Regular Perturbation Theory for differential equations

Ex: $y'' + (1 - \epsilon x) y = 0$; $y(0) = 1$
$y'(0) = 0$

Step-1: Let $y = y_0 + \epsilon y_1 + \epsilon^2 y_2 + \cdots$
Substituting, we have

$O(1)$: $y_0'' + y_0 = 0$ with $y_0(0) = 1$
$y_0'(0) = 0$

$O(\epsilon)$: $y_1'' + y_1 = x y_0$ with $y_1(0) = 0$
$y_1'(0) = 0$

$O(\epsilon^2)$: $y_2'' + y_2 = x y_1$ with $y_2(0) = y_2'(0) = 0$

Solving the equations:-

$O(1)$: $y_0(x) = c_1 \cos x + c_2 \sin x$

Using $y_0(0) = 1$, $y_0'(0) = 0$, we get

$$\boxed{y_0(x) = \cos x}$$

$O(\epsilon)$: $y_1'' + y_1 = x \cos x$

$$y_1(x) = \frac{1}{8}\Big[2x \cos x \cos 2x - \sin x + 2x^2 \sin x + \cos 2x \cdot \sin x - \cos x \cdot \sin 2x + 2x \sin x \cdot \sin 2x\Big]$$

$$y_{pert}(x) = y_0(x) + \epsilon y_1(x) + O(\epsilon^2)$$

Submit on Tuesday:

Assignment:- $$\frac{dV}{dx} + V(1-\epsilon V) = 0$$

$$V(0) = 2$$

Obtain soln. till $O(\epsilon^2)$.

ie, determine V_0, V_1, V_2 in

$$V(x) = V_0(x) + \epsilon V_1(x) + \epsilon^2 V_2(x) + O(\epsilon^3)$$

Using Wolfram Alpha or any other symbolic package to determine exact soln:-

dv/dx + (1- epsilon.v).v = 0 ; v(0) = 2

Plot exact & perturbation solution. for $\epsilon = 0.1$

Submit on Friday:-

Ex: $$y'' + \lambda y + \epsilon y^2 = 0$$

$$y(0) = 0; \quad y(\pi) = 0$$

If $$y = y_0 + \epsilon y_1 + \epsilon^2 y_2 + \cdots$$

$O(1)$: $y_0'' + \lambda y_0 = 0 \implies$ We can find $y_0(x)$

$\underline{\underline{O(\epsilon)}}$: $y_1'' + \lambda y_1 + y_0(x)^2 = 0 \implies$ Linear in $y_1(x)$

$\underline{\underline{\text{Hint}}}$: Treat $\lambda > 0$, $\lambda = 0$, $\lambda < 0$ Separately.

You will have to determine λ as well.