# H1d/a and H1d/b
# Set of Weirs for H1d



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# SECTION 1.0 INTRODUCTION

In hydraulic engineering, weirs often regulate flow in rivers and other open channels. When made to correct mathematical dimensions, you can calculate discharge volume over the weir from the upstream water level. Figure 1 shows a rectangular notch weir and a V-shaped notch weir, in vertical plates. These notches usually have sharp edges so the water leaves the plate cleanly as it passes through the notch.

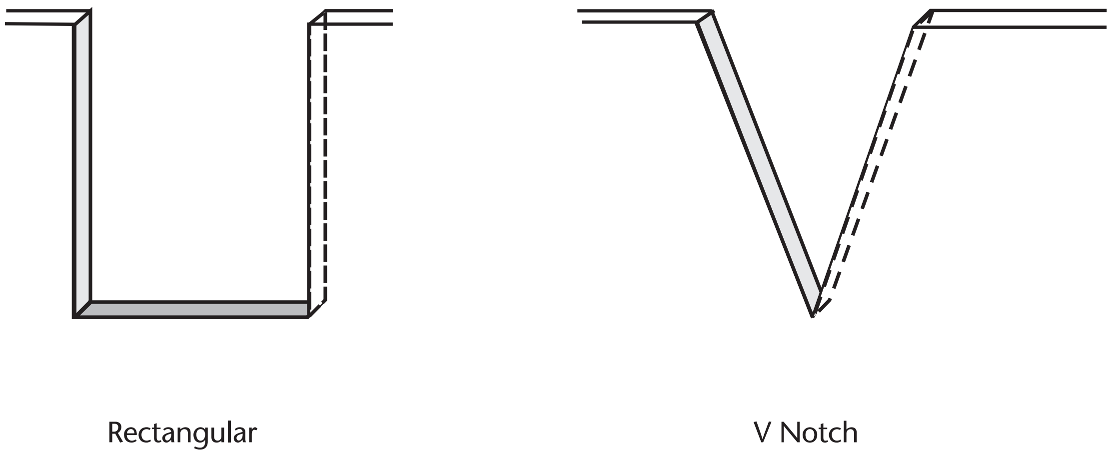



*Figure 1 Rectangular and V-Notches*

This equipment allows students to find the relationships between head and discharge for different weirs.

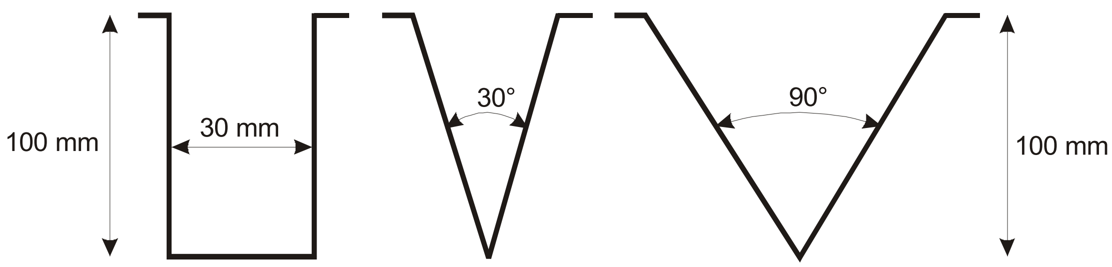
## 1.1 Description of Apparatus

The TQ Set of weirs (H1d/a) and the TQ Advanced set of weirs (H1d/b) apparatus works with the TQ H1d Volumetric Hydraulic Bench. The H1d/a has two 'V-notch' and one rectangular weir. The H1d/b has a Cipoletti weir, a broad crested weir and a linear head/flow weir. The H1d/a set of weirs pack includes a precision height gauge to measure upstream water levels.

## 1.2 Routine Care and Maintenance

After use remove the weirs and height gauge from the H1d tank and dry with a lint-free cloth.
For care and maintenance of the H1d tank, consult the H1d User Guides.

