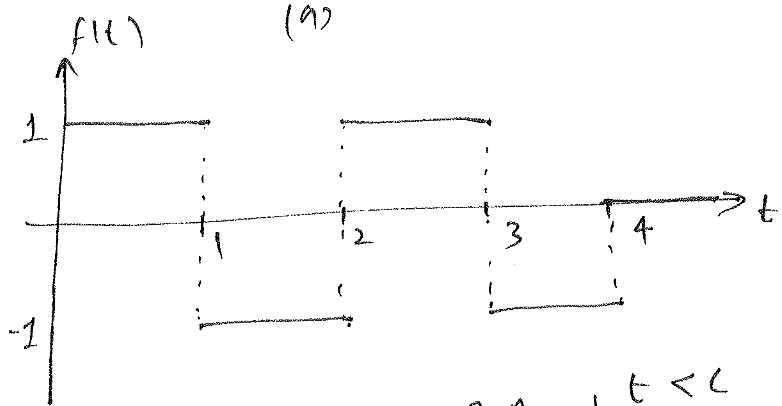
MATH 215/255 HW#9

Section 6.3 #8

(a)

$$f(t) = \begin{cases} 1 & 0 \le t < 1 \\ -1 & 1 \le t < 2 \\ 1 & 2 \le t < 3 \\ -1 & 3 \le t < 4 \\ 0 & t \ge 4 \end{cases}$$



(b) Recall that $u_c(t) = \begin{cases} 0 & , t < c \\ 1 & , t > c \end{cases}$

$$\Rightarrow \quad f(t) = 1 - 2u_1(t) + 2u_2(t) - 2u_3(t) + u_4(t)$$

(Detailed steps!)

$f_1 = 1 \rightarrow$ constant function from 0 to ∞

$f_2 = 1 - 2u_1(t) \rightarrow$ jump from 1 to -1 at $t = 1$ and stay at -1 forever.

$f_3 = f_2 + 2u_2(t) \rightarrow$ Go from -1 to $+1$ at $t = 2$

$= 1 - 2u_1(t) + 2u_2(t)$

$f_4 = f_3 - 2u_3(t) \rightarrow$ Jump from $+1$ to -1 at $t = 3$

$= 1 - 2u_1(t) + 2u_2(t) - 2u_3(t)$

$f_5 = f_4 + u_4(t) \rightarrow$ Jump from -1 to 0 and stay there

Note that f_5 is your final required function

$$\Rightarrow f_5(t) = f(t) = 1 - 2u_1(t) + 2u_2(t) - 2u_3(t) + u_4(t)$$

Section 6.3 # 13

$$f(t) = \begin{cases} 0 & t < 2 \\ t-2 & t \geq 2 \end{cases}$$

$$= u_2(t)\,(t-2)^2$$

Recalls, from # 13 in the table:

$$\mathcal{L}\{u_c(t)\, g(t-c)\} = e^{-cs} G(s)$$
$$= e^{-cs}\mathcal{L}\{g(t)\}$$

Here $c = 2$

$$g(t-c) = g(t-2) = (t-2)^2$$

$$\Rightarrow \quad g(t) = t^2$$

$$\therefore \quad G(s) = \mathcal{L}\{g(t)\} = \frac{2}{s^3}$$

$$\therefore \quad \mathcal{L}\{f(t)\} = \mathcal{L}\{u_2(t)\, g(t-2)\} = e^{-2s}\mathcal{L}\{g(t)\} = \frac{e^{-2s}\cdot 2}{s^3}$$

Section 6.3 # 15

$$f(t) = \begin{cases} 0 & t < \pi \\ t-\pi & \pi \leq t < 2\pi \\ 0 & t \geq 2\pi \end{cases}$$

Jump of $(t-\pi)$ at $t = \pi$

$$\Rightarrow \quad f(t) = (t-\pi)\, u_\pi(t) + h(t)\, u_{2\pi}(t)$$

unknown jump size.

