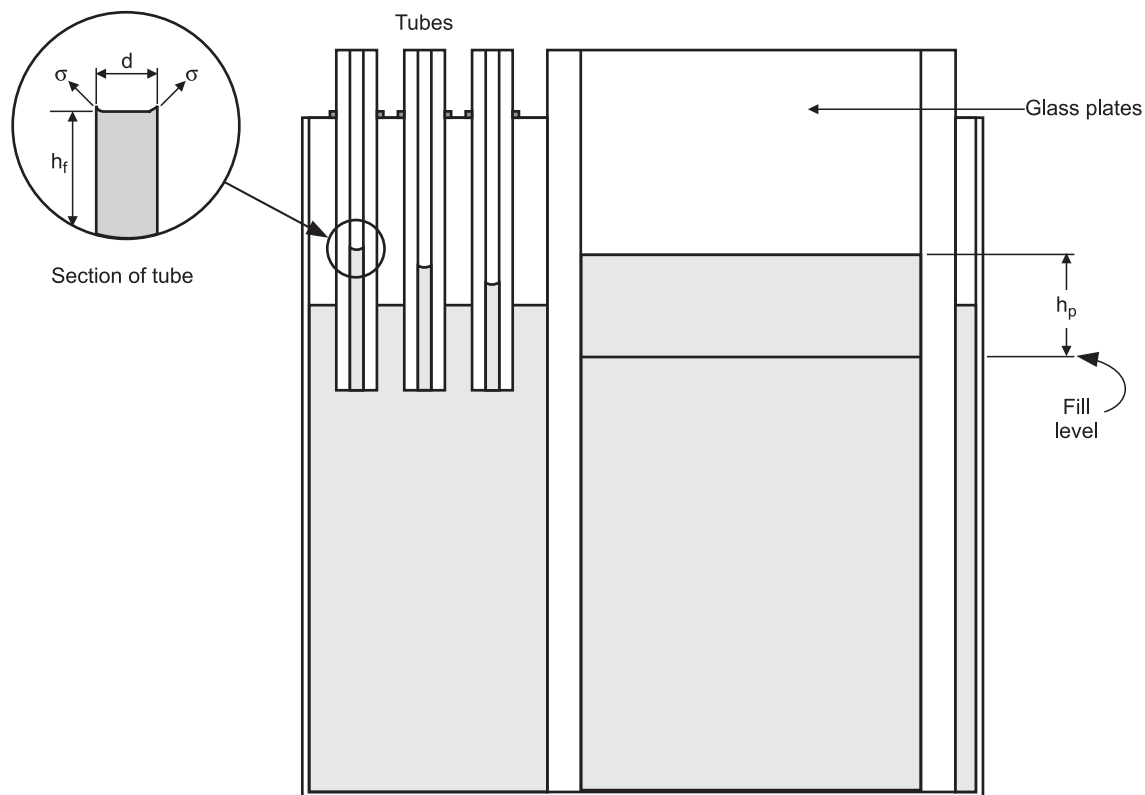


Figure 6 Capillarity Apparatus

A quantitative analysis can be carried out as follows.

An enlarged section of a tube is shown in Figure 6. The surface tension σ (force per unit length) produces an upwards force around the periphery of the tube. We will assume that the water is drawn up at the edges, such that it forms a tangent to the surface, and the force acts vertically on the water. The total force F is:

$$F = \pi d \sigma \quad (1)$$

For equilibrium, this force is balanced by the weight W of the column of water which is approximately:

$$W = \frac{\pi d^2}{4} \rho g h_t \quad (2)$$

Solving for h_t between Equations (1) and (2) we obtain:

$$h_t = \frac{4\sigma}{\rho g d} \quad (3)$$

Similarly, for capillary action between plates distance b apart, we obtain:

$$h_p = \frac{2\sigma}{\rho g b} \quad (4)$$

The actual heights obtained in an experiment will depend very much on the cleanliness of the tubes or plates and on whether any impurities are present in the water. The effect can be very variable and it is not worth calculating values of height from an accepted value of σ as the heights are unlikely to agree with observations. However, it is worth checking that the heights are in the correct proportions. If σ and ρ are constant, we should find that $\frac{h_t d}{4}$ and $\frac{h_p b}{2}$ are constant (i.e. equal to $\frac{\sigma}{\rho g}$) for various tubes and various plate separations.

A typical set of results is as follows:

| | | |
|--------------------|-----------------------|-------------------------|
| 1.6 mm tube: | $h_t = 7 \text{ mm}$ | $\frac{h_t d}{4} = 2.8$ |
| 0.8 mm tube: | $h_t = 12 \text{ mm}$ | $\frac{h_t d}{4} = 2.4$ |
| 0.4 mm tube: | $h_t = 23 \text{ mm}$ | $\frac{h_t d}{4} = 2.3$ |
| Plates 0.35 apart: | $h_p = 14 \text{ mm}$ | $\frac{h_p b}{2} = 2.5$ |

Table 1 Typical Results

The results in Table 1 demonstrate that the levels vary in the predicted manner and that σ was roughly constant. In most hydrostatic calculations, the effect of surface tension can be ignored, but in cases concerning liquid in small tubes it may need to be taken into account. This is referred to in detail in the experiments on manometry later in this section.

4.5 Viscosity

As explained in the introduction to this section, viscosity is one of the most important properties of fluids since it determines the behaviour whenever relative movement between fluids and solids occurs. In a simple case in which a section of fluid is acted on by a shear stress τ , it can be shown that a velocity gradient is produced which is proportional to the applied shear stress. The constant of proportionality is the coefficient of viscosity μ and the equation is usually written:

$$\tau = \mu \frac{du}{dy} \quad (5)$$

where $\frac{du}{dy}$ is the velocity gradient normal to the plane of the applied stress.

Equation (5) is derived in most text books and represents a model of a situation in which layers of fluid move smoothly over one another. This is termed **viscous** or **laminar** flow. For such conditions, experiments show that Equation (5) is valid and the μ is constant for any given temperature. For other conditions at higher velocities, when turbulent eddies are formed and mixing takes place between the layers, the behaviour cannot be represented so simply and we will not consider these cases here.

Equation (5) shows that if fluid flows over an object, there will be a velocity gradient in the flow adjacent to the surface, and a shear force transmitted to the fluid which tends to resist its motion. Similarly, if an object moves through a fluid, velocity gradients will also be set up and a force generated on the object which tends to resist its motion. In all such cases, a knowledge of μ is required to calculate the forces involved. It should be noted that μ varies with temperature, so values for a given fluid are usually tabulated for various temperatures. In the SI system μ has units of Ns/m^2 .

In fluid mechanics the term μ/ρ often appears and this is called the **Kinematic Viscosity** and is denoted by:

$$\nu = \frac{\text{Coefficient of Viscosity } \mu}{\text{Density } \rho} \quad (6)$$

Kinematic viscosity is very often more convenient to use and has units of m^2/s which are often easier to work with.

There are many experimental methods which can be used to determine μ and these are generally less direct than measuring the parameters in Equation (5). One common method is to consider the rate at which a smooth sphere will fall through a liquid for which it is required to determine the viscosity. Under equilibrium conditions, the shear or 'friction' forces on the sphere will equal its weight, and the sphere will fall at a constant velocity μ , called the **terminal velocity**. An equation due to Stokes defines the terminal velocity and this is called **Stokes' Law**.

The equation can be written:

$$\mu = \frac{gd^2}{18\nu} \left(\frac{\sigma}{\rho} - 1 \right) \quad (7)$$

where:

d is the diameter of the sphere
 σ is the density of the sphere
 ρ is the density of the fluid
 ν is the kinematic viscosity of the fluid.

This equation is only applicable for viscous flow, for which a variable called **Reynolds Number** is below a certain value where:

$$\text{Reynolds Number } Re = \frac{\rho u d}{\mu} = \frac{u d}{\nu} \quad (8)$$

The limiting value of Re is often taken as 0.2 and, above this value, the errors in applying Equation (7) becomes significant.

In considering Equation (7), it is clear that the velocity decreases as v increases, and this can be demonstrated for a range of different liquids. It is also possible to determine v (or μ) from Equation (7) and this can be done using the falling sphere viscometer supplied with the apparatus.

Demonstration of Varying Viscosity of Liquids

For this simple demonstration, the three graduated jars can be used, together with the set of steel balls supplied. The cylinders should be filled with three different liquids, for example, water, oil and glycerine. Insert the ball guide (28) into the top of each cylinder in turn. Comparisons can be made by dropping balls (10) of the same size into each cylinder and observing the time taken to reach the bottom. By comparing different sized balls, it can be shown that the velocity depends on diameter as shown in Equation (7).

Determination of Viscosity

The viscosity of relatively high viscosity fluids, such as oil, glycerine, castor oil and so on, can be determined. Fill each of the three graduated jars with different fluids.

NOTE: The oil supplied with the H314 is for maintenance of the piston of the pressure measurement instruments, not for the viscosity experiments.

Test each fluid in turn by:

- Inserting the ball guide.
- Set the upper timing band marker approximately 20 mm below the level of the base of the ball guide.
- Set the lower timing band marker to approximately 200 mm below the first.
- Drop the ball into the fluid and time the descent between the markers using the stopwatch (12).
- Measure the distance between the markers.
- Measure the temperature of the liquid.

Note:

- Moveable timing band markers are used to allow practical timings for very viscous fluids where less than a 200 mm fall is required.
- A vertical reference for the timing band markers is provided by the volume scale on the jar.

| Liquid | Specific gravity at 200°C | Kinematic viscosity ($\nu \times 10^5$) m^2/s at 20°C | Typical time to fall 200 mm(s) | |
|------------|---------------------------|---|--------------------------------|-------------|
| | | | 1.6 mm ball | 3.2 mm ball |
| Water | 1.0 | 0.1 | 0.02 | 0.005 |
| Medium oil | 0.89 | 12 | 2.8 | 0.7 |
| Thick oil | 0.90 | 30 | 6.8 | 1.7 |
| Glycerine | 1.26 | 65 | 10 | 2.5 |
| Caster oil | 0.96 | 100 | 20 | 5 |

Table 2 Viscosity Data for Typical Liquids

It can be seen that thick oils, glycerine and castor oil are the most suitable and that the best accuracy (i.e. longest times) is obtained with the smallest balls.

Typical results obtained with fairly thick lubricating oil, using a 1.6 mm ball, are as follows:

| | |
|----------------------|------------|
| Actual ball diameter | = 1.59 mm |
| Temperature of oil | = 18°C |
| Time to fall 200 mm | = 4.2 secs |

The density of the ball was taken as 7800 kg/m³ and that of the oil as 900 kg/m³. Hence, $\frac{\sigma}{\rho} = 8.7$.

The velocity was $\frac{0.2}{4.2} = 0.048$ m/s.

From Equation (7):

$$v = \frac{gd^2}{18\mu} \left(\frac{\sigma}{\rho} - 1 \right)$$

$$= \frac{9.81 \times 1.59^2 \times 10^{-6} \times (8.7 - 1)}{18 \times 0.048}$$

Therefore:

$$v = 22.1 \times 10^{-5} \text{ m}^2/\text{s}$$

These results are in reasonable agreement with those expected from the data given in Table 2.

4.6 Hydrostatic Principles

In the study of hydrostatics, we are primarily concerned with the pressures and forces produced on solid boundaries by static fluids and, in particular, liquids. It can generally be assumed that the density of fluid does not vary significantly with pressure. Density does vary with temperature, but for most purposes this variation can also be ignored. For water, it can be assumed that the density is constant and equal to 1000 kg/m³. In order to determine the effects of forces produced by static liquids, we need to know the pressure at each point in the liquid and the direction of the forces produced.