## 1.3 Notch Details



# SECTION 2.0 THEORY OF FLOW OVER SHARP-EDGED WEIRS

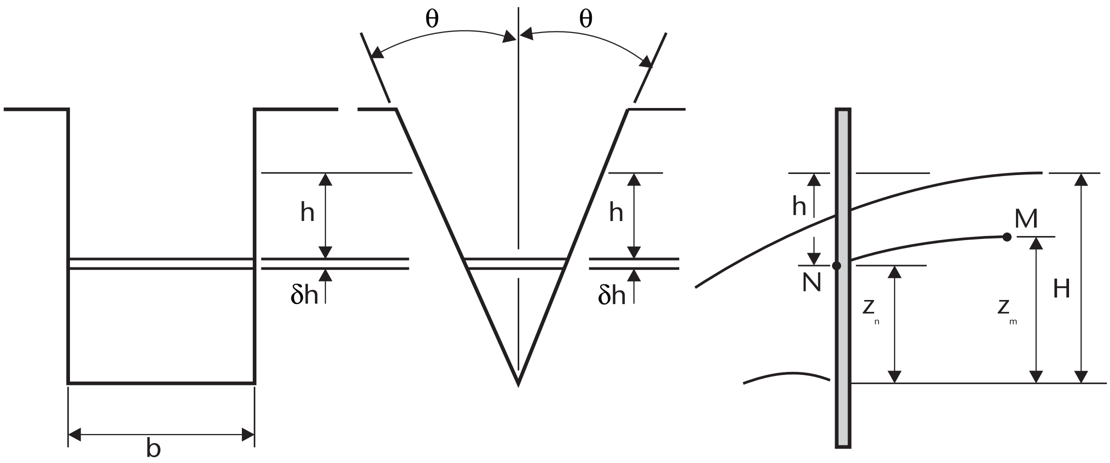



*Figure 2 Flow over Rectangular and V-Notches*

Figure 2 shows the important dimensions you need to find flow over rectangular and V- notches. Consider the motion of a particle of fluid from a position M some distance upstream of the weir to its subsequent position N in the plane of the vertical weir plate. If there is no energy loss, according to Bernoulli's equation:

$$\frac{u_M^{\,2}}{2g}+\frac{P_M}{w}+z_M=\frac{u_N^{\,2}}{2g}+\frac{P_N}{w}+z_N$$

**(2-1)**

Where:
u = velocity
P = pressure
w = specific weight
z = height
g = acceleration due to gravity

Provided that the approach channel has a much larger cross-sectional area than the notch, the fluid in the vertical plane containing M will be comparatively at rest. It will then be in an almost hydrostatic condition, for which the total head of all points has the same value H (relative to the datum shown). Making the further (and less justifiable) assumption that $P_N = 0$, i.e. the static pressure is atmospheric at N, Equation (2-1) simplifies to:

$$\frac{u_N^{\;2}}{2g} + z_N = H$$

**(2-2)**

Now $$H - z_N = h$$

**(2-3)**

as may be seen from the figure, so

$$\frac{u_N^{\;2}}{2g} = h$$

**(2-4)**

This velocity is the same as that which would be attained by a particle falling freely from the level of the upstream surface to the position of $N$.

The discharge over each weir may now be found by integration. For the rectangular weir of width $b$, the area of an element having height $\delta h$ is $b\delta h$, giving the flow rate $\delta Q$ through it as

$$\delta Q = u_N b\delta h = \sqrt{2gh}\; b\delta h$$

**(2-5)**

The total flow rate, $Q$, obtained by integrating between zero and $H$ gives a result which neglects the lowering of the surface level in the plane of the weir, and is:

$$Q = \int_0^H \sqrt{2gh}\; b\delta h$$

or $$Q = \frac{2}{3}\sqrt{2g}\; b\, H^{3/2}$$

**(2-6)**

for the rectangular weir

For the V-notch of angle $2\theta$, the width of an element is $2(H - h)\tan\theta$, so that the area of the element having height $\delta h$ is $2(H - h)\tan\theta\,\delta h$. The flow rate through it is:

$$\delta\theta = u_N\, 2(H - h)\tan\theta\delta h = \sqrt{2gh}\; 2(H - h)\tan\theta\delta h$$

**(2-7)**

so that, integrating as above,

$$Q = \int_0^H \sqrt{2gh}\; 2(H - h)\tan\theta\delta h$$

or

$$Q = \frac{8}{15}\sqrt{2g}\,\tan\theta H^{5/2}$$

**(2-8)**

for the V-notch.

There is, in fact, a notable contraction of the stream as it passes through the notch. This is observed in the vertical plane, where the upper surface slopes downwards over the notch and the lower surface springs from the crest of the notch in an upward direction. Contraction is also noted in the horizontal plane, where the water leaves the edges of the weir in a curve, which reduces the width of the stream. This contraction is similar to that produced at a sharp-edged orifice and has the same effect of reducing the discharge. It is therefore usual to rewrite the equations in the form:

$$Q = C_d\,\frac{2}{3}\sqrt{2g}\,b\,H^{3/2}$$

**(2-9)**

for the rectangular notch, and

$$Q = C_d\,\frac{8}{15}\sqrt{2g}\,\tan\theta H^{5/2}$$

**(2-10)**

for the V notch, in which $C_d$ is a coefficient of discharge of the notch which is not necessarily independent of $H$ and may be determined by experiment.

A convenient way of finding $C_d$ and the exponent of $H$ in either of these expressions is as follows. Either Equation (2-9) or (2-10) may be written in the form:

$$Q = kH^n$$

**(2-11)**

or $$\log Q = \log k + n \log H$$

**(2-12)**

If experimental results are plotted on a graph having an X-axis of log $H$ and Y-axis of log $Q$, then they will lie on a straight line. The slope n will intercept at log k on the axis of log $Q$ (See Figure 3). This is conditional on $k$ and $n$ being constant over the range of the results.