Since $f(t) = 0$ for $t \geq 2\pi$, & recall $u_\pi(t) = 1$ for $t \geq 2\pi$
& $u_{2\pi}(t) = 1$ for $t \geq 2\pi$,

we have $0 = (t-\pi) + h(t)$ for $t \geq 2\pi$

$\Rightarrow \quad h(t) = \quad -(t-\pi)$

$\therefore \quad f(t) = (t-\pi)u_\pi(t) - (t-\pi)u_{2\pi}(t)$

We again use # 13 in the table: $\mathcal{L}\{u_c(t)f(t-c)\} = e^{-cs}F(s)$

~~Here~~ The second term is not in the required form.

Rewriting $f(t)$, we have

$$f(t) = (t-\pi)u_\pi(t) - (t-2\pi+\pi)u_{2\pi}(t)$$

$$= (t-\pi)u_\pi(t) - (t-2\pi)u_{2\pi}(t) - \pi u_{2\pi}(t)$$

~~$\mathcal{L}\{f(t)\} = (e^{-\pi s}F(s)),$~~

Using $\mathcal{L}\{g(t-c)u_c(t)\} = e^{-cs}G(s)$, we have

$$\mathcal{L}\{f(t)\} = e^{-\pi s}\mathcal{L}\{t\} - e^{-2\pi s}\mathcal{L}\{t\} - \pi\mathcal{L}\{u_{2\pi}(t)\}$$

$$= \frac{e^{-\pi s}}{s^2} - \frac{e^{-2\pi s}}{s^2} - \frac{\pi e^{-\pi s}}{s}$$

Note that $g(t-\pi) = t-\pi$
$\Rightarrow g(t) = t$

Section 6.3 # 21

$$F(s) = \frac{2(s-1)e^{-2s}}{s^2-2s+2}$$

We write $F(s) = \dfrac{2(s-1)e^{-2s}}{(s-1)^2+1} = e^{-2s}H(s)$

Now $\mathcal{L}^{-1}\left\{\dfrac{2(s-1)}{(s-1)^2+1}\right\} = \mathcal{L}^{-1}\{H(s)\} = 2e^t\cos t = h(t)$
(from # 10)

Since $\mathcal{L}\{u_c(t)\,h(t-c)\} = e^{-cs}H(s)$;

$$\Rightarrow \mathcal{L}^{-1}\{e^{-cs}H(s)\} = u_c(t)\,h(t-c)$$

$$\Rightarrow \mathcal{L}^{-1}\{e^{-2s}H(s)\} = u_2(t)\,h(t-2)$$

where $h(t) = 2e^t\cos t$

$$\therefore \; f(t) = 2u_2(t)\,e^{(t-2)}\cos(t-2)$$

Section 6.3 # 22

$$F(s) = \frac{2e^{-2s}}{s^2-4}$$

We write $F(s) = e^{-2s}G(s)$. where $G(s) = \dfrac{2}{s^2-4}$

Now $g(t) = \mathcal{L}^{-1}\{G(s)\} = \sinh(2t)$ (using # 7)

Since $\mathcal{L}\{g(t-c)\,u_c(t)\} = e^{-cs}G(s)$, we have

$$\mathcal{L}^{-1}\{e^{-cs}G(s)\} = u_c(t)\,g(t-c)$$

$$\Rightarrow \mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\{e^{-2s}G(s)\} = u_2(t)\,g(t-2)$$

$$= u_2(t)\sinh[2(t-2)]$$

Section 6.3 # 24

$$F(s) = \frac{e^{-s}}{s} + \frac{e^{-2s}}{s} - \frac{e^{-3s}}{s} - \frac{e^{-4s}}{s}$$

$$\mathcal{L}^{-1}\{F(s)\} = u_1(t) + u_2(t) - u_3(t) - u_4(t)$$ (using # 12)

Section 1.3 # 25

(i) Prove that if $s > ca$, then $\mathcal{L}\{f(ct)\} = \frac{1}{c} F\left(\frac{s}{c}\right)$

where $F(s) = \mathcal{L}\{f(t)\}$ exists for $s > a \geq 0$

Proof: Recall the definition of Laplace transform.