Consider the column of liquid in Figure 7. The cross-sectional area is constant and equal to A, the height is h, and the liquid is homogeneous and therefore of constant density ρ . The downward force at plane 2 is the sum of the pressure p_1 acting over area A and the weight of water in the column. For equilibrium, this must be balanced by pressure p_2 acting upwards over area A, so we may write:

$$p_2 A = p_1 A + \rho ghA$$

or

$$p_2 = p_1 + \rho gh \quad (9)$$

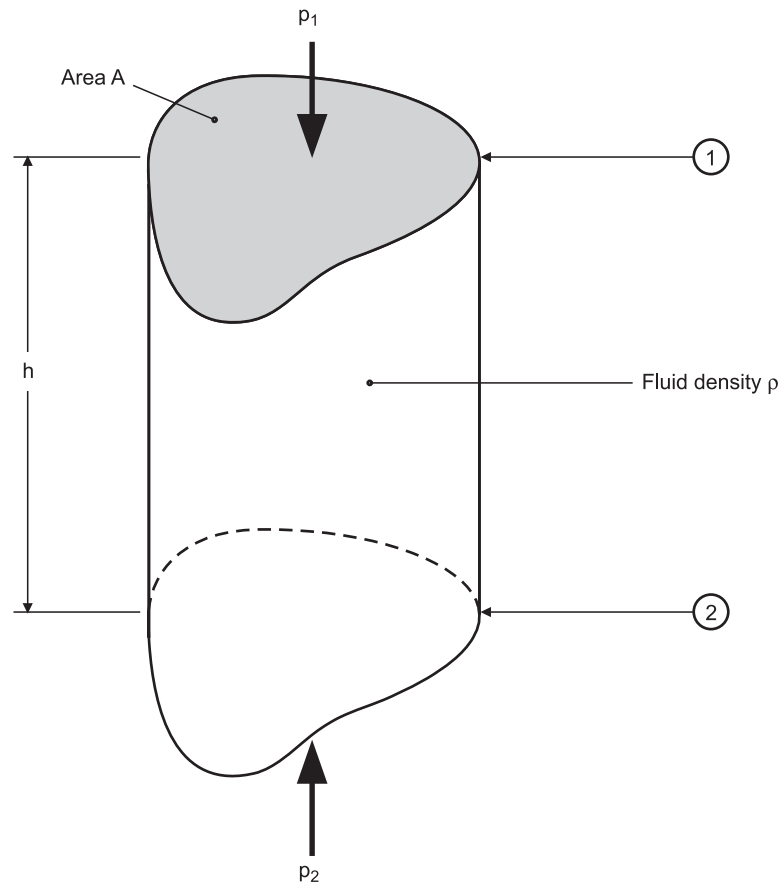


Figure 7 Hydrostatic Pressure due to a Column of Fluid

Notice that the area cancels out and plays no part in determining p_2 so, if the area is very small, we can consider p_2 as the pressure at a point in the liquid. Also note that we have considered p_2 as a pressure acting on the fluid at plane 2. If the liquid column extends downwards, then p_2 must be the pressure exerted upwards by the liquid just below plane 2. Similarly, the liquid at plane 2 must exert a pressure p_2 in the downward direction. We may therefore deduce that the pressure at a point acts equally upwards and downwards. In fact, the pressure at a point in a liquid acts equally in all directions and this is known as **Pascal's Law**. Most text books give proof of this by considering the pressure forces acting on a triangular prism of fluid. The derivation need not be given here.

There are two further important facts about pressure forces in fluids which are related to Pascal's Law.

- a) Pressure forces acting between liquids and solid boundaries always act normal to the plane of the boundary if the liquid is at rest. If they did not, shear forces would be produced and, as discussed in Section 4.1, the liquid would then move.
- b) The pressure is the same at all points in any horizontal plane in a liquid at rest. If this were not the case, there would be a sideways force on an element of liquid and the liquid would move.

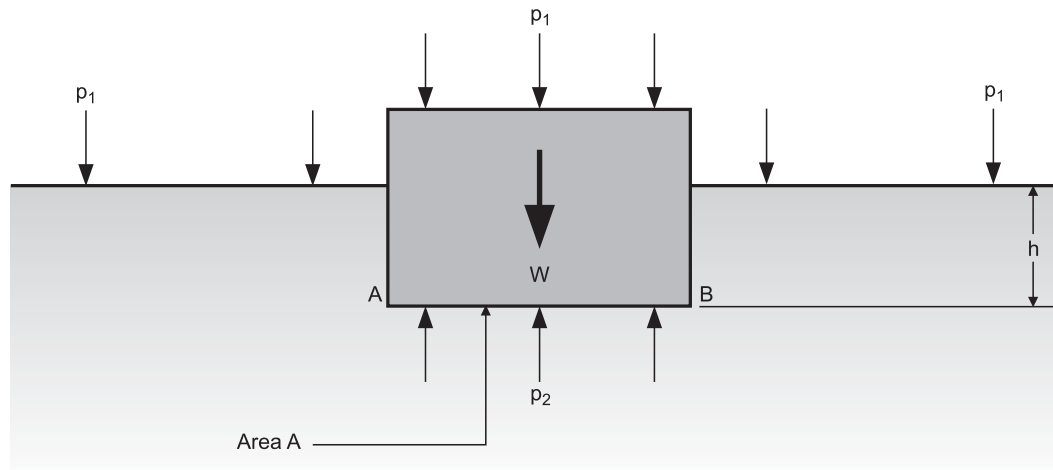


Figure 8 Upthrust on a Floating Body

We may now consider the case of a body immersed in a liquid. Figure 8 shows a rectangular body of a plan area A floating in a liquid with its bottom horizontal. The pressure p_2 acting upwards on the bottom is given by Equation (9) and is equal over area A . For equilibrium, the net upwards force must balance the weight W , hence:

$$W = (p_1 + \rho gh)A - p_1 A = \rho ghA \quad (10)$$

We may also note that the volume of water displaced is $h \times A$ and its weight is ρghA . This is equal to the net upwards force of upthrust given by Equation (10). In fact, this is a simple case of a general rule known as **Archimedes' Law** which states that: *"The upthrust is equal to the weight of water displaced."* This is true for any body, irrespective of its shape, provided that the body and the liquid are at rest.

Liquid Level Apparatus (Pascal's Law)

The apparatus (13) consists of vertical tubes of different sizes, shapes and cross-sections. The tubes are joined by a horizontal pipe at the bottom. The apparatus is permanently connected to the header tank and will thus be filled as the header tank is filled. The header tank is filled using the bilge pump (44) and the 800ml beaker (1). Ensure that the drain valve is closed.

Before filling the apparatus, students should be asked what the levels might be in each of the tubes. Some may well suggest that the levels will be different due to the different cross-sectional areas. Filling the apparatus to various levels will soon show that the levels are always the same in all the tubes. The only slight variation which may occur is due to surface tension and this can form a separate point for discussion. Various conclusions can be drawn by considering the conditions in the apparatus.

- There is no flow along the horizontal pipe between the tubes, so the pressure in the horizontal pipe must be constant.
- The pressure at the bottom of each tube is the same, and the height of water in each is the same. Therefore, a given height of liquid always produces the same pressure, irrespective of the area of the tube and the weight of water contained in it.
- The same pressure is produced by a certain height of liquid, irrespective of the shape of the tube containing the liquid.
- Pressure is transmitted down the bent tube in the same way as in the other tubes. Since the tube is bent, the pressure must act at different angles, following the shape of the tube, and this indicates that pressure acts equally in all directions.

Fluid Upthrust (Archimedes' Law)

“Every body experiences an upthrust equal to the weight of liquid displaced.” The validity of Archimedes' Law can be demonstrated using the cylindrical body (14) attached to moulding above the three-beam balance.

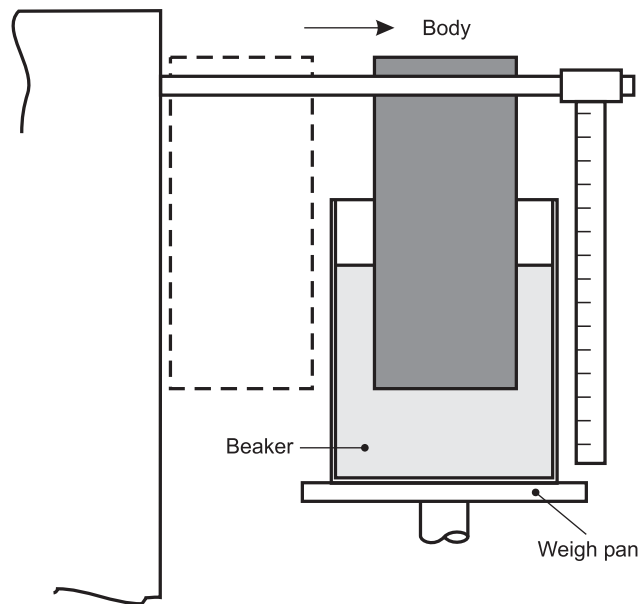


Figure 9 Apparatus for Determination of Fluid Upthrust

Slide the body out from its parked position until it is over the weigh pan of the scale and measure its diameter. Place the body in a beaker by rotating it through 90° , fitting the beaker, and then letting the body and beaker come back to the vertical, allowing the beaker to rest on the pan, as shown in Figure 9.

- Fill the beaker with water until the body is just in the liquid.
- Balance the scale and record the weight.
- Read the height of liquids on the hanging scale.
- Repeat steps b) and c) until the beaker is full (approximately six steps).

From the volume of water displaced by the cylinder, the height of liquid and the resultant weight on the balance, the principle of Archimedes' can be verified.

To drain the beaker after the experiment, slide the body and beaker together off the support and pour water through the working surface.

4.7 Buoyancy, Flotation and Stability of Floating Bodies

In the preceding experiment we have seen that the upthrust, or buoyancy force, is equal to the weight of water displaced. It should be noted that this depends only on the immersed volume of the body and not on its density or weight. However, the question of whether a body will float or sink does depend on these factors. The maximum buoyancy force is produced when the body is fully immersed and if this is less than its total weight it will sink. Conversely, if it is greater, the body will find an equilibrium position with only parts of its volume immersed, such that the buoyancy force just balances the weight of the body, i.e. the body will float. The criterion for floating, therefore, is that the average density must be less than that of water.

Simple Demonstrations

Various simple demonstrations can be carried out using one of the plastic beakers and the weights supplied with the apparatus.

- a) Attempt to float the beaker in the upper tank by placing it in the water, bottom downwards. It is unstable and tips over, and will float or sink depending on whether water gets into it or not. The beaker is 'top heavy' and will not float in the upright position.
- b) Place a few weights in the centre of the bottom of the beaker and again place it in the water. It is now more stable and will float in a more upright position, although it is lower in the water due to the extra weight. The extra stability is produced by lowering the centre of gravity.
- c) If further weights are added, the stability will further improve and the beaker will float lower down in the water.
- d) If sufficient weights are added, the beaker will sink. The volume is approximately 450 ml, so approximately 450 g should be required to make it sink. Note that at this point, the average density of the beaker is equal to that of water, even though much of the volume is filled with air.

Stability of a Floating Body

The type of behaviour demonstrated above can be quantified and analysed using the Stability of a Floating Body apparatus (15) supplied with the bench. The stability will be found to depend on the position of the centre of gravity and, in particular, its position in relation to the centre of buoyancy. This leads to a definition of the metacentric height as a measure of stability.

The question of the stability of a body, such as a ship, which floats in the surface of a liquid, is one of obvious importance. Whether the equilibrium is stable, neutral or unstable is determined by the height of its centre of gravity, and, in this experiment, the stability of a pontoon may be determined with its centre of gravity at various heights. A comparison with calculated stability may also be made.