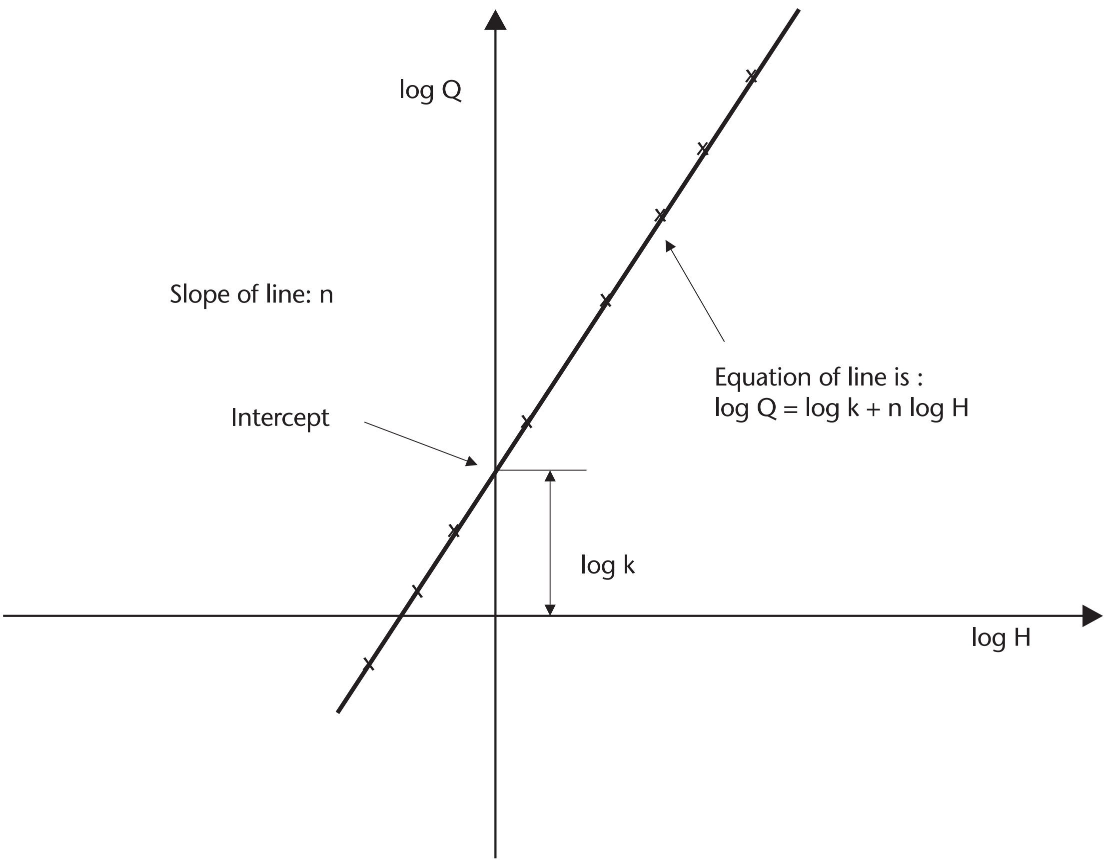



*Figure 3 Finding the Coefficient of discharge for Rectangular and V-Notches.*

# SECTION 3.0 CIPOLETTI AND PROPORTIONAL NOTCHES

## 3.1 The Cipoletti (Trapezoidal) Notch

The most familiar kinds of sharp-edged notched weirs used for flow measurement are rectangular and V-shaped. However, notches of other shapes have been suggested, and among these is one developed by Cipoletti, as shown in Figure 4. This slight variation of a rectangular profile is intended to produce a constant value of discharge coefficient over a wide range of heads.

Experiments on rectangular notches show that the value of the discharge coefficient falls slightly with increasing head, and thus with increasing value of ratio H/b. By using sloping sides, the mean width of the notch increases with head, providing compensation for the reduction in discharge coefficient.

To check the validity of this concept, the Cipoletti notch must be tested in exactly the same manner as the rectangular one, and the two head-discharge characteristics compared on a logarithmic basis.