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt \quad \text{--- } (*)$$

$$\therefore \mathcal{L}\{f(ct)\} = \int_0^\infty e^{-st} f(ct)\, dt$$

Let $w = ct$ be the new variable

$$\Rightarrow dt = \frac{1}{c} dw$$

$$\Rightarrow \mathcal{L}\{f(ct)\} = \frac{1}{c}\int_0^\infty e^{-\left(\frac{s}{c}\right) w} f(w)\, dw$$

↳ The integral is identical to $(*)$ except that s is now replaced with $\frac{s}{c}$

$$\therefore \mathcal{L}\{f(ct)\} = \frac{1}{c} F\left(\frac{s}{c}\right)$$ valid for $\frac{s}{c} > a$

$\Rightarrow s > ca$

(ii) Prove that if $k > 0$, then

$$\mathcal{L}^{-1}\{F(ks)\} = \frac{1}{k} f\left(\frac{t}{k}\right)$$

Now $\mathcal{L}\{\frac{1}{k} f\left(\frac{t}{k}\right)\} =$ $F(ks)$.

$$\Rightarrow \mathcal{L}\{\frac{1}{k} f\left(\frac{t}{k}\right)\} = \frac{1}{k}\int_0^\infty f\left(\frac{t}{k}\right) e^{-st} dt$$

Let $\omega = \dfrac{t}{k} \Rightarrow dt = k\,d\omega$

$$\Rightarrow \mathcal{L}\left\{ \frac{1}{k} f\left(\frac{t}{k}\right) \right\} = \int_0^\infty f(\omega)\, e^{-(sk)\omega}\, d\omega$$

$$= F(sk)$$

we need $k > 0$ so that $\int_0^\infty (\ )\,dt \to \int_0^\infty (\ )\,d\omega$

(iii) Show that if a, b are constants with $a > 0$, then

$$\mathcal{L}^{-1}\left\{ F(as+b) \right\} = \frac{1}{a} f\left(\frac{t}{a}\right) e^{-bt/a}$$

Proof: $$\mathcal{L}\left\{ \frac{1}{a} f\left(\frac{t}{a}\right) e^{-bt/a} \right\} = \int_0^\infty \left[\frac{1}{a} f\left(\frac{t}{a}\right) e^{-\frac{bt}{a}}\right] e^{-st}\, dt$$
$$= I$$

Let $\omega = \dfrac{t}{a} \Rightarrow dt = a\,d\omega$ (Require $a > 0$ so that the limits $\int_0^\infty dt \to \int_0^\infty d\omega$)

Then $$I = \int_0^\infty f(\omega)\, e^{-b\omega}\, e^{-sa\omega}\, d\omega$$

So $$I = \int_0^\infty f(\omega)\, e^{-(as+b)\omega}\, d\omega = F(as+b)$$

Section 6.3 #27

$$F(s) = \frac{2s+1}{4s^2+4s+5} = \frac{2\left(s+\frac{1}{2}\right)}{4\left[s^2+s+\frac{1}{4}+1\right]} = \frac{2\left(s+\frac{1}{2}\right)}{4\left(s+\frac{1}{2}\right)^2+4}$$

$$\Rightarrow \quad F(s) = \frac{1}{2}\left[\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+1}\right]$$

$$= \frac{1}{2} H(s+\tfrac{1}{2}) \quad \text{where} \quad H(s+\tfrac{1}{2}) = \frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+1}$$

we can directly use #10 in the table to get the answer. Alternately, using the result of 25 (iii),

$$H(as+b) = \mathcal{L}\left\{\frac{1}{a} h\left(\frac{t}{a}\right)e^{-\frac{bt}{a}}\right\}$$