The arrangement of the apparatus is shown in Figure 10. A pontoon of rectangular form floats in water and carries a plastic sail, with five rows of V-slots at equally-spaced heights on the sail. The slots' centres are spaced at 7.5 mm intervals, equally disposed about the centre sail line. An adjustable weight, consisting of two machined cylinders which can be screwed together, fits into the V-slots on the sail; this can be used to change the height of the centre of gravity and the angle of list of the pontoon. A plum bob is suspended from the top centre of the sail and is used in conjunction with the scale fitted below the base of the sail to measure the angle of list.

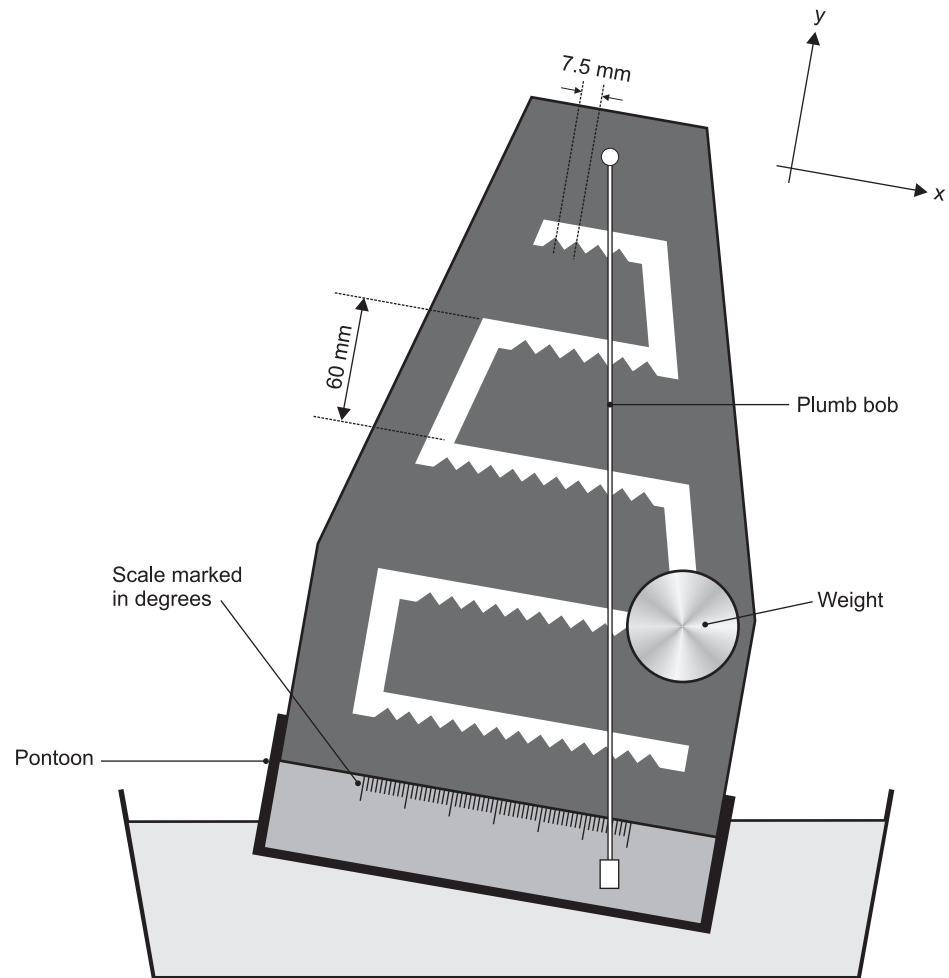


Figure 10 Arrangement of the Floating Pontoon

Consider the rectangular pontoon shown floating in equilibrium on an even keel, as shown in the cross-section of Figure 11(a). The weight of the floating body acts vertically downwards through its centre of gravity G and this is balanced by an equal and opposite buoyancy force acting upwards through the centre of buoyancy B , which lies at the centre of gravity of the liquid displaced by the pontoon.

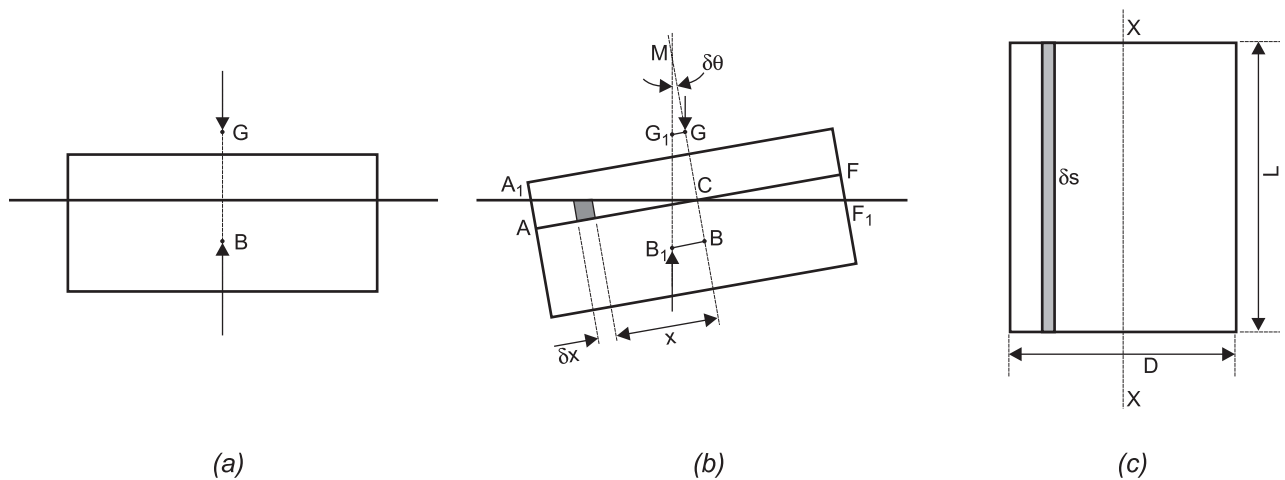


Figure 11 Derivation of the Stability of a Floating Pontoon

To investigate the stability of the system, consider a small angular displacement $\delta\theta$ from the equilibrium position as shown in Figure 11(b). The centre of gravity of the liquid displaced by the pontoon shifts from B to B_1 . The vertical line action of the buoyancy force is shown in the diagram and intersects the extension of line BG at M, the metacentre.

The equal and opposite forces through G and B_1 exert a couple on the pontoon, and provided that M lies above G, as shown in Figure 11(b), this couple acts in the sense of restoring the pontoon to even keel, i.e. the pontoon is stable. If, however, the metacentre M lies below the centre of gravity G, the sense of the couple is to increase the angular displacement and the pontoon is unstable. The special case of the neutral stability occurs when M and G coincide.

Figure 11(b) shows clearly how the metacentric height GM may be established experimentally using the adjustable weight (of mass ω) to displace the centre of gravity sideways from G.

Suppose the adjustable weight is moved a distance δx_1 from its central position. If the weight of the whole floating assembly is W, then the corresponding movement of the centre of gravity of the whole, in a direction parallel to the base of the pontoon, is $\frac{\omega}{W}\delta x_1$. If this movement produces a new equilibrium position at an angle of list $\delta\theta$, then in Figure 11(b), G_1 is the position of the centre of gravity of the whole, i.e.

$$GG_1 = \frac{\omega}{W}\delta x_1 \quad (11)$$

Now, from the geometry of the figure:

$$GG_1 = GM \cdot \delta\theta \quad (12)$$

Eliminating GG_1 , between these equations we derive:

$$GM = \frac{\omega}{W} \cdot \frac{\delta x_1}{\delta\theta} \quad (13)$$

or in the limit:

$$GM = \frac{\omega}{W} \left(\frac{dx_1}{d\theta} \right) \quad (14)$$

The metacentric height may thus be determined by measuring $\left(\frac{dx_1}{d\theta} \right)$ knowing ω and W.

Quite apart from experimental determinations, BM may be calculated from the mensuration of the pontoon and the volume of liquid which it displaces. Referring again to Figure 11(b), it may be noted that the restoring moment about B, due to shift of the centre of buoyancy to B_1 , is produced by additional buoyancy represented by triangle AA_1C to one side of the centre line, and reduced buoyancy represented by triangle FF_1C to the other. The element shaded in Figure 11(b) and Figure 11(c) has an area δs in plan view and a height $x\delta\theta$ in vertical section, so that its volume is $x\delta s\delta\theta$. The weight of liquid displaced by this element is $w x\delta s\delta\theta$, where w is the specific weight of the liquid, and this is the additional buoyancy due to the element. The moment of this elementary buoyancy force about B is $w x^2\delta s\delta\theta$, so that the total restoring moment about B is given by the expression:

$$w\delta\theta \int x^2 ds$$

where the integral extends over the whole area s of the pontoon at the plane of the water surface. The integral may be referred to as I , where:

$$I = \int x^2 ds \quad (15)$$

the second moment of area of s about the axis XX.

The total restoring moment about B may also be written as the total buoyancy force, wV , in which V is the volume of liquid displaced by the pontoon, multiplied by the lever arm BB_1 . Equating this product to the expression for total retiring moment derived previously:

$$wV \cdot BB_1 = w\delta\theta \int x^2 ds$$

Substituting from Equation (15) for the integral and using the expression:

$$BB_1 = BM \cdot \delta\theta \quad (16)$$

which follows from the geometry of Figure 11(b), leads to:

$$BM = \frac{I}{V} \quad (17)$$

This result, which depends only on the mensuration of the pontoon and the volume of liquid which it displaces, will be used to check the accuracy of the experiment. It applies to a floating body of any shape, provided that I is taken about an axis through the centroid of the area of the body at the plane of the water surface, the axis being perpendicular to the plane in which angular displacement takes place. For a rectangular pontoon, B lies at a depth below the water surface equal to half the total depth of immersion, and I may readily be evaluated in terms of the dimensions of the pontoon as:

$$I = \int_{-D/2}^{D/2} x^2 L dx = \frac{1}{12} LD^3 \quad (18)$$

Experimental Procedure

The total mass of the apparatus (including the two magnetic weights and the adjustable weight) is stamped on a label affixed to the sail housing. The total mass (m) is the sum of all the mass values.

The weight (W) of the pontoon = $m \times g$

Where g = acceleration due to gravity.

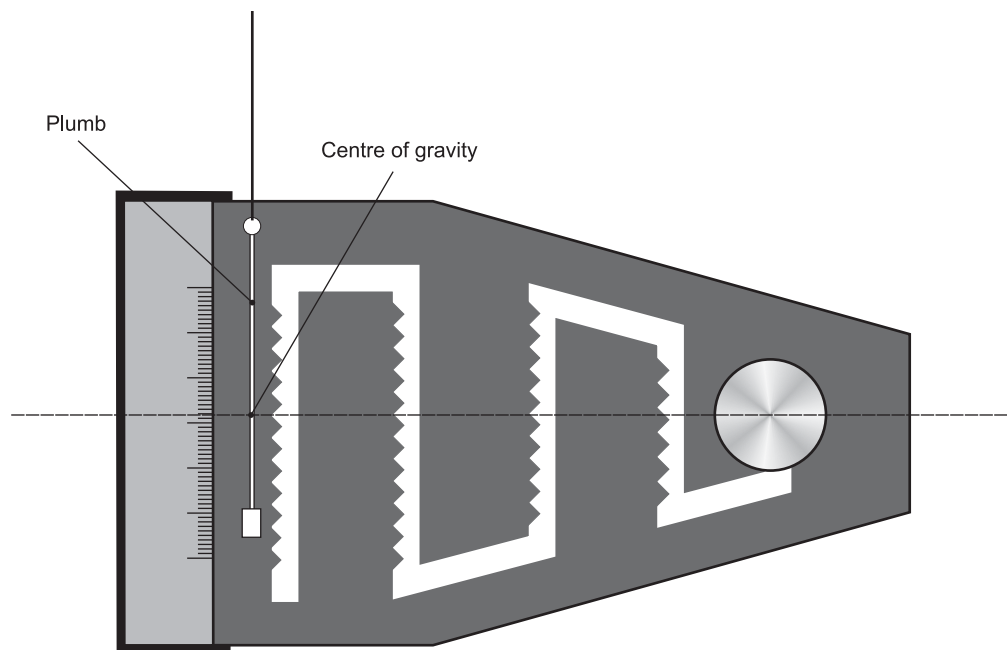


Figure 12 Method of Finding the Centre of Gravity

With reference to Figure 12, the height of the centre of gravity may be found as follows:

- a) Fit the two magnetic weights to the base of the pontoon.
- b) Fit the thick knotted cord, with the plumb weight, through the hole in the sail, ensuring that the plumb weight is free to hang down on the side of the sail which has the scored centre line.
- c) Clamp the adjustable weight into the V-slot on the centre line of the lowest row and suspend the pontoon from the free end of the thick cord. Mark the point where the plumb line crosses the sail centre line (for example, with typist's correcting fluid or a similar marking fluid).
- d) Repeat step c) for the other four rows.