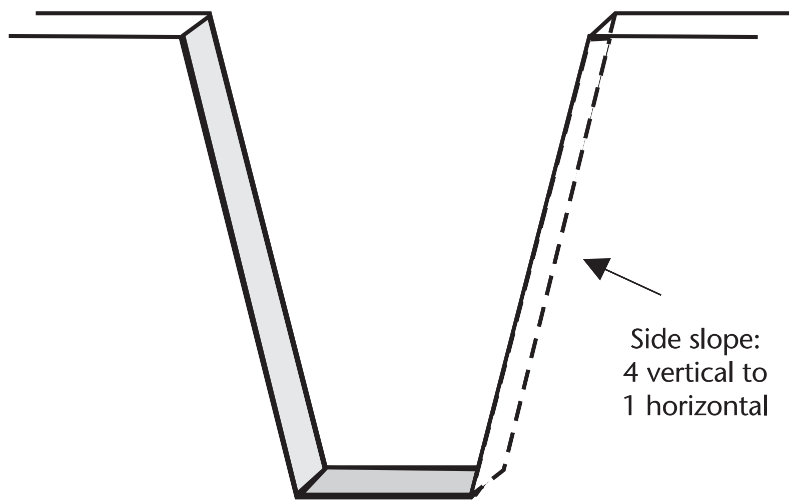



*Figure 4 Profile of the Cipoletti Notch*

## 3.2 The Linear Head/Flow (Proportional) Weir

The linear head/flow (proportional) weir (may be referred to as a Sutro Weir) is an ingenious design for producing a flow rate $Q$ which is linearly proportional to Head $H$. Therefore, in the expression $Q = KH^n$ the value of $n$ is to be $n$ = 1. Now we know already that for a notch of constant width (a rectangular notch), $n$ = 1.5 nearly, and for a V notch, where the width increases with head, $n$ = 2.5 nearly. It would therefore be expected that to obtain a result of $n$ = 1, the notch width would need to decrease in some way with increasing head. This indeed proves to be the case. The required shape is sketched in Figure 5 Notch profile.

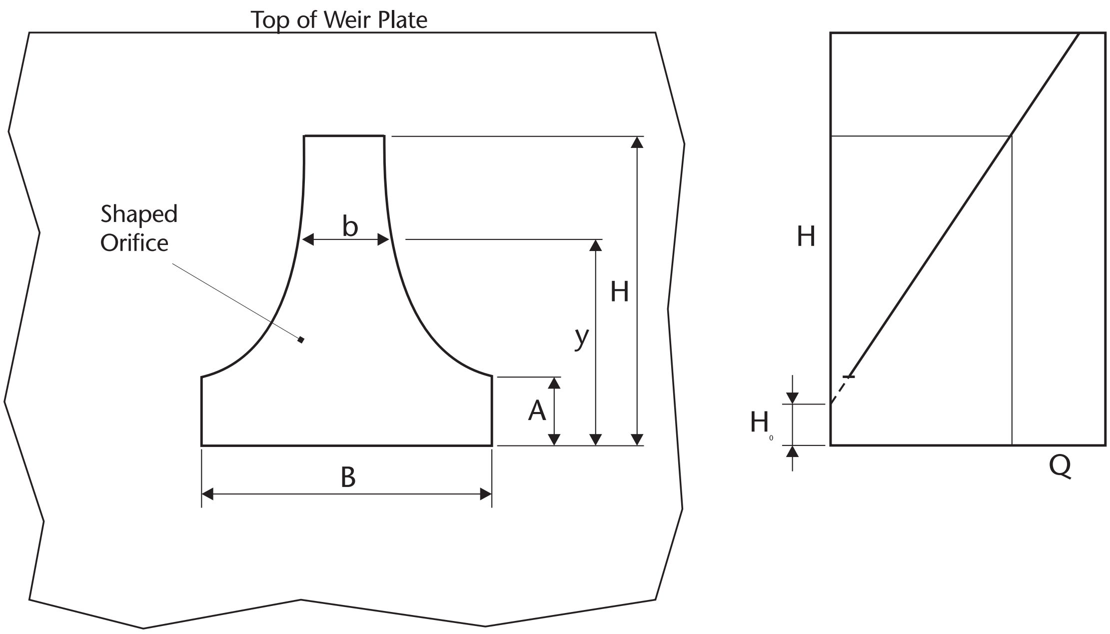



*Figure 5 The Proportional or Linear Head/Flow notch*

**Discharge Rate:**

When the flow over the crest is small, the notch behaves as if it were a simple rectangular one.

For higher flow rates:

$$Q = K(H - H_0)$$

**(3-1)**

in which the theoretical value of $H_0$ is given by:

$$H_0 = H/3 \tag{3-2}$$

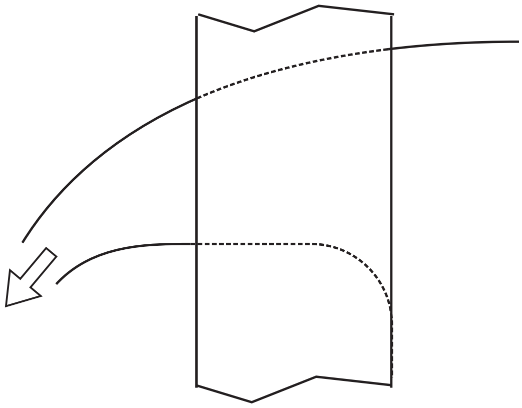
and K is the constant defined as:

$$K = C_d B\sqrt{2gA} \tag{3-3}$$

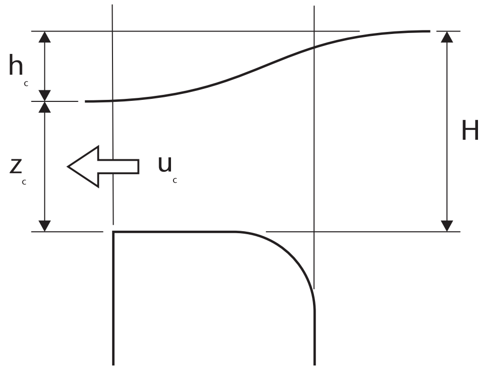
$C_d$ is a discharge coefficient to be determined by experiment. A value of $H_0$ may also be determined experimentally, and checked against the theoretical value given by Equation (3-2).