Here $a = 1, \quad b = 1/2$

$$\Rightarrow \quad H(s+\tfrac{1}{2}) = \mathcal{L}\left\{h(t)e^{-t/2}\right\}$$

where $h(t) = \mathcal{L}^{-1}\{H(s)\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} = \cos t$

$$\therefore \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2}\mathcal{L}^{-1}\{H(s+\tfrac{1}{2})\}$$

$$= \frac{1}{2}e^{-t/2}\cos t$$

Section 6.3 # 28

$$F(s) = \frac{1}{9s^2-12s+3} = \frac{1}{9(s^2-\frac{4}{3}s)+3}$$

Complete the square:

$$F(s) = \frac{1}{9(s^2-\frac{4}{3}s+\frac{4}{9}-\frac{4}{9})+3} = \frac{1}{9(s^2-\frac{4s}{3}+\frac{4}{9})-1}$$

So $F(s) = \dfrac{1}{9\left(s-\frac{2}{3}\right)^2 - 1} = \dfrac{1}{9\left[\left(s-\frac{2}{3}\right)^2 - \frac{1}{9}\right]}$

$$= \frac{1}{3}\left[\frac{1/3}{\left(s-\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2}\right]$$

$$= \frac{1}{3} H\left(s - \frac{2}{3}\right)$$

where $H(s) = \dfrac{1/3}{s^2 - (1/3)^2}$

Using # 14, $\mathcal{L}^{-1}\{H(s-c)\} = e^{ct} h(t)$

$$\Rightarrow \mathcal{L}^{-1}\left\{H\left(s-\frac{2}{3}\right)\right\} = e^{\frac{2}{3}t} h(t)$$

$$= e^{\frac{2}{3}t} \mathcal{L}^{-1}\{H(s)\}$$

$$= e^{\frac{2}{3}t} \sinh\left(\frac{t}{3}\right) \quad (\text{from } \# 7)$$

$$\therefore \mathcal{L}^{-1}\{F(s)\} = \frac{1}{3} e^{\frac{2}{3}t} \sinh\left(\frac{t}{3}\right)$$

Recall $\sinh(t) = \dfrac{e^t - e^{-t}}{2}$ $\Rightarrow \sinh\left(\frac{t}{3}\right) = \dfrac{e^{t/3} - e^{-t/3}}{2}$

$$\Rightarrow \mathcal{L}^{-1}\{F(s)\} = \frac{1}{3} e^{\frac{2t}{3}} \left[\frac{e^{t/3} - e^{-t/3}}{2}\right] = \frac{1}{6} e^{t/3}\left(e^{2t/3} - 1\right)$$

Section 6.3 # 29

$$F(s) = \frac{e^2 e^{-4s}}{2s - 1}$$

$$F(s) = \frac{e^2}{2} \frac{e^{-4s}}{(s-\frac{1}{2})} = \frac{e^2}{2} e^{-4s} G(s)$$

where $G(s) = \frac{1}{s-\frac{1}{2}}$

Recall $\mathcal{L}^{-1}\left\{\frac{1}{s-\frac{1}{2}}\right\} = e^{t/2} = g(t)$

$\therefore \mathcal{L}^{-1}\left\{e^{-4s} G(s)\right\} = u_4(t)\, g(t-4)$

Thus $f(t) = \mathcal{L}^{-1}\{F(s)\} = e^{\frac{1}{2}(t-4)} \frac{e^2}{2} u_4(t)$

So $f(t) = \frac{e^{t/2}}{2} u_4(t)$ which is same as

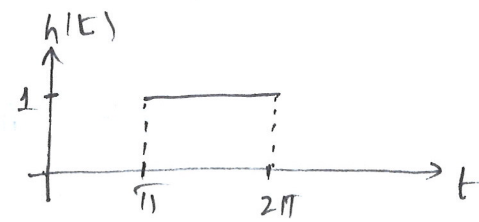
$f(t) = \frac{e^{t/2}}{2} u_2(t/2)$

Section 6.4 # 2

$y'' + 2y' + 2y = h(t)$;