With the adjustable weight situated in the centre of one of the rows, allow the pontoon to float in water and position the two magnetic weights on the base of the pontoon to trim the vessel. When the vessel has been trimmed correctly, the adjustable weight may be moved to positions either side of the centre line for each of the five rows. At each position, the displacement can be determined by the angle the plumb line from the top of the sail makes with the scale on the sail housing.

Results and Calculations

| | | |
|--|---|----------------|
| Total weight of floating assembly (W) | = | N |
| Adjustable weight (ω) | = | N |
| Breadth of pontoon (D) | = | mm |
| Length of pontoon (L) | = | mm |
| Second moment of area $I = \frac{LD^3}{12} \times 10^{-12}$ | = | m ⁴ |
| Volume of water displaced $V = \frac{W}{10^3 \rho}$ | = | m ³ |
| Height of metacentre above centre of buoyancy $BM = \frac{I}{V}$ | = | m |
| Depth of immersion of pontoon = $\frac{V \times 10^6}{LD}$ | = | m |
| Since L and D are in mm depth of centre of buoyancy $CB = \frac{V \times 10^6}{2LD}$ | = | m |

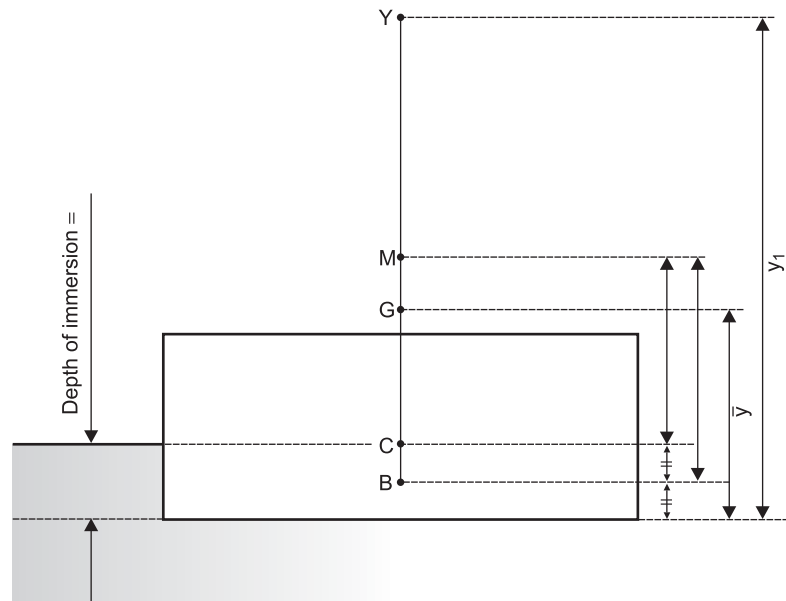


Figure 13 Standard Dimensions of the Pontoon

It is recommended that Figure 13 is marked up appropriately and referred to each time the apparatus is used. Note that when measuring the heights \bar{y} and y_1 , as it is only convenient to measure from the inside floor of the pontoon, the thickness of the sheet metal bottom should be added to \bar{y} and y_1 measurements. The position of G (and hence the value of \bar{y}) and a corresponding value of y was marked earlier in the experiment when the assembly was balanced.

The height of \bar{y} of G above the base will vary with the height y of the adjustable weight above the base, according to the equation:

$$\bar{y} = y_1 \frac{\omega}{W} + A \quad (19)$$

where A is a constant, which pertains to the centre of gravity of the pontoon and the height of the adjustable weight.

Using one set of results for the centre of gravity of the pontoon and the height of the adjustable weight, \bar{y} and y_1 can be measured and the constant A calculated. This can then be used in calculations for subsequent heights of \bar{y} and y_1 which can be checked against the markings.

Values of angles of list produced by lateral movement of the adjustable weight height y_1 should be recorded in the form of Table 3. A graph (Figure 14) for each height y_1 , of lateral position of adjustable weight against angle of list, can then be plotted.

Note: Decide which side of the sail centre line is to be termed negative and then term list angles on that side negative.

| Height of adjustable weight y_1 (mm) | Angles of list for adjustable weight lateral displacement from sail centre line x_1 (mm) | | | | | | | | | | |
|--|--|-----|-----|-----|-----|---|----|----|----|----|----|
| | -75 | -60 | -45 | -30 | -15 | 0 | 15 | 30 | 45 | 60 | 75 |
| | | | | | | | | | | | |

Table 3 Values of List Angles for Height and Position of Adjustable Weight

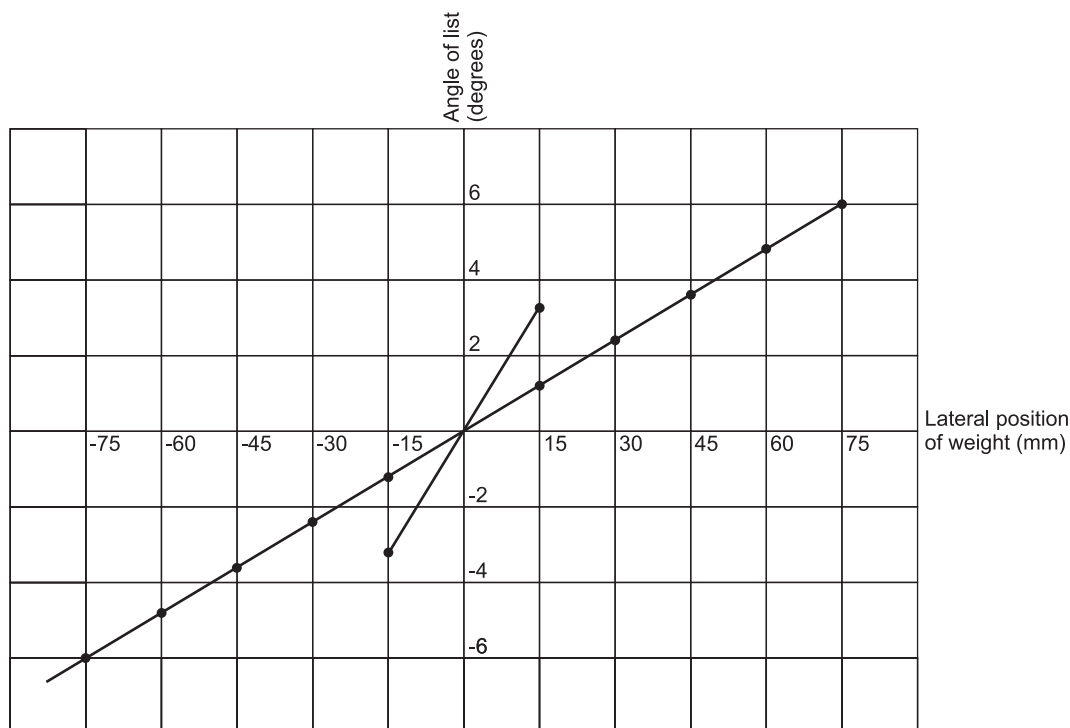


Figure 14 Variation of Angle List with Lateral Position of Weight

From Figure 14, for the five values of y_1 , the corresponding values of $\frac{dx_1}{d\theta}$ can be extracted. Using Equation (14), values of GM can be obtained. Using Equation (19) and knowing the immersion depth, values of CG can be derived. Also, since $CM = CG + GM$, values of CM can be calculated. These values should be calculated and arranged in tabular form as shown in Table 4.

| Height of adjustable weight y_1 (mm) | Height of G above water surface CG (mm) | $\frac{dx_1}{d\theta}$ (mm/°) | Metacentric height CM (mm) | Height of M above water surface CM (mm) |
|--|---|-------------------------------|----------------------------|---|
| | | | | |

Table 4 Derivation of Metacentric Height from Experimental Results

The values of $\frac{dx_1}{d\theta}$ can now be plotted against CG, the height of G above the water line. Extrapolation of this plot will indicate the limiting value of CG above which the pontoon will be unstable (i.e. when $\frac{dx_1}{d\theta}$ is zero and $CG = M$).

Periodicity of a Floating Body

It may be noted during the course of a previous experiment that if a stable floating body is displaced it will oscillate about its normal position with a certain periodic time. When a body is displaced and released, the only force acting is the restoring moment given by:

$$M = W BM \sin\theta$$

where W is the weight of the body, BM the metacentric height and θ the angle through which the body is displaced.

By Newton's Second Law of Motion, we may equate M to the product of the moment of inertia I of the body about the axis of rotation, and the angular acceleration, thus:

$$M = -I \frac{d^2\theta}{dt^2}$$

For small angles, $\sin\theta \rightarrow \theta$ and we may write:

$$\frac{d^2\theta}{dt^2} = -\frac{WBM}{I}\theta$$

This is the equation for simple harmonic motion for which the period of oscillation is:

$$t = 2\pi \sqrt{\frac{I}{WBM}} \quad (20)$$

The periodic times may be determined for the pontoon used in the previous experiment for various values of metacentric height BM . It will be found that t increases as the metacentric height decreases. A graph of t against $\frac{1}{\sqrt{BM}}$ will produce a straight line and the slope can be used to determine I in Equation (20).

4.8 Forces on Plane Surfaces: Centre of Pressure

The centre of pressure may be defined as: “The point in a plane at which the total fluid thrust can be said to be acting normal to that plane.”

The apparatus permits the moment due to the total fluid thrust on a wholly or partially submerged plane surface to be measured directly and compared with theoretical analysis.

Water is contained in a quadrant tank assembly (16) as part of a balance. The cylindrical sides of the quadrant have their axes coincident with the centre of rotation of the tank assembly, and therefore the total fluid pressure acting on these surfaces exerts no moment about that centre. The only moment present is that due to the fluid pressure acting on the plane surface. This moment is measured experimentally by applying weights (17) to a weight hanger mounted on the opposite side to the quadrant tank.

A second tank, situated on the same side of the assembly as the weight hanger, provides a trimming facility. A scale on the quadrant tank is used to measure the level of the water below the pivot (h).

The apparatus will require trimming in order to bring the submerged plane to the vertical in line with the pendulum rule. This is achieved by gently pouring water into the trim tank until the desired position is achieved. Should it be necessary to remove any water, a pipette (29) is supplied which is dipped into the tank and water held by putting a finger on the top of the pipette and transferring the water to the reservoir. Add weight to the weight hanger. Pour water into the quadrant tank until a 0° balance is restored. Note the weight and the level of the water (h). Repeat the procedure for the full range of weights.

Using this method, the particular case of $\theta = 0^\circ$ can be investigated.