The head-flow relationship is shown in Figure 5, where Head $H$ is plotted vertically and flow rate $Q$ horizontally. Experimental values shown in this way should lie on a straight line. From this line, values of $H_0$ and $C_d$ can be obtained, using Equations (3-1), (3-2) and (3-3).

## SECTION 4.0 THEORY OF FLOW OVER BROAD-CRESTED WEIRS



*(a) Flow over a Broad-Crested Weir*




*(b) Idealised Flow*

*Figure 6 The Broad-Crested Weir*

The weir shown in Figure 6(a) differs from a notch, in that the crest is not sharp, but is long in relation to the head $H$. This gives it the term 'broad-crested'. In the acceleration along the length of the crest, the flow tends towards a uniform condition. Figure 6(b) shows an idealisation in which the flow has become uniform and horizontal at the outlet section C from the crest. At this section, the depth of flow is $z_c$ and the uniform flow velocity is $u_c$. The flow rate is given by:

$$Q = u_c b z_c$$

**(4-1)**

where b is the width of the weir (in the direction normal to the figure).

In this, $u_c$ may be found by applying Bernoulli's equation to the streamline in the water surface:

$$u_c = \sqrt{2gh_c} = \sqrt{2g(H - z_c)}$$

**(4-2)**

Substituting for $u_c$ in Equation (4-2) from Equation (4-1):

$$Q = \sqrt{2g(H - z_c)}\, b z_c$$

**(4-3)**

This result appears to show that the flow rate $Q$ depends on both the two variables $H$ and $z_c$ . However, it is obvious that $z_c$ is not truly independent, as it must depend in some way on $H$.
With any chosen fixed value of $H$ on a given weir, $z_c$ will settle to some value somewhere in the range between zero and $H$. This value is in fact the one that maximises $Q$ for the chosen value of $H$.

For fixed values of $g$, $b$ and $H$, $Q$ has a maximum when:

$$\frac{d}{dz_c}\sqrt{H-z_c}\, z_c = 0$$

Performing the differentiation:

$$-\tfrac{1}{2}\left(H-z_c\right)^{-1/2} z_c + \left(H-z_c\right)^{1/2} = 0$$

**(4-4)**

which leads to:

$$z_c = \tfrac{2}{3}H$$

Substituting in Equation (4-3) leads to the result

$$Q = \frac{2}{3\sqrt{3}}\sqrt{2g}\, bH^{3/2}$$

**(4-5)**

This is the theoretical flow rate. To take account of the simplifications used in deriving this, the discharge coefficient $C_d$ is introduced as before:

$$Q = C_d \frac{2}{3\sqrt{3}}\sqrt{2g}\, bH^{3/2}$$

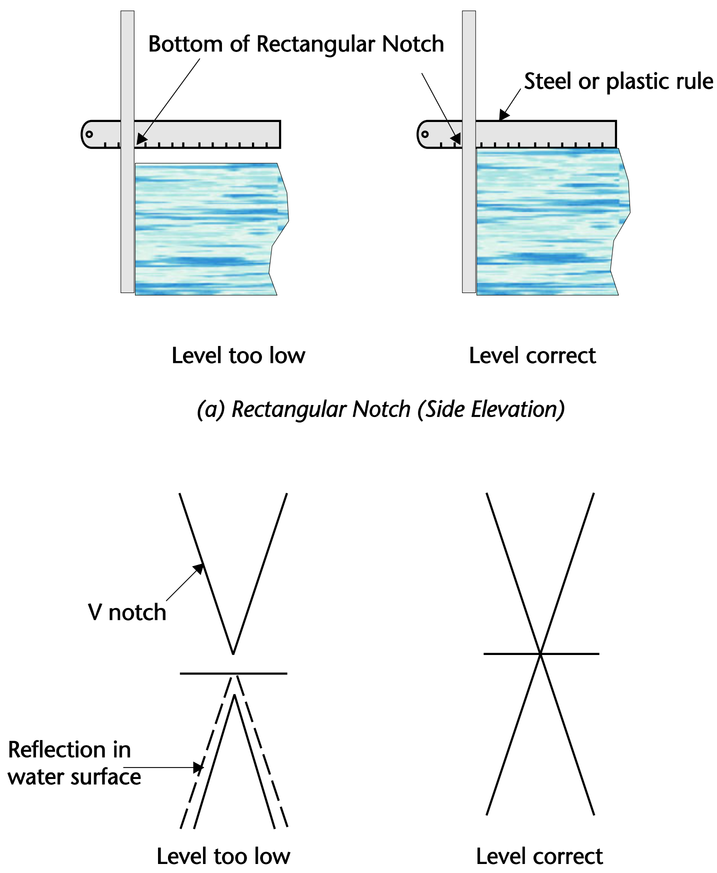
**(4-6)**

The value of $C_d$ is found by experiment, having a value which is usually less than unity. It may vary slightly with $H$, as the ratio of head to crest breadth changes. A logarithmic plot would not be a straight line of slope 1.5. It is therefore recommended that Equation (4-6) be used to compute the value of $C_d$ for each pair of observations of $H$ and $Q$. The resultant values of $C_d$ are then plotted against the ratio of head to crest breadth.