$y(0) = 0$; $y'(0) = 1$

$$h(t) = \begin{cases} 1 & \pi \le t < 2\pi \\ 0 & 0 \le t < \pi \;\&\; t \ge 2\pi \end{cases}$$



Take Laplace transform on both sides

Clearly $h(t) = u_\pi(t) - u_{2\pi}(t)$

$\mathcal{L}\{y'' + 2y' + 2y\} = \mathcal{L}\{u_\pi(t) - u_{2\pi}(t)\}$

$\Rightarrow [s^2 Y(s) - s(y(0)) - y'(0)] + 2[sY(s) - y(0)] + 2Y(s) = \frac{e^{-\pi s}}{s} - \frac{e^{-2\pi s}}{s}$

$\Rightarrow (s^2 + 2s + 2)Y(s) - 1 = \frac{e^{-\pi s}}{s} - \frac{e^{-2\pi s}}{s}$

So, $\quad Y(s) = \dfrac{1}{s^2+2s+2} + F(s)\, e^{-\pi(s)} - F(s)\, e^{-2\pi s}$

where $\quad F(s) = \dfrac{1}{s(s^2+2s+2)}$

$$\Rightarrow \quad Y(s) = \frac{1}{(s+1)^2+1} + F(s)\, e^{-\pi s} - F(s)\, e^{-2\pi s}$$

Using #9 and #13, we have

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = e^{-t}\sin t + u_{\pi}(t)\, f(t-\pi) - u_{2\pi}(t)\, f(t-2\pi)$$

where $\quad f(t) = \mathcal{L}^{-1}\{F(s)\}$

$$= \mathcal{L}^{-1}\left\{ \frac{1}{s(s^2+2s+2)} \right\}$$

Now $\quad \dfrac{1}{s(s^2+2s+2)} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+2s+2}$

$$1 = A(s^2+2s+2) + s(Bs+C)$$

s^2 : $\quad A+B = 0 \quad \Rightarrow \quad B = -1/2$

s^1 : $\quad 2A+C = 0 \quad \Rightarrow \quad C = -1$

s^0 : $\quad 2A = 1 \quad \Rightarrow \quad A = 1/2$

$$F(s) = \frac{1}{s(s^2+2s+2)} = \frac{1}{2}\cdot\frac{1}{s} - \frac{1}{2}\,\frac{(s+2)}{s^2+2s+2}$$

$$= \left(\frac{1}{2}\right)\frac{1}{s} - \frac{1}{2}\left[\frac{s+1}{(s+1)^2+1} + \frac{1}{(s+1)^2+1} \right]$$

Thus $f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2} - \frac{1}{2}e^{-t}\cos t - \frac{1}{2}e^{-t}\sin t$

$\therefore \quad y(t) = e^{-t}\sin t + u_\pi(t)\, f(t-\pi) - u_{2\pi}(t)\, f(t-2\pi)$

We can write this as

$$f(t-\pi) = \frac{1}{2} - \frac{1}{2}e^{-(t-\pi)}\cos(t-\pi) - \frac{1}{2}e^{-(t-\pi)}\sin(t-\pi)$$

$$= \frac{1}{2} + \frac{1}{2}e^{-(t-\pi)}\cos t + \frac{1}{2}e^{-(t-\pi)}\sin t$$

and $$f(t-2\pi) = \frac{1}{2} - \frac{1}{2}e^{-(t-2\pi)}\cos t - \frac{1}{2}e^{-(t-2\pi)}\sin t$$

Thus

$$y(t) = e^{-t}\sin t + u_\pi(t)\left[\frac{1}{2} + \frac{1}{2}e^{-(t-\pi)}\cos t + \frac{1}{2}e^{-(t-\pi)}\sin t\right]$$
$$+ u_{2\pi}(t)\left[\frac{1}{2} - \frac{1}{2}e^{-(t-2\pi)}\cos t - \frac{1}{2}e^{-(t-2\pi)}\sin t\right]$$