Readings should be tabulated in the form outlined in Table 5 and the results calculated in line with the theory given.

| ω (g) | $M = \frac{W \times 9.81 \times R_3}{10^3}$ (Nm) | h (mm) | h (m) | h^3 (m ³) | $M + \frac{BR_2^2 h}{2}$ (Nm) |
|-----------------|---|-----------|----------|----------------------------|----------------------------------|
| | | | | | |

Table 5 Format of Results Table – Centre of Pressure for the Particular Case of $\theta = 0^\circ$

The following analysis is applied to the general condition of a plane surface at various angles when it is wholly or partially submerged in a fluid.

Let breadth of quadrant = B
and weight per unit volume = ω

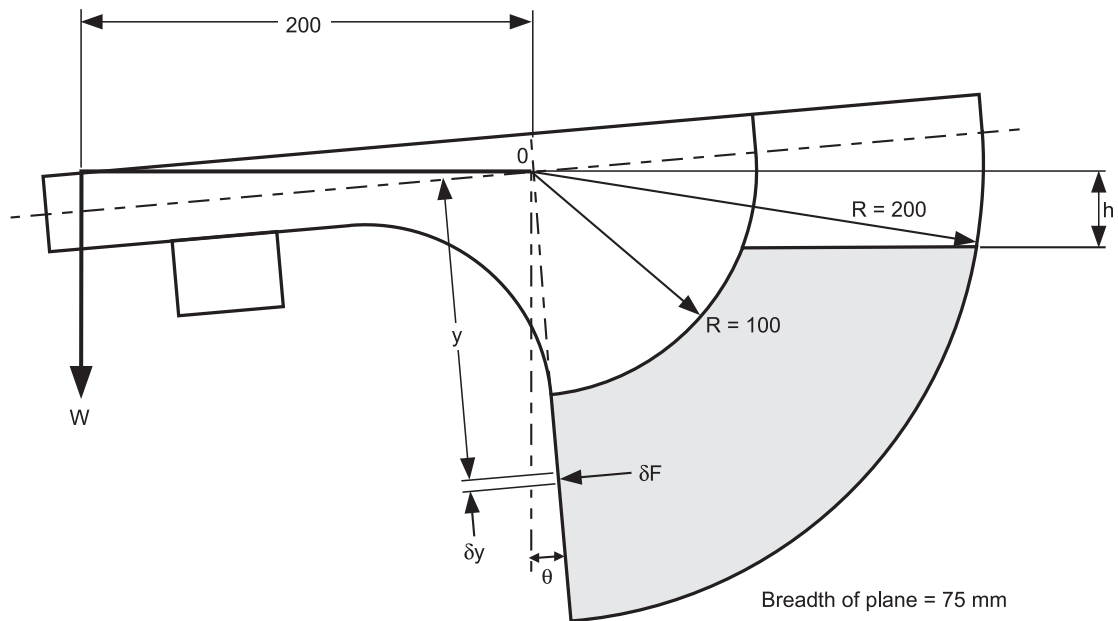


Figure 15 Pressure Forces on a Plane Surface

Referring to Figure 15, consider an element at start depth y , width δy .

Therefore, force on element $\delta F = \omega(y \cos \theta - h)B \delta y$ and moment of force on element about $O = \omega B(y \cos \theta - h)y \delta y$.
Therefore:

$$\text{Total moment about } O = M = \omega B \int (\cos \theta y^2 - hy) dy$$

Case 1: Plane Fully Submerged

Limits R_1 and R_2

$$M = \omega B \int_{R_1}^{R_2} (\cos \theta y^2 - hy) dy$$

$$M = \omega B \left[\frac{\cos \theta y^3}{3} - \frac{hy^2}{2} + c \right]_{R_1}^{R_2}$$

$$M = \frac{\omega B \cos \theta}{3} (R_2^3 - R_1^3) - \frac{\omega B}{2} (R_2^2 - R_1^2) h \quad (21)$$

This equation is of the form of $y = mx + c$

A plot of M against h will yield a straight line graph of gradient $-\frac{\omega B}{2} (R_2^2 - R_1^2)$. The value of ω can now be calculated.

Case 2: Plane Partially Submerged

Limits R_2 and $h \sec \theta$

Hence:

$$M = \omega B \int_{h \sec \theta}^{R_2} (\cos \theta y^2 - hy) dy$$

$$M = \left[\frac{\omega B \cos \theta y^3}{3} - \frac{hy^2}{2} + c \right]_{h \sec \theta}^{R_2}$$

$$M = \frac{\omega B \cos \theta}{3} (R_2^3 - h^3 \sec^3 \theta) - \frac{\omega B h}{2} (R_2^2 - h^2 \sec^2 \theta)$$

$$M = \frac{\omega B \cos \theta R_2^3}{3} - \frac{\omega B R_2^2 h}{2} + \frac{\omega B \sec^2 \theta h^3}{6}$$

Rearranging:

$$M + \frac{\omega B R_2^2 h}{2} = \frac{\omega B \sec^2 \theta h^3}{6} + \frac{\omega B \cos \theta R_2^3}{3}$$

Obtain ω from Case 1 and plot h^3 against $M + \frac{\omega B R_2^2 h}{2}$

Figure 16 and Figure 17 show the general form of the graphs expected from this experiment.

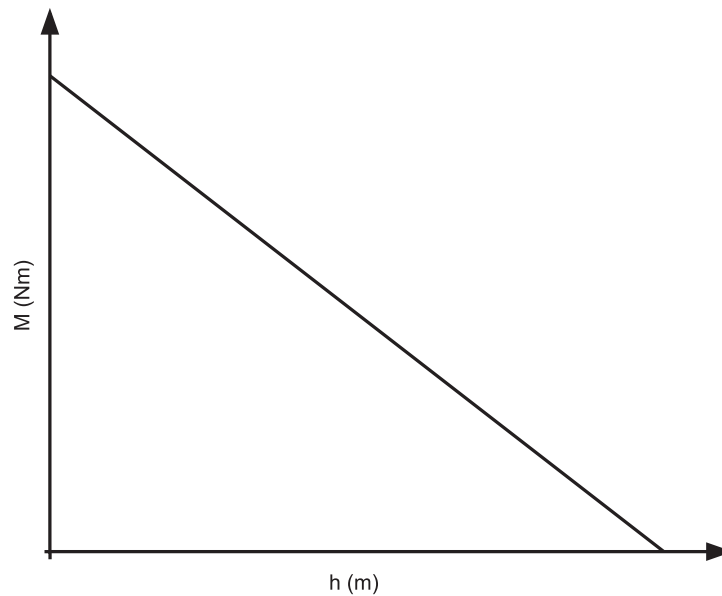


Figure 16 Centre of Pressure Graph: Plane Fully Submerged

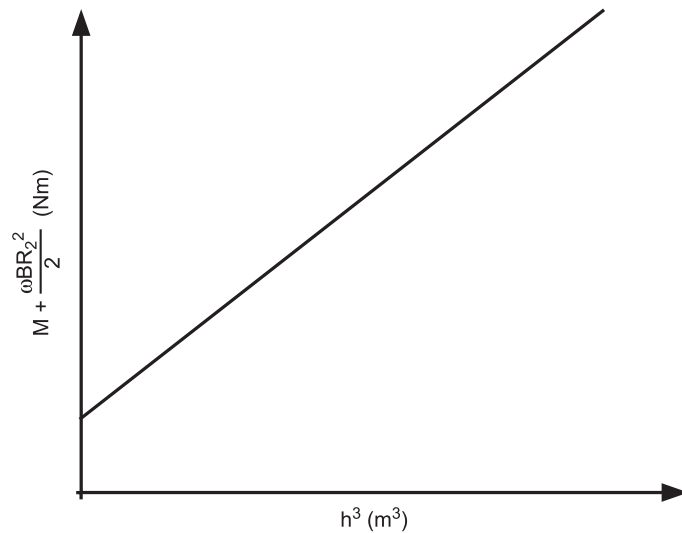


Figure 17 Centre of Pressure Graph: Plane Partially Submerged

4.9 Pressure Measurement

Pressure measurement is important, not only in fluid mechanics but in virtually every branch of engineering. There is a wide range of methods for measuring pressure and many of these employ hydrostatic principles. The following experiments illustrate some of the methods and, in particular, how columns of fluid can be used in various forms to measure pressure.

Bourdon Pressure Gauge

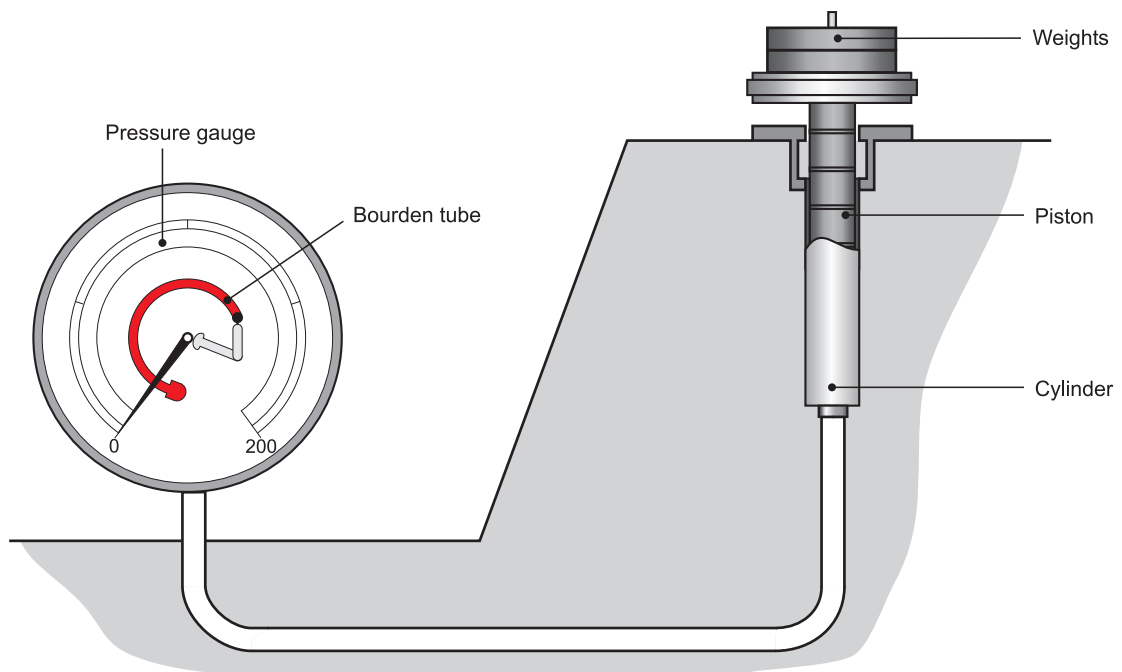


Figure 18 Principle of the Bourdon Pressure Gauge Calibrator

The pressure gauge apparatus has a large Bourdon pressure gauge and facilities for calibrating the gauge using dead weights.

The mechanism of the gauge may be seen through the transparent dial of the instrument, as illustrated in Figure 18. A tube, having a thin wall of oval cross-section, is bent to a circular arc encompassing about 270 degrees. It is rigidly held at one end, where the pressure is admitted to the tube, and is free to move at the other end, which is sealed. When pressure is admitted, the tube tends to straighten, and the movement at the free end operates a mechanical system which moves a pointer round the graduated scale – the movement of the pointer being proportional to the pressure applied. The sensitivity of the gauge depends on the material and dimensions of the Bourdon tube; gauges with a very wide selection of pressure ranges are commercially available.

To check the accuracy of a Bourdon gauge, the procedure is to load it with known pressures by a dead weight tester using oil to transmit the pressure. This experiment, however, works satisfactorily with water instead of oil.