## SECTION 5.0 EXPERIMENT PROCEDURE

Apply the grease (supplied) to the edges of the weir to help seal it into the slot on the H1D.

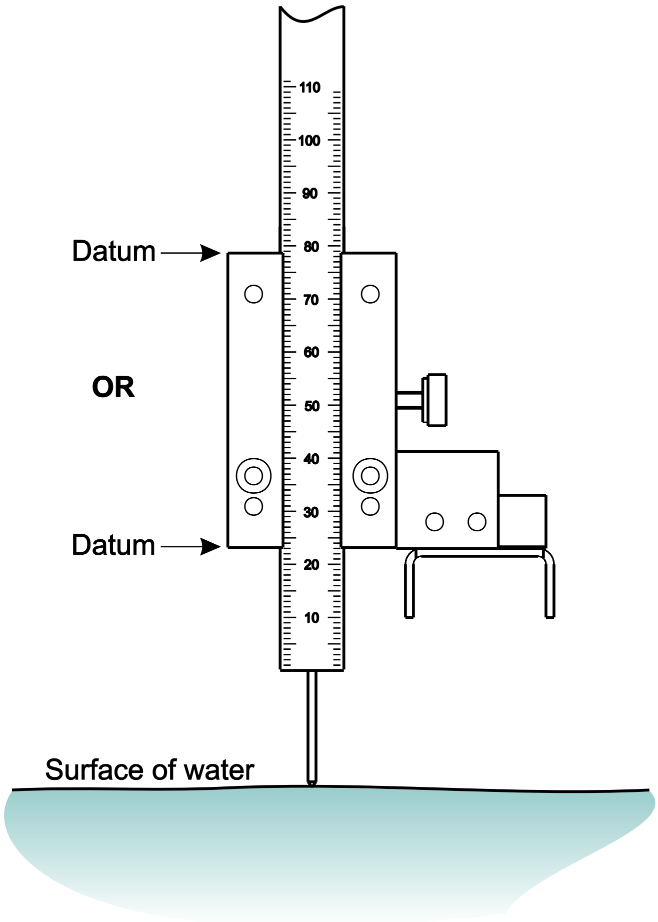
Make sure that the water in the weir channel is level with the bottom of the notch. To do this, add water into the small reservoir in the top of the H1D until it is level with the bottom of the weir. For the rectangular notch, you can check this as shown in Figure 7(a) by carefully holding a flat steel or plastic rule on the crest. For the V notch, the reflection of the V in the surface indicates whether the level is correct or not (See Figure 7(b)). When the level is correct fit the height gauge. Adjust the height gauge so its tip is just touching the surface of the water and note the indicated value (use either the top or the bottom of the gauge). This is the datum for all your other height measurements.




*(a) Rectangular Notch (Side Elevation)*

*(b) V notch*

*Figure 7 Correct Level for Rectangular and V-Notches*




Now use the height gauge to measure the level of water upstream as you alter the flow in stages, starting from maximum flow. At each stage, use the hydraulic bench to measure the actual flow or discharge rate (Q). Stop readings when the water flow is so low that the water does not pour cleanly over the weir (it starts to run down the weir face). This is normally at a head of approximately 10 mm for a rectangular notch and about 20 mm for a V notch.  Eight sets of readings for each notch should be sufficient.

Record the width of the rectangular notch and the angle of the V notches (best found by measuring the depth and width of the V).

## SECTION 6.0 RESULTS AND CALCULATIONS

Results given in this section are typical of those obtainable from the equipment supplied. The results should only be used as a guide as slight differences between units may occur.

Table 1 shows the format of a suitable results table and includes columns for recording measurements of head, H and discharge, Q, together with log H and log Q. From results obtained, plot a graph of discharge rate Q against head H and log Q against log H.

| Gauge Reading mm | H mm | Q $m^3/s$ x $10^{-4}$ | Log Q | Log H |
|---|---|---|---|---|
| **Datum** 3.93 | 0 | - | - | - |
| 62.61 | 58.68 | 7.62 | -3.1180 | -1.2315 |
| 57.12 | 53.19 | 6.70 | -3.1739 | -1.2742 |
| 49.92 | 45.99 | 5.43 | -3.2652 | -1.3373 |
| 43.07 | 39.14 | 4.28 | -3.3686 | -1-4074 |
| 39.34 | 35.41 | 3.68 | -3.4342 | -1.4509 |
| 31.98 | 28.05 | 2.58 | -3.5884 | -1.5520 |
| 28.01 | 25.08 | 2.06 | -3.6861 | -1.6007 |
| 22.38 | 18.45 | 1.44 | -3.8402 | 1.7350 |
| 17.06 | 13.13 | 0.78 | -4.1051 | -1.8817 |

*Table 1 Results with Rectangular Notch*

Figure 8 and figure 9 show the form of the graphs expected. The slope n and intercept k on the axis of the log Q scale should be determined and used to derive the relationship between the discharge rate Q and the head H. See Section 2 for the related theory.