Section 6.4 #6

$$y'' + 3y' + 2y = u_2(t)$$

$$y(0) = 0, \quad y'(0) = 1$$

Take Laplace transform to get

$$\left[s^2 Y(s) - sy(0) - y'(0)\right] + 3\left[sY(s) - y(0)\right] + 2Y(s) = \frac{e^{-2s}}{s}$$

Then $$(s^2+3s+2)\,Y(s) = 1 + \frac{e^{-2s}}{s}$$

So $$Y(s) = \frac{1}{s^2+3s+2} + F(s)\,e^{-2s}$$

where $$F(s) = \frac{1}{s(s^2+3s+2)}$$

So $$Y(s) = \frac{1}{(s+2)(s+1)} + F(s)\,e^{-2s}$$

$$= \underbrace{\frac{1}{s+1} - \frac{1}{s+2}}_{\text{(Using Partial Fractions)}} + F(s)\,e^{-2s}$$

Also $$F(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} = \frac{1}{s(s+1)(s+2)}$$

So $$A(s+1)(s+2) + B\,s(s+2) + C\,s(s+1) = 1$$

we get $A = 1/2$

$B = -1$

$C = 1/2$

$\therefore$ $$f(t) = \mathcal{L}^{-1}[F(s)]$$

$$= \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}$$

$\therefore$ So $$y(t) = \mathcal{L}^{-1}\{Y(s)\} = e^{-t} - e^{-2t} + u_2(t)\,f(t-2)$$

$$= e^{-t} - e^{-2t} + u_2(t)\left[\frac{1}{2} - e^{-(t-2)} + \frac{1}{2}e^{-2(t-2)}\right]$$

Section 6.4 #9

$$y'' + y = g(t) \; ; \quad y(0) = 0 \; ; \; y'(0) = 0$$

$$g(t) = \begin{cases} t/2 & 0 \leq t < 6 \\ 3 & t \geq 6 \end{cases}$$

We rewrite $g(t)$ as :

$$g(t) = \frac{t}{2} + \begin{cases} 0 & 0 \leq t < 6 \\ 3 - \frac{t}{2} & t \geq 6 \end{cases}$$

$$= \frac{t}{2} + u_6(t)\left(3 - \frac{t}{2}\right)$$

$$= \frac{t}{2} + u_6(t)\left(\frac{6-t}{2}\right)$$

$$= \frac{t}{2} + u_6(t)\, f(t-6)$$

where

$$f(t-6) = \frac{6-t}{2}$$

$$\Rightarrow f(t) = \frac{-t}{2}$$

Now $\mathcal{L}\{g(t)\} = \frac{1}{2s^2} + e^{-6s}\left(\frac{-1}{2s^2}\right)$

Taking Laplace transform of the equation :

$$\left[s^2 Y(s) - s\,y(0) - y'(0)\right] + Y(s) = \frac{1}{2s^2} - \frac{1}{2s^2} e^{-6s}$$

with $y(0) = 0$, $y'(0) = 1$, we have

$$(s^2+1)\, Y(s) = 1 + \frac{1}{2s^2} - \frac{1}{2s^2} e^{-6s}$$

$$\Rightarrow Y(s) = \frac{1}{s^2+1} + F(s) - F(s) e^{-6s}, \quad \text{where } F(s) = \frac{1}{2s^2(s^2+1)}$$

Then $\quad y(t) = \mathcal{L}^{-1}\{Y(s)\} = \sin t + f(t) - u_6(t)\, f(t-6)$

where $\quad f(t) = \mathcal{L}^{-1}\{F(s)\}$

$$= \mathcal{L}^{-1}\left\{ \frac{1}{2}\left[\frac{1}{s^2} - \frac{1}{s^2+1}\right]\right\}$$ By Partial fractions

So $\quad f(t) = \frac{1}{2}(t - \sin t).$

Then
$$y(t) = \sin t + \frac{1}{2}(t - \sin t) - u_6(t)\left[\frac{1}{2}(t-6 - \sin(t-6))\right]$$

$$\Rightarrow \quad y(t) = \frac{t}{2} + \frac{1}{2}\sin t - u_6(t)\left[\frac{t}{2} - 3 - \frac{1}{2}\sin(t-6)\right]$$