- a) Remove the piston from the unit. The piston is delivered lightly oiled and should be wiped only when the unit is to be used.
- b) Fill the cylinder with water, and remove air trapped in the transparent tube by gently tapping the unit. A small amount of air left in the system will not affect the experiment.
- c) Top up with water and insert the piston into the cylinder, allowing air and excess water to escape through the top hole in the side of the cylinder. Allow the piston to settle.

**NOTE**

Always smear the whole piston surface lightly with oil (supplied) after use.

Do not attempt to polish the piston rod with emery cloth, or any harsh abrasive. Use only a mixture of powdered chalk and oil to remove discoloration.

Experimental Procedure

Ensure the cylinder is vertical. Masses are added in approximately eight increments up to a maximum of 5.2 kg. **On no account should more than the supplied masses be loaded.** Always load the masses gradually. Do not drop them onto the platform. The pressure gauge reading should be recorded at each increment of loading. To prevent the piston sticking, rotate the piston gently as each mass is added. Reverse the procedure, taking readings as the masses are removed. **The cross-sectional area and the mass of the piston should be recorded.**

Results and Calculations

The actual hydrostatic pressure (p) in the system due to a mass of M kg (including the piston mass) applied to the piston is given by:

$$p = \frac{M \times 9.81}{A} \times 10^{-3} \text{ kN/m}^2$$

where A is the piston area in m^2 .

The observed and derived results are tabulated in the form of Table 6.

Table 6 Results for Increasing and Decreasing Pressure

Two different kinds of error may normally be expected in a gauge of this type. Firstly, there is the possibility of

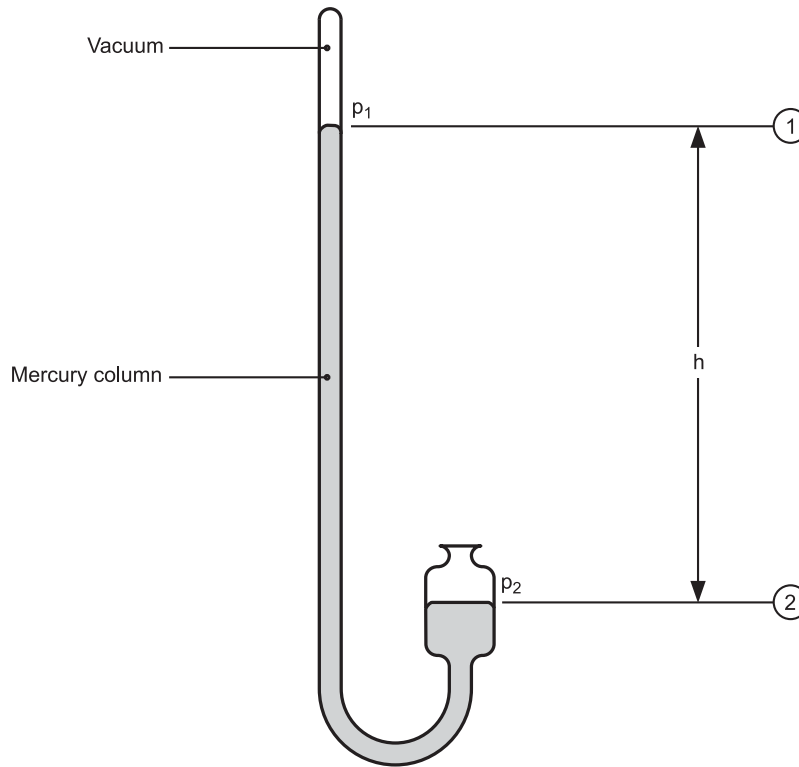


Figure 19 Simple Mercury Barometer

If the top of the column is distance h above the free surface at plane 2, the pressure at that plane is given by:

$$p_2 = p_1 + \rho gh \quad (22)$$

where p_2 is an absolute pressure.

In our case, $p_1 = 0$, so the height h is a measure of the absolute pressure at plane 2. The only pressure acting on the free surface is atmospheric pressure, so the height h is an absolute measure of atmospheric pressure. Atmospheric pressure varies from day to day, but a typical value gives a mercury column height of 760 mm.

Liquid Column Manometers

Columns of liquid can be used in a wide range of configurations for measuring pressures in both static and moving fluids. The barometer represents a special case in which absolute pressure is measured but, in general, liquid columns are used to measure differential pressures; that is to say the difference in pressure between two points in a fluid system. Strictly speaking, the term manometer relates to all methods of measuring pressure but, in normal usage, it is taken to refer to liquid columns and particularly those in the form of U-tubes.

Figure 20 shows the general case of a U-tube manometer measuring the differential pressure between two points in a system containing fluid (liquid or gas) of density ρ_1 . The U-tube is filled with a heavier fluid (liquid) of density ρ_2 and the differential pressure is measured in terms of the difference in height $\Delta h = (h_4 - h_3)$ of the two columns.

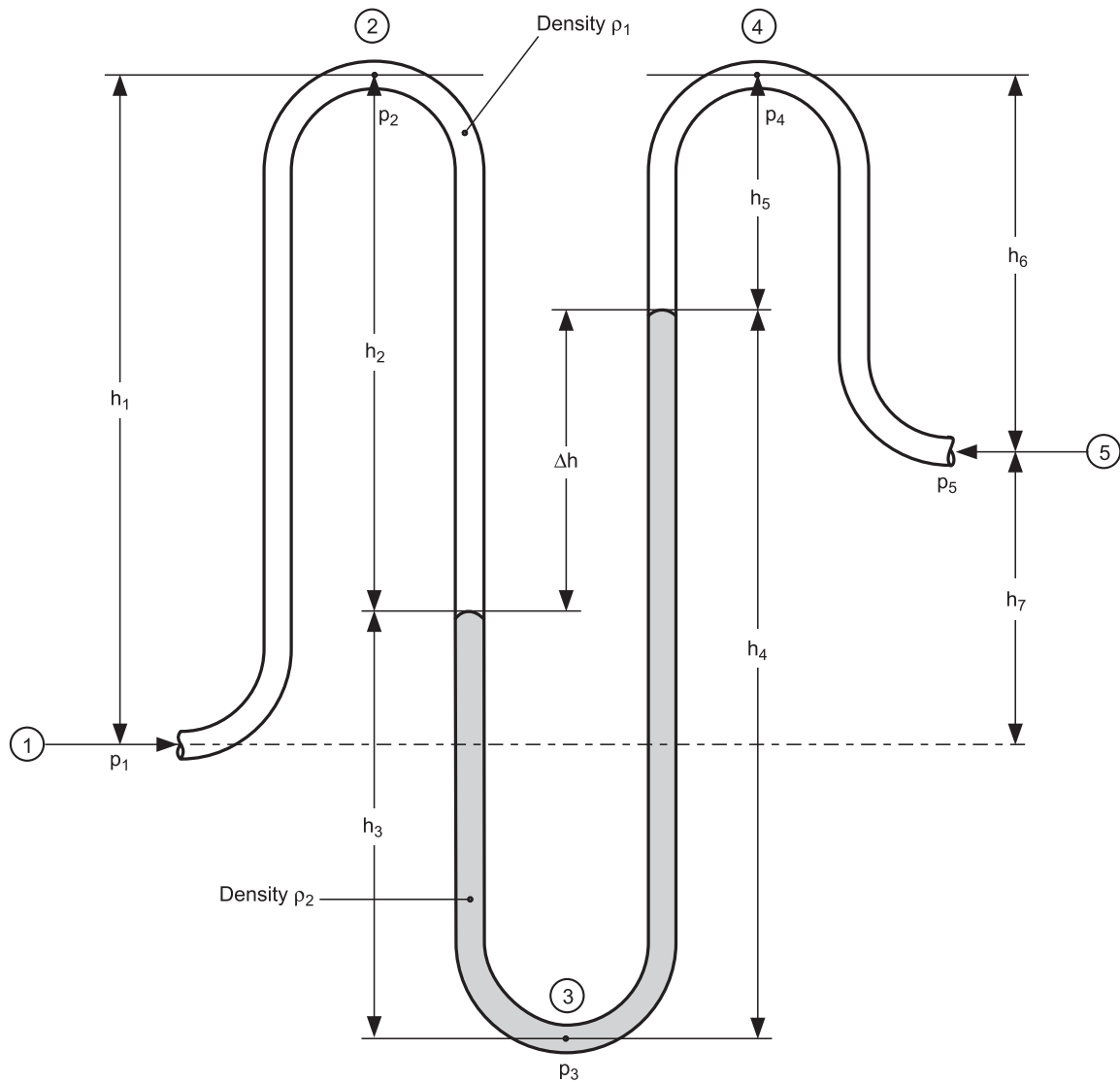


Figure 20 Pressure in a U-tube Manometer

The remainder of the U-tube and the connecting tubes are filled with the working fluid of density ρ_1 . First consider the pressure at point 1 due to the column h of working fluid:

$$p_1 = p_2 + \rho_1 g h_1$$

or

$$p_2 = p_1 - \rho_1 g h_1$$

Now consider the pressure at the bottom of the U-tube due to fluid in the left-hand column:

$$p_3 = p_2 + \rho_1 g h_2 + \rho_2 g h_3 \quad (23)$$

and substituting for p_2 we obtain:

$$p_3 = p_1 - \rho_1 g (h_1 - h_2) + \rho_2 g h_3$$

Similarly, for the right-hand column and connecting tube:

$$p_3 = p_5 - \rho_1 g (h_6 - h_5) + \rho_2 g h_4 \quad (24)$$

Then, on equating the right-hand sides of Equations (23) and (24) and rearranging we obtain:

$$(p_1 - p_5) = \rho_2 g(h_4 - h_3) + \rho_1 g(h_1 - h_6) - \rho_1 g(h_2 - h_5) \quad (25)$$

Finally, substituting $h_4 - h_3 = h_2 - h_5 = \Delta h$ and $h_1 - h_6 = h_7$ we obtain:

$$(p_1 - p_5) = (\rho_2 - \rho_1)g\Delta h + \rho_1 gh_7 \quad (26)$$

This represents the general case where the pressure tapings are at different heights and the density of the working fluid is significant compared to that of the manometer fluid. It can be seen that the difference in height Δh of the manometer columns gives a measure of $(p_1 - p_5)$, but a correction has to be made for the different heights of the pressure tapings. This is most important and must be remembered whenever a manometer is used with liquid rather than gas as the working fluid in the connecting tubes.

Notice also that the apparent density of the manometer fluid is reduced by ρ_1 , the density of the working fluid. A common case is the mercury under water manometer where the apparent density is $13600 - 1000 = 12600 \text{ kg/m}^3$ instead of 13600 kg/m^3 for a mercury/air manometer. There are two important cases which lead to a simplification of Equation (26).

Tappings at the same height

If the tapings are at the same height, the last term in Equation (26) becomes zero giving:

$$(p_1 - p_5) = (\rho_2 - \rho_1)g\Delta h \quad (27)$$

Thus, for this case, the pressure difference is proportional to Δh multiplied by the apparent density $(\rho_2 - \rho_1)$.

