ME7100: Advanced Topics in Mathematical Tools
Assignment-1

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Due date: 30th January 2022, before the class begins. Neatly written solutions (handwritten will suffice) along with printed graphs (if any) to be submitted on my desk before the class begins. Submission at the end of the class will automatically attract a 10% mark penalty.

Problem 1: Solve the following algebraic equations using perturbation techniques you learnt in class for $\varepsilon \ll 1$. Obtain all solutions correct upto 2nd order.

1. $x^2 + x + 6\varepsilon = 0$.
2. $x^3 - \varepsilon x - 1 = 0$.
3. $\varepsilon x + 3y = 10, \quad 4x + 2y = 7$. Obtain x and y simultaneously correct to $O(\varepsilon^2)$.
4. $x^3 - x + \varepsilon = 0$.

Problem 2: First obtain the exact solution of

$$x^2 - \pi x + 2 = 0$$

Use a calculator and write down the exact solution upto six decimal places. Now pretend that $\pi = 3+\varepsilon$. Obtain the perturbation solution of

$$x^2 - (3+\varepsilon)x + 2 = 0$$

up to 3rd order. It is sufficient to obtain the solution of the smaller root. Finally, set the value of $\varepsilon = \pi - 3$ to obtain the approximate solution. How does the obtained approximate solution compare with the exact solution, i.e. what is the exact value of the error?

Problem 3 Solve the following non-algebraic equation using regular perturbation technique for $\varepsilon \ll 1$:

$$x^2 - 1 = \varepsilon e^x.$$

Here is some help. If we set $\varepsilon = 0$, the leading order solutions are $x = -1$ and $x = +1$. Using regular perturbation technique, obtain the approximate solution of the first solution, i.e., when $x_0 = -1$. You will have to substitute the perturbation expansion for x in $e^x = e^{x_0+\varepsilon x_1+\varepsilon x^2+\dots}$ and simplify into a polynomial expression. Obtain the solution upto 2nd order.

Problem 4 Solve the following algebraic equations using perturbation techniques you learnt in class for $\varepsilon \ll 1$. Obtain all solutions correct upto 1st order. Note that some of the below equations may have both regular and singular roots. Obtain all the roots.

1. $\varepsilon x^2 + x - 1 = 0$.
2. $\varepsilon x^3 - x + 1 = 0$.
3. $\varepsilon x^3 + x^2 - 2x + 1 = 0$.
4. $(1-\varepsilon)x^2 - 2x + 1 = 0$. (Hint: You will have a non-integral power for ε. You will get a repeated root for leading term which will lead to difficulties).

Problem 5 Consider potential flow around a slightly distorted sphere whose surface is given by

$$r = R(\theta, \varepsilon) \equiv 1 + \varepsilon P_2(\cos\theta)$$

where P_2 is a Legendre function and $\varepsilon \ll 1$. The flow outside the sphere is given by the potential flow equations:

$$\begin{aligned} \nabla^2\phi = 0 \quad & \text{in} \quad r \geq R(\theta, \varepsilon), \\ \text{subject to} \quad \phi = 1 \quad & \text{on} \quad r = R(\theta, \varepsilon), \\ \text{and} \quad \phi \to 0 \quad & \text{as} \quad r \to 1 : \end{aligned}$$

This problem can be solved with regular perturbation expansion

$$\phi(r;\theta;\varepsilon) = \phi_0(r;\theta) + \varepsilon\phi_1(r;\theta) + \varepsilon^2\phi_2(r;\theta) + ...$$

Simply derive the system of equations at various orders of ε. There is no need to solve the problem.