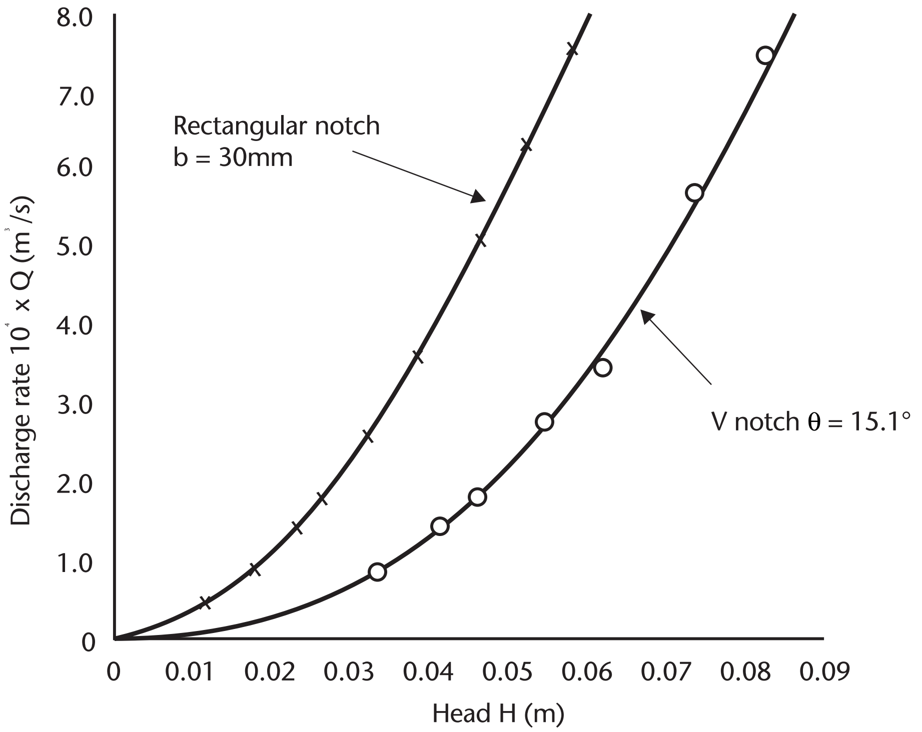



*Figure 8 Variations of Discharge with Head for Rectangular and V-Notch*

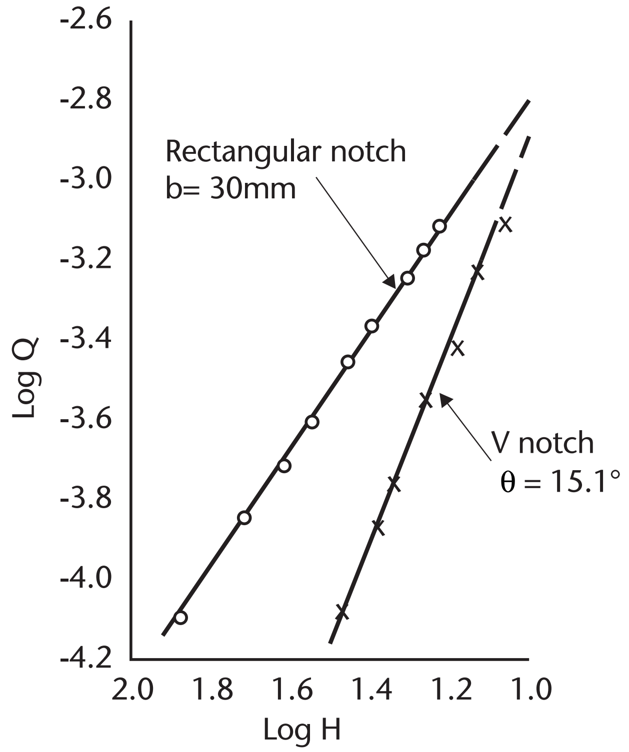



*Figure 9 Variations of Log Q with Log H for Rectangular and V Notches.*

## 6.1 Rectangular Notch - Typical Calculation

Width of notch $b$ = 30mm

From graph of log $H$ against log $Q$: Slope $n$ = 1.50

Intercept on log $Q$ axis (i.e. when log $H$ = 0) = 2.72 (by extrapolation). The relationship between log $Q$ and log $H$ is thus:

$$\log Q = 2.72 + 1.50 \log H$$

**(2-13)**

so that the relationship between $Q$ and $H$ is:

$$Q = 0.0525\, H^{1.50}$$

**(2-9)**

Comparing this with the expression derived previously,

$$Q = C_d \frac{2}{3}\sqrt{2g}\, bH^{3/2}$$

Note that the exponent of $H$ is the same in both, and that $C_d$ is given by:

$$C_d = \frac{0.0525}{2/3\sqrt{2g}\ b} = \frac{0.0525}{2/3 \times \sqrt{2 \times 9.81} \times 0.030}$$

$$C_d = 0.59$$

## 6.2 V Notch - Typical Calculations ($\theta$ = 15°)

Width across top of V = 55 mm

Depth of V = 102 mm

$$\tan\theta = \frac{55}{2 \times 102} = 0.270 = 15.1°$$

| Gauge Reading mm | H mm | Q $m^3/s$ x $10^{-4}$ | Log Q | Log H |
|---|---|---|---|---|
| 1.94 | 0 | - | - | - |
| 85.88 | 83.94 | 7.46 | - 3.1273 | - 1.0760 |
| 75.28 | 73.34 | 5.77 | - 3.2388 | - 1.1347 |
| 65.35 | 63.41 | 3.96 | - 3.4023 | - 1.1978 |
| 56.37 | 54.43 | 2.78 | - 3.5560 | - 1.2648 |
| 45.98 | 44.04 | 1.69 | - 3.7721 | - 1.3561 |
| 42.79 | 40.35 | 1.34 | - 3.8729 | - 1.3888 |
| 35.60 | 33.60 | 0.83 | - 4.0799 | - 1.4729 |

*Table 2 Results with V Notch $\theta$ = 15°*

From graph of log H against log Q: Slope n = 2.50

Intercept on log Q axis = 1.60 (by extrapolation)

The relationship between log Q and log H is thus

$$\log Q = 1.60 + 2.50 \log H$$

so that the relationship between Q and H is

$$Q = 0.398\, H^{2.50} \tag{2-14}$$

Comparing this with the expression derived previously,

$$Q = C_d \frac{8}{15}\sqrt{2g}\ \tan\theta\, H^{5/2} \tag{2-10}$$

we note that the exponent of H is the same in both, and that $C_d$ is given by :

$$C_d = \frac{0.398}{8/15\sqrt{2g}\tan} = \frac{0.398}{8/15 \text{ x } \sqrt{2 \text{ x } 9.81} \text{ x } 0.270}$$

$$C_d = 0.62$$

## 6.3 Discussion of Results

The results show that discharge over the rectangular weir may be represented by the equation:

$$Q = C_d \frac{2}{3}\sqrt{2g}\, b\, H^{3/2}$$

**(2-9)**

in which: $C_d$ = 0.59 over the range of the experiment

The expression $\frac{2}{3}\sqrt{2g}\, bH^{3/2}$ which appeared in the equation is the calculated discharge, neglecting losses and the contraction of the jet as it passes through the notch. $C_d$ is the empirical factor to take account of these effects.

The equation for discharge over the V notch can now be represented by the equation:

$$Q = C_d \frac{8}{15}\sqrt{2g}\tan\theta\, H^{5/2}$$

**(2-10)**

in which: $C_d$ = 0.62 over the range of the experiment

In this case, $\frac{8}{15}\sqrt{2g}\ \tan\theta H^{5/2}$ represents the calculated discharge, and $C_d$ again takes account of losses and of contraction of the jet.
Note that, over the range of experiments, slightly different exponents of H, with corresponding different values of $C_d$, could be fitted to the results. For example, the equation,

$$Q = C_d \frac{2}{3}\sqrt{2g}\,bH^{1.47}$$

**(2-15)**

in which $C_d$ = 0.54 fits the results for the rectangular weir almost as well as Equation (2-9) with $C_d$ = 0.59. A wider range of $H$ would be required to differentiate between the various alternatives. However, without any evidence to suggest that the exponents 1.50 and 2.50 (for the rectangular V notches) do not apply exactly in practice, it is reasonable and convenient to take these values. Moreover, the values of $C_d$ associated with these exponents are close to the value obtained for flow through an orifice.

## SECTION 7.0 QUESTIONS FOR FURTHER DISCUSSION

1. What suggestions do you have for improving the apparatus?

2. How would you interpret results which, when plotted logarithmically, as on Figure 9, fall on a line which is not straight but slightly curved?

3. To what extent does the experiment confirm the theoretical treatment? Has the dependence on b (for the rectangular notch) or $\theta$ (for the V notch) been established?

   A suggested project would be;
   *Plan a series of tests to explore the dependence on b, using a set of notches or by partially covering the width of the one supplied with a sharp-edged metal strip.*

   What range of b should be chosen?
   What is the best way to present the results?

4. Is there a recognised change to the form of Equation (2-9) which allows for the effect of the contractions at the side?

5. A V notch with an angle, of 45° is supplied. What effect does this change in angle from the 15° standard have on the equations?
   Why?