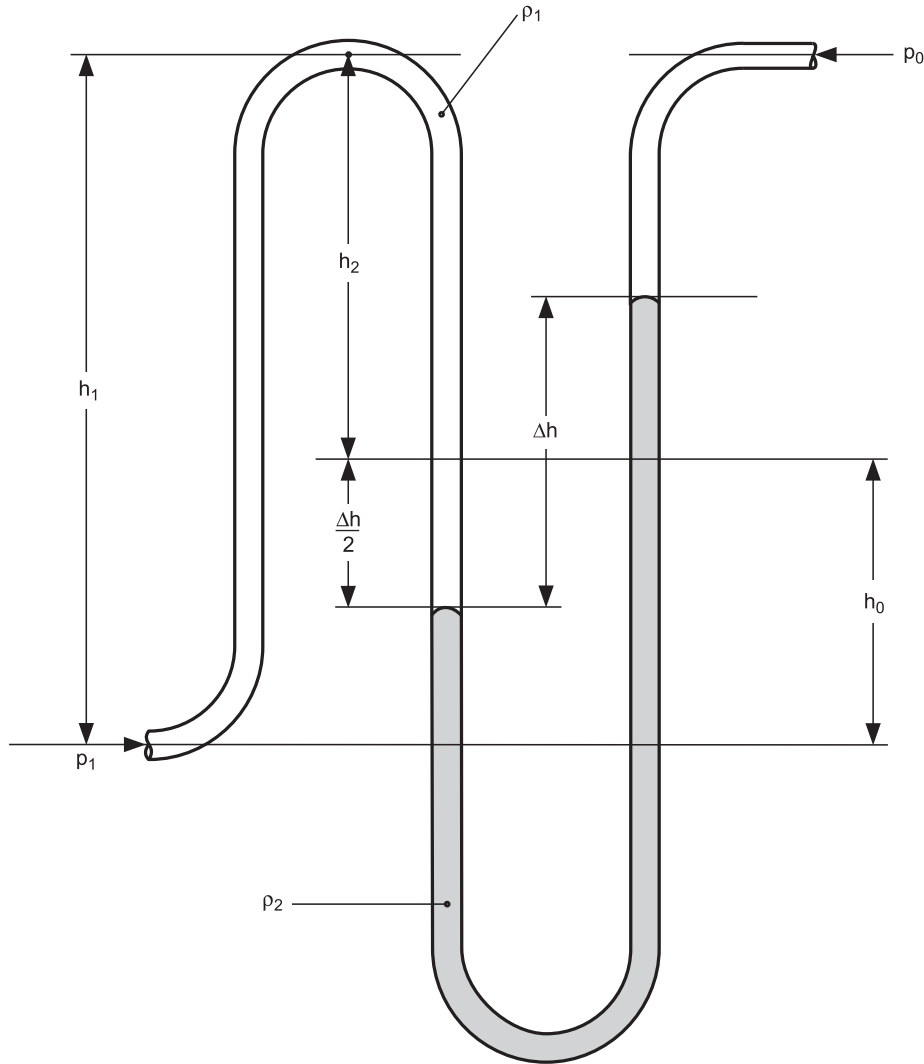
Gas as the working fluid

If a gas is the working fluid, its density can usually be taken as negligible compared to that of the manometer fluid and Equations (26) and (27) reduce to:

$$(p_1 - p_5) = \rho_2 gh \Delta h \quad (28)$$

In this case, the pressure difference depends directly on Δh and the actual density of the manometer fluid.

Equations (27) and (28) are quite general. The following two cases are worthy of further discussion.

Manometers open to atmospheric pressure

Figure 21 Manometer Open to Atmospheric Pressure

The manometers fitted to the H314 apparatus all have one column which is open to atmospheric pressure and this provides a common datum pressure. In the case of a liquid/air manometer, Equation (28) applies except that the pressure difference is measured relative to a fixed datum of atmospheric pressure p_0 , rather than a varying pressure p_5 . In the case of the mercury/water manometer, only one column has water above the mercury and Equation (25) applies in a simplified form. If the right-hand column is open to atmospheric pressure p_0 and there is no water in the connecting pipe as shown in Figure 21, Equation (25) becomes:

$$(p_1 - p_0) = \rho_2 g \Delta h + \rho_1 g (h_1 - h_2) \quad (29)$$

It may be noted that the second term on the right-hand side is a correction for the varying column of water in the connecting tube. This variable correction can be eliminated by substituting $h_1 - h_2 = h_0 - \frac{\Delta h}{2}$, where h_0 is the height of the columns above the pressure tapping when the pressure difference is zero (see Figure 21). Equation (29) can then be written:

$$(p_1 - p_0) = \left[\rho_2 - \frac{\rho_1}{2} \right] g \Delta h + \rho_1 g h_0 \quad (30)$$

where the second term on the right-hand side is now a constant.

It should be noted that this is based on the assumption that each column moves the same distance from level h_0 when displaced and this means that Equation (30) only applies if the tubes are of the same cross-sectional area. The significance of this is explained in the next case.

Manometer with limbs of different cross-sectional areas

In the preceding cases, the pressure difference is measured in terms of the height Δh , which is usually obtained as the difference between two measured heights. This is often laborious and it would be easier if the pressure difference could be obtained in terms of a single direct reading. One possible solution is to take the initial level of the columns at zero pressure difference as a datum and then measure the deflection of one column relative to this. If the limbs were of equal cross-sectional area, each column would move by $\Delta h/2$ and the reading could be doubled to obtain Δh . This is not often done in practice due to the loss of accuracy which results, but this can be overcome by using limbs of different cross-sectional areas.

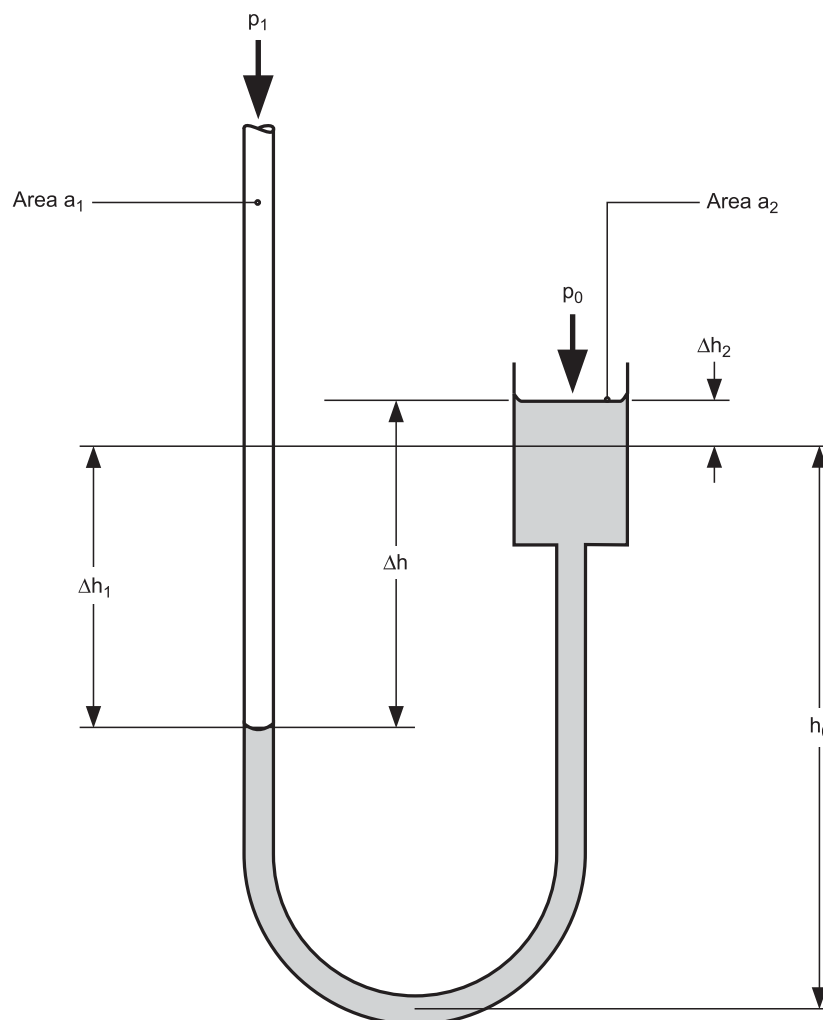


Figure 22 Manometer with Unequal Limbs

Consider the manometer shown in Figure 22 which has limbs of different cross-sectional areas. From the results of previous experiments (see page 11, Measuring Beaker) it will be appreciated that the differing areas do not affect the pressures in the columns, so for the case shown (a liquid/air manometer), Equation (28) still applies. The significant point is that the columns move different distances from the datum level h_0 . The volume of liquid displaced is the same for each column so we can write:

$$\Delta h_1 a_1 = \Delta h_2 a_2$$

or alternatively:

$$\Delta h_2 = \Delta h_1 \frac{a_1}{a_2} \quad (31)$$

If we make a_2 large compared to a_1 then nearly all of the change in height will occur in column 1 and the deflection can be measured to a greater accuracy. Since $\Delta h = \Delta h_1 + \Delta h_2$ we can obtain h by substituting using Equation (31):

$$\Delta h = \Delta h_1 \left(1 + \frac{a_1}{a_2} \right) \quad (32)$$

This arrangement is the basis of a common form of direct-reading, 'single-limb' manometer. The second limb is in the form of a reservoir tank of large cross-sectional area, such that the change in level in it is small compared to that in the narrow limb. Equation (32) can be used to define a scale for the narrow limb which is calibrated to read Δh directly from the level in that limb. Alternatively, Δh_1 can be measured and a small correction applied using Equation (32) to obtain Δh , or if a_2 is made sufficiently large, the measurement could be taken as a direct reading of h without introducing significant errors.

Manometer Experiments

The equations and underlying principles described previously can be demonstrated using the manometers fitted to the rear panel of the H314 apparatus. The manometers should be filled and all air bubbles removed from the water lines.

Comparison of different fluids

- With the isolating valve (V3) to the air/water manometer open, attach the air pump to the Schrader valve and gently pump until a pressure is registered. This will be shown on both the mercury and water manometers.
- Compare the readings of each manometer until the water manometer approaches the limit of its range.
- Isolate the manometer and increase the pressure until the mercury manometer is at full capacity.
- To vent both manometers, insert the top of the Schrader valve cap into the centre of the valve and vent the mercury manometer. Open the isolating valve for the water manometer and again vent the air via the Schrader valve.

4.10 Static Head due to Friction

The tube at the base of the Pascal's tube is of small bore (3 mm diameter) and is used to demonstrate the static head due to friction as water flows through it.

- Fill the header tank to a mark and allow the Pascal's tube to fill so that all show the same level.
- Open valve (V1) a small amount and create a flow through the small bore pipe. The variation of static head along the pipe can be demonstrated and seen to vary as the flow is increased.

4.11 Hare's Tube Apparatus

This apparatus enables the specific gravity of a liquid to be determined when compared with water. This is done as follows:

- Fill one 100 ml beaker with water and another 100 ml beaker with the other liquid. Ensure the two liquids are at the same level.
- Place the filled beakers under the vertical glass tubes, ensuring the tubes are at the same height, so the tube goes into the liquid.
- With the hand pump hose clip tight, compress the pump and release so the liquid is drawn up the glass tubes. Repeat if required.

d) Record the two liquid levels, and calculate the unknown liquid's specific gravity as shown:

$$\text{Specific gravity (relative density)} = \frac{\text{Height of water}}{\text{Height of other liquid}}$$

To release the vacuum, undo the hand pump hose clip.