Section 6.4 #10

$$y'' + y' + \frac{5}{4}y = g(t) \; ; \quad y(0) = 0 \; ; \; y'(0) = 0$$

$$g(t) = \begin{cases} \sin t & 0 \le t < \pi \\ 0 & t \ge \pi \end{cases}$$

Now
$$g(t) = \sin t + \begin{cases} 0 & 0 < t < \pi \\ -\sin t & t \ge \pi \end{cases}$$

$$= \sin t + \begin{cases} 0 & 0 \le t < \pi \\ \sin(t-\pi) & t \ge \pi \end{cases}$$ Since $\sin(t-\pi) = -\sin t$

This gives
$$g(t) = \sin t + u_\pi(t)\sin(t-\pi)$$
$$= \sin t + u_\pi(t)\, f(t-\pi)$$
where $\quad f(t) = \sin t$

Now $\quad Y(s) = \mathcal{L}\{y(t)\}$ gives

$$\left[s^2 Y(s) - s\,y(0) - y'(0)\right] + \left[s\,Y(s) - y(0)\right] + \frac{5}{4} Y(s) = \mathcal{L}\{g(t)\}$$

with $\quad y(0) = 0 \quad \& \quad y'(0) = 0,$ we have

$$\left(s^2 + s + \frac{5}{4}\right) Y(s) = \frac{1}{s^2+1} + \frac{e^{-\pi s}}{s^2+1}$$

$$\therefore \quad Y(s) = \frac{1}{(s^2+1)\left(s^2+s+\frac{5}{4}\right)} + e^{-\pi s}\frac{1}{(s^2+1)\left(s^2+s+\frac{5}{4}\right)}$$

This gives

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{F(s) + e^{-\pi s} F(s)\right\}$$

where $\quad F(s) = \dfrac{1}{(s^2+1)\left(s^2+s+\frac{5}{4}\right)}$

$$\therefore \quad y(t) = f(t) + u_\pi(t)\, f(t-\pi)$$

Now $\quad F(s) = \dfrac{1}{(s^2+1)\left(s^2+s+\frac{5}{4}\right)} = \dfrac{As+B}{s^2+1} + \dfrac{Cs+D}{s^2+s+\frac{5}{4}}$

So $\quad (As+B)\left(s^2+s+\frac{5}{4}\right) + (Cs+D)(s^2+1) = 1$

So

$s^3 : \quad A + C = 0 \quad$ — (1)

$s^2 : \quad A + B + D = 0 \quad$ — (2)

$s^1 : \quad \frac{5}{4} A + B + C = 0 \quad$ — (3)

$s^0 : \quad \frac{5}{4} B + D = 1 \quad$ — (4)

We get $A = \frac{-16}{17}$, $B = \frac{4}{17}$, $C = \frac{16}{17}$, $D = \frac{12}{17}$

Thus

$$F(s) = \frac{-16}{17} \frac{s}{s^2+1} + \frac{4}{17} \frac{1}{s^2+1} + \frac{\frac{16}{17} s + \frac{12}{17}}{(s+\frac{1}{2})^2+1}$$

$$\Rightarrow F(s) = \frac{-16}{17} \frac{s}{s^2+1} + \frac{4}{17} \frac{1}{s^2+1} + \frac{\frac{16}{17}(s+\frac{1}{2})}{(s+\frac{1}{2})^2+1} + \frac{\frac{4}{17}}{(s+\frac{1}{2})^2+1}$$

$$\Rightarrow f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{-16}{17} \cos t + \frac{4}{17} \sin t + \frac{16}{17} e^{-t/2} \cos t + \frac{4}{17} e^{-t/2} \sin t \quad \text{——} \ (*)$$

finally

$$y(t) = f(t) + u_\pi(t) \ f(t-\pi)$$

where $f(t)$ is given in $(*)$.