4.12 Depth Gauge (also known as a 'Hook' Gauge)

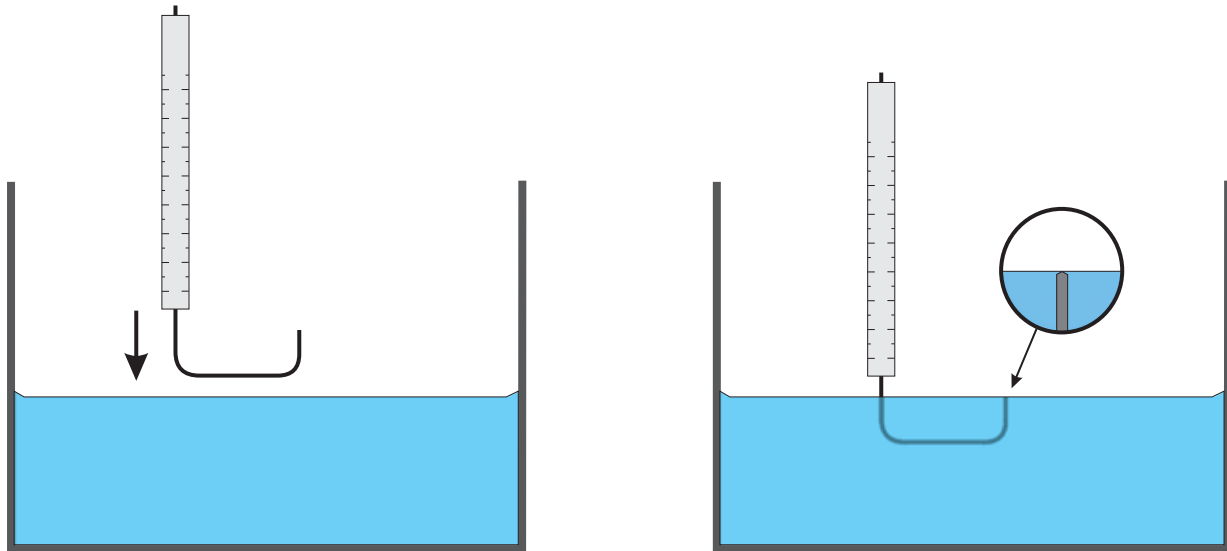


Figure 23 Using the Depth Gauge

The Depth Gauge in the header tank accurately measures changes in liquid level. It uses a pointed hook to 'touch' the surface of the liquid and a scale to help you measure differences in liquid height. To use it you must choose one of two methods and be consistent. If you are not consistent, the small meniscus of liquid that forms around the tip will affect your results.

Method 1: Gently lower the hook into the liquid and note when the tip just passes under the surface of the liquid. Note the reading on the scale. Repeat for different liquid levels and subtract the differences in scale readings to find the change in liquid level.

Method 2: Gently lower the hook into the liquid until it is completely submerged, then slowly raise it until the tip just breaks the liquid surface from underneath. Repeat for different liquid levels and subtract the differences in scale readings to find the change in liquid level.

Air Valves

TecQuipment's

Fluid Mechanics Products

Instruction Sheets

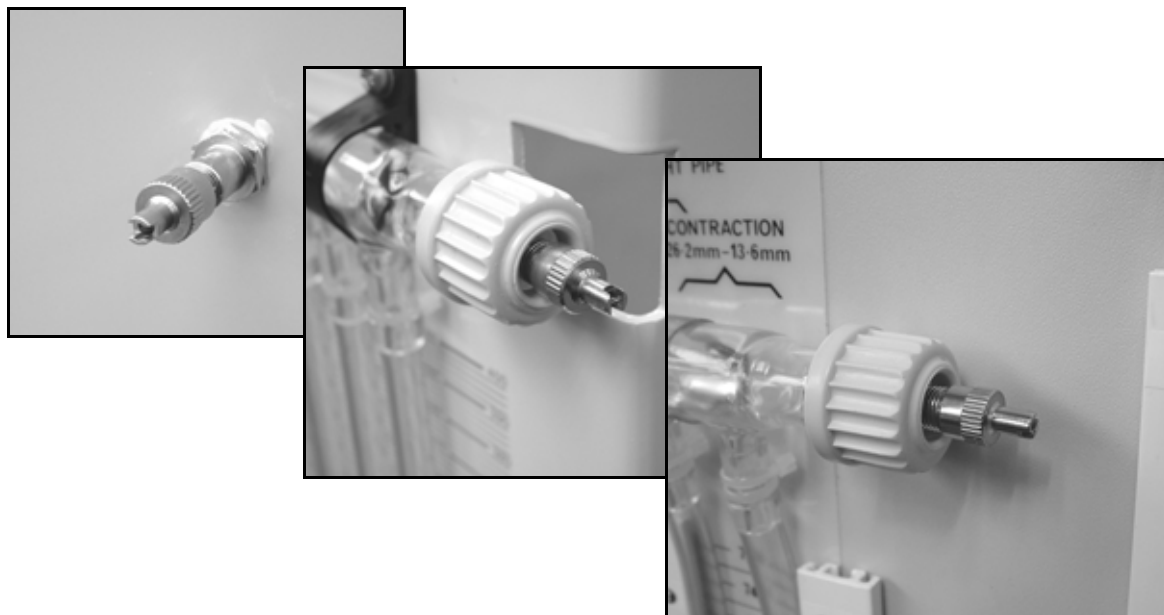


Figure 24 Typical Air Valves on Some of TecQuipment's Products

Many of the products in TecQuipment's Fluid Mechanics range use air valves at the tops of manometers or piezometers. The valves keep the air in the manometer tubes to allow you to offset the pressure range of the manometer or piezometer.

The valves are similar to valves used in vehicle tyres and include a special cap. The hand pump supplied with the equipment is similar to those used for bicycle tyres, except that TecQuipment remove the cross-shape part of the flexible pipe.

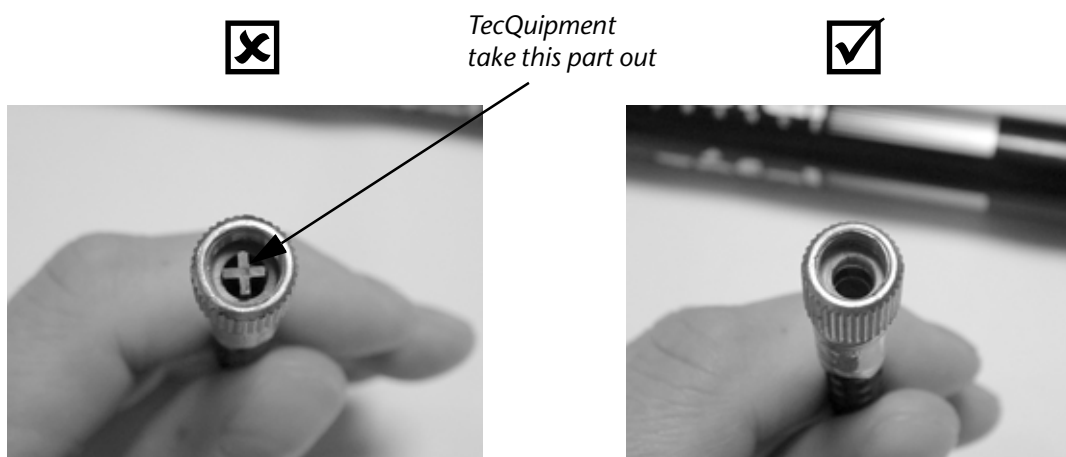


Figure 25 TecQuipment Remove the Cross-shape Part of the Flexible Pipe

Normally, when you connect the flexible pipe to an air valve, the cross-shape piece in the flexible pipe pushes open the valve as you pump air with the hand pump. With TecQuipment fluid mechanics products, this could allow water back out through the valve. For this reason TecQuipment remove the cross-shape piece. Without the cross-shape piece, only pressurised air can go through the valve in one direction, and no water can come back out.



Figure 26 The Hand Pump and Flexible Pipe

When you first use the hand pump with the air valve, you may find it hard to push air through the valve. This is because the valve is new and you do not have the cross-shape piece to help push it open. The valve will open more easily after you have pumped air through it a few times.

You may need some practice to use the air valve. To do it correctly:

1. Unscrew the cap from the valve.

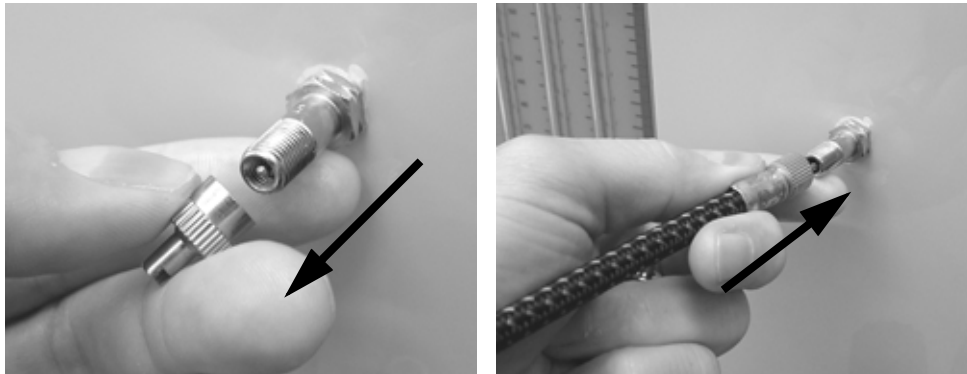


Figure 27 Unscrew the Cap and Fit the Pipe

2. Connect the flexible pipe to the valve.
3. Connect the hand pump to the flexible pipe.
4. Using complete strokes, **slowly and firmly** pump the hand pump to force air into the manometer or piezometer.
5. Unscrew the hand pump and flexible pipe and refit the valve cover.
6. To let air back out through the air valve, use the end of the special cap to press on the inner part of the valve (see Figure 28).

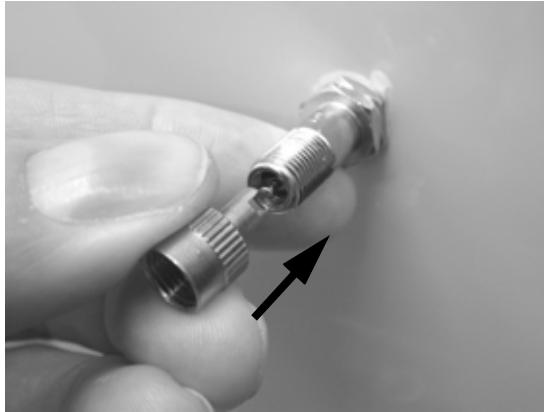


Figure 28 To Let Air Out - Use the End of the Special Cap to Press the Inner Part of the Valve



Take care when you let air back out from the air valve. Water may come out!

Clean up any water spills immediately.

If using the hand pump is too difficult, the valve may be stuck. If you need to check the valve is working, use the special cap to unscrew the valve, then gently press the end of the valve. It should move easily and return back to its original position (see Figure 29).

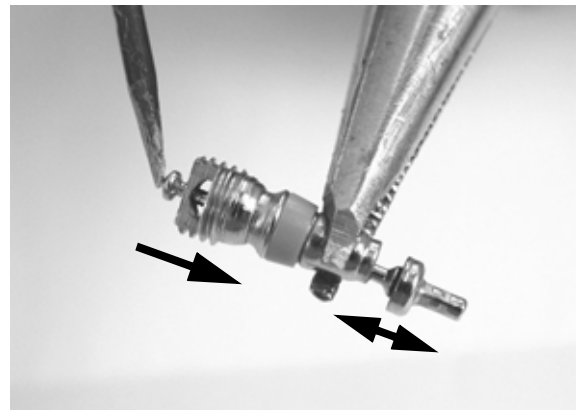
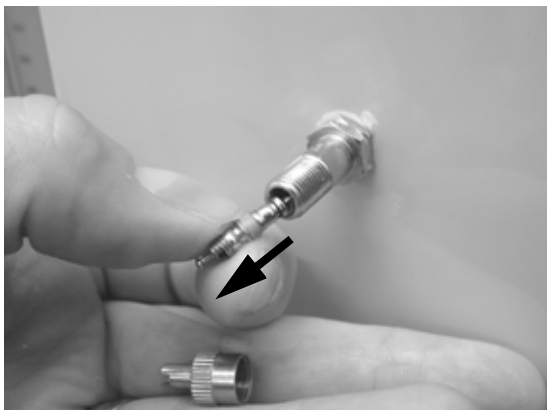


Figure 29 Unscrew the Valve and Check it

If the valve does not move easily, then contact TecQuipment Customer Services for help.

Telephone: +44 115 9722611

Fax: +44 115 973 1520

Email: customer.care@tecquipment.com

TecQuipment 0809 